

中央大学基幹理工学部 幾何トポロジーセミナー

日時： 2026 年 12 月 11 日（金） 17:10–18:10

場所： 中央大学後楽園キャンパス 6 号館 12 階 61125A 室

講師： Boris Botvinnik (University of Oregon)

タイトル： Generalized positive scalar curvature on spin^c manifolds

アブストラクト： This talk is based on joint work with J. Rosenberg and P. Piazza. Let (M, L) be a compact non-spin spin^c manifold. Fix a Riemannian metric g on M and a connection A on L , and let D_L be the associated spin^c Dirac operator. Let $R_g^{\text{tw}} := R_g + 2ic(\Omega)$ be the twisted scalar curvature (which takes values in the endomorphisms of the spinor bundle), where R_g is the scalar curvature of g and $2ic(\Omega)$ comes from the curvature 2-form Ω of the connection A . Then the Lichnerowicz–Schrödinger formula for the square of the Dirac operator takes the form $D_L^2 = \nabla^* \nabla + \frac{1}{4} R_g^{\text{tw}}$. It turns out that a closed non-spin simply connected spin^c manifold of dimension $n \geq 5$ admits a pair (g, A) such that $R_g^{\text{tw}} > 0$ if and only if the index $\alpha^c(M, L) := \text{ind } D_L$ vanishes in KU_n . We introduce a scalar-valued generalized scalar curvature $R_g^{\text{gen}} := R_g - 2|\Omega|_{\text{op}}$, where $|\Omega|_{\text{op}}$ is the pointwise operator norm of Clifford multiplication $c(\Omega)$ acting on spinors. We show that the positivity condition on the operator R_g^{tw} is equivalent to the positivity of the scalar function R_g^{gen} . We prove a corresponding trichotomy theorem concerning the curvature R_g^{gen} and study its implications. We also show that the space of pairs (g, A) with $R_g^{\text{gen}} > 0$ has non-trivial topology, and we address a conjecture concerning the non-triviality of the “index difference” map. Finally, we address the spin^c version of the Gromov–Lawson–Rosenberg conjecture.