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The effect of yardstick competition on public goods supply under vertical political externality

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Abstract

In this study, we examine the effect of yardstick competition on the level of public good provision under shared accountability, which is the co-financing of public goods by upper and lower governments. It is well known that partial expenditure decentralization, where different levels of government share costs, leads to the under-provision of public goods compared with the socially optimal level (Joanis (2014)). This occurs because rent-maximizing politicians have a free-riding incentive (a vertical political externality) to place the cost burden on the other level of government. We investigate whether the introduction of yardstick competition, which allows voters to compare public service levels across jurisdictions, can mitigate this under-provision. Our analysis yields three key findings. First, yardstick competition alleviates the under-provision of public goods. Second, this positive effect is decreased by the distortion caused by the asymmetric vertical political externality—what Joanis (2014) terms "shared accountability." Third, when asymmetric vertical political externality exists and yardstick competition is sufficiently prevalent, the efficacy of yardstick competition regarding achieving the efficient provision of public goods is limited and suppressing vertical political externality is more effective.

JEL classification: H77; H41; D72
Keywords: Shared Accountability; Yardstick Competition; Vertical Political Externality; Public Goods; Electoral Discipline

1. Introduction

The purpose of this paper is to clarify whether the supply of a public good, which is subject to a vertical political externality (shared accountability) that arises from joint production, can be brought closer to a Pareto improvement through the disciplinary effect of yardstick competition on politicians.

Since the seminal works of Tiebout (1956) on "voting with the foot" and Oates (1972) on the "decentralization theorem," the desirability of decentralization has been widely debated from both economic and political perspectives. However, in the real world, decentralization is often incomplete, existing in a state between full centralization and full decentralization. Brueckner (2009) termed this state "partial fiscal decentralization," defining it as follows: "spending authority is devolved to the subnational level while financing relies on transfers from the central government." This situation is common in developing countries where the lower levels of governments cannot finance their fiscal operations with their tax revenues and thus rely on transfers from the upper level of government. Despite this, developed countries can also be considered to be in a state of partial fiscal decentralization because the upper level of government is often deeply involved in the provision of local public goods.

Joanis (2014) focused on a specific form of this "partial expenditure decentralization" (PED), where upper and lower levels of governments co-finance and share the "expenditure authority" (the authority to provide public goods). Joanis (2014) demonstrated that when (1) politicians exist at both the upper level (e.g., prefectural assembly members) and lower level (e.g., municipal assembly members), and (2) these politicians can choose to either allocate fiscal revenues to public goods or appropriate them as political rents, voters face asymmetric information (we call this "limited voter information"). Because voters cannot perceive the politicians' choices regarding the allocation between public good funding and political rents, inefficiencies in public good provision arise from the resulting vertical political externality[1].

The theoretical validity of Joanis's (2014) argument —that vertical political externality causes inefficiencies in public good provision —was empirically verified by Jametti and Joanis (2020) using fiscal expenditure data from Canadian provincial and federal governments. Although it has been shown that vertical political externality can foster inefficient public good provision, the self-interested, rent-seeking behavior of politicians makes it unlikely that they will voluntarily remedy the situation. Therefore, a different mechanism or policy instrument is required to enhance resource allocation efficiency. We investigate whether introducing yardstick competition among governments at the same level can ameliorate the problem of inefficient public good provision by enabling voter oversight.

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[1] Public goods are jointly supplied by upper and lower levels of government. However, voters can only observe the total amount of the public good supplied and cannot perceive the individual contribution (funding) of each level of government. This lack of accountability creates an incentive for politicians at both levels to free-ride on each other's fiscal burden, a phenomenon known as vertical political externality. As a result, the supply of the public good becomes inefficient.

Yardstick competition, first formalized by Besley and Case (1995), has been modeled in various forms. These models capture the fact that voters in each local jurisdiction can observe neither the costs of the public goods they enjoy nor the political rents that politicians can extract during their terms of office. Consequently, to enable voters to decide whether to re-elect an incumbent, they compare their politicians with those in other jurisdictions using observable indicators, such as the quality or quantity of public goods. This comparison forces incumbents into a competitive mechanism, where each incumbent maximizes its rent while considering the actions of others and the influence on the incumbent's probability of re-election. Because incumbents seek to place their jurisdiction in a favorable position relative to others to obtain a new mandate, yardstick competition serves to reduce the inefficiencies that stem from the information asymmetry between politicians and voters[2].

In this paper, we clarify the effect of yardstick competition on the total amount of public goods produced by joint provision, and thus resource allocation efficiency in the presence of vertical political externality. According to Joanis (2014), the "shared accountability" that results from this joint provision creates incentives for free riding among politicians at different levels, which leads to an under-provision of public goods in equilibrium. By contrast, yardstick competition allows voters to discipline politicians at the same level (both upper and lower) through comparisons of the total amount of public goods with other jurisdictions. We theoretically analyze whether the inefficiency in public good provision that arises from joint provision can be improved by the discipline voters impose through yardstick competition.

A key feature of our analysis is the use of a two-period model with *a Tullock-type contest success function* to analyze joint provision. Although the two-period Tullock contest model has often been adopted in symmetric or asymmetric yardstick competition models, to the best of our knowledge, no prior theoretical analysis has been conducted to consider the effect of yardstick competition on public goods production using "joint provision"[3]. Our analysis reveals three main findings: first, yardstick competition alleviates the under-provision of public goods. Second, this positive effect is attenuated by the distortion caused by the asymmetric vertical political externality—what Joanis (2014) terms " shared accountability." Third, when asymmetric vertical political externality exists and yardstick competition is sufficiently prevalent, the efficacy of yardstick competition in achieving the efficient provision of public goods is limited, and suppressing vertical political externality is more effective.

The remainder of this paper is organized as follows: In Section 2, we present a centralized provision

[2] A recent research direction for yardstick competition is the influence of inter-jurisdictional "differences."

[3] Allers (2012) theoretically demonstrates that when local governments differ in their revenue capacity or expenditure needs, politicians in fiscally wealthy local governments can provide high-quality services while keeping local taxes low. Interestingly, these politicians, despite earning high political rents, are more likely to be re-elected. Consequently, such fiscal disparities lead to a yardstick bias, which in turn hinders the correction of information asymmetry. Recent research has been dedicated to investigating which asymmetries generate yardstick bias and how the improvement of efficiency is thereby impeded (Di Liddo and Giuranno (2016), Farah (2019), Di Liddo and Vinella (2021), Di Liddo and Giuranno (2024)).

model in which a public good is supplied by a single-tier government as a benchmark result. In Section 3, we present the theoretical consequences of a decentralized model featuring joint provision. Finally, in Section 4, we summarize the conclusions and discuss future research directions.

2. Social Planner Model

Assume there are two $(j = 1,2)$ regions in a country[4]. In each period, a politician (legislator) in the upper-level government (subscript c) and a politician (legislator) in the lower-level government (subscript l) each provide a public good, $g_{ij} \geq 0$ $(i = c\ or\ l)$, in each region i. Assume that tax revenue T_i is given in each region and the government incurs a cost of τ to produce one unit of the public good. The inputs (or investments) from the upper and lower-level governments produce the public good g_{ij} according to a constant elasticity of substitution (CES) production function[5]. This type of public good production, where inputs are provided by both upper and lower-level governments, is called PED[6]:

$$G_j = \left(\frac{\sqrt{g_{cj}}}{2} + \frac{\sqrt{g_{lj}}}{2}\right)^2, (j = 1,2), \tag{1}$$

where the properties of the function G_j are $\partial G_j/\partial g_{ij} > 0$, $\partial^2 G_j/\partial g_{ij}\partial g_{ij} < 0$, and $\partial^2 G_j/\partial g_{ij}\partial g_{\neq ij} < 0$. Following Joanis (2014), all (representative) individuals are assumed to value the public good according to the following linear utility function:

$$u_j = G_j. \tag{2}$$

Moreover, the social welfare function for the country is defined as the sum of the utilities of each region:

$$SW = u_j + u_{\neq j}. \tag{3}$$

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[4] The qualitative analytical results remain unchanged, even if the two-region model is extended to a multi-region model.

[5] The assumption of joint supply by the upper-level and lower-level governments, which is the source of the vertical political externality, is highly compatible with the reality of public education service provision in Japan. In Japan, although municipalities are responsible for providing compulsory education, teacher salaries are funded by subsidies provided by the national government and prefectures. As a result, for instance, the salaries of teachers additionally hired to reduce class sizes may be paid by both the upper-level government (national or prefectural) and lower-level government (municipalities). However, no clear rules exist for the cost-sharing ratio, which varies by municipality. In reality, voters do not know the breakdown of the financial contributions that each government has made to implement the policy. In this sense, the assumption of our theoretical model – that both levels of government share the responsibility for providing public services and that the allocation of this responsibility is determined endogenously – is considered applicable to the actual state of public education service supply in Japan.

[6] Joanis (2014) uses a general CES production function, $G_j = \left(\theta_c g_{cj}^{\rho} + \theta_l g_{lj}^{\rho}\right)^{1/\rho}$. To simplify the analysis, we specify the parameters as $\theta_c = \theta_l = \rho = 1/2$.

First, we derive the benchmark Pareto-efficient public good supply. Let g_{cj} represent the upper-level government's input (investment) into public good provision and g_{lj} represent the lower-level government's input. We assume that each government obtains government revenue, T_{ij}, where the total revenue is $4T$ because of the existence of four symmetric governments. Thus, the budget constraint for the social planner is given by

$$4T - \tau(g_{cj} + g_{c\neq j}) - \tau(g_{lj} + g_{l\neq j}) = 0. \quad (4)$$

For simplification, assuming $T = \tau,$ the social planner chooses g_{cj} and g_{lj} to maximize Equation (3) subject to the constraint in Equation (4). This yields a socially optimal public good level of

$$G^{FB} = 1. \quad (5)$$

3. Political Agency Model of Shared Accountability

Following Joanis (2014) and Di Liddo and Giuranno (2016), we analyze the public good supply problem using a two-period model, considering yardstick competition in the presence of vertical political externality. We assume that elections for both the upper and lower-level governments are held simultaneously at the end of each period. Each government receives a government revenue of T_{ij} in every period. Both the upper and lower-level governments are assumed to have a balanced budget, where government revenue is allocated to public expenditure and political rent. This implies that $T_{ij} = s_{ij} + \tau g_{ij}$ holds for any period, where s_{ij} represents the political rent for the upper and lower-level governments in each region.

The governments, both at the upper and lower levels, aim to maximize the expected rent per capita over two periods. This is represented by the equation $B_{ij} = s_{ij} + P_i s_{ij}$, where P_i is the politician's expected (perceived) probability of being re-elected.

The political game unfolds as follows:

1. The incumbent politician at each level sets the amount of spending (contribution) on local public goods[7].
2. Voters observe the total level of public goods in their region, G_j, and in other regions, $G_{\neq j}$.
3. Elections for both upper and lower government politicians are held simultaneously[8].

[7] In this paper, we assume that politicians, as representatives of their respective constituencies, are only politically concerned with the provision of public goods in their districts.

[8] In Japan, roughly 42% of all local governments hold elections for prefectural and municipal assembly members on the same day.

4. If re-elected, the politician earns the maximum rent in the second period.

When voting, constituents can only observe the total level of public goods, G_j; the contributions from the upper level of government (g_{cj}) and the lower level of government (g_{lj}) are unobservable. Therefore, voters engage in retrospective voting based on the observed total level of public goods.

Regarding the probability of re-election, we assume that it is determined by a contest success function, as is common in much of the previous research on yardstick competition. The contest success function is useful because it captures the fundamental relationship between voter behavior and the policy choices of rent-seekers. We formalize the contest success function using the following equation, in line with Bodenstein and Ursprung (2005)[9]:

$$P_{ij} = \frac{\omega_{ij} G_j}{\omega_{ij} G_j + G_{\neq j}} \ (i = c, l), \tag{6}$$

where the parameter $\omega_{ij} > 1$ represents the relative difficulty of elections, which measures the intensity of yardstick competition. If ω_{ij} is sufficiently large, a politician's re-election is guaranteed; a situation in which yardstick competition is virtually eliminated[10]. By contrast, if $\omega_{ij} = 1$, the performance of a politician is completely comparable with that of politicians in other regions, which represents a situation in which yardstick competition functions perfectly. The properties of this function are $\partial P_{ij}/\partial g_{ij} = P_{g_{ij}} > 0$ and $\partial P_{ij}/\partial g_{ij} g_{ij} = P_{g_{ij} g_{ij}} < 0$.

3.1. Public Good Supply under Yardstick Competition and Vertical Political Externality

In this section, we derive the public good supply level at the Nash equilibrium when public expenditures for public good provision are made by both the upper-level and lower-level governments. The upper and lower-level politicians, chosen by the incumbent in the first period, choose the public good level to maximize their net expected rent, as represented by the following objective function:

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[9] For an n-region model, the re-election probability function is given by $P_{ij} = \frac{\omega_{ij} G_{ij}}{\omega_{ij} G_{ij} + \frac{1}{n-1} \sum_{\neq i}^{n-1} G_{\neq i \neq j}}$.

[10] It is possible to interpret ω_{ij} not only as a parameter but also as a policy variable that influences the relative difficulty of elections. For instance, if ω_{ij} is viewed as the *district magnitude*, $\omega_{ij} > 1$ can be satisfied under a single-member or multi-member district system. In Japanese local assembly elections, both prefectural and municipal assembly elections adopt a multi-member district system, and both types of local assembly members are elected from the same electoral district, but with different numbers of representatives. It is a common belief that municipal assembly members, who have more representatives per electoral district than their prefectural counterparts, face a relatively lower difficulty in elections (i.e., a relatively upper probability of re-election). Even in the analysis in this paper, in which we treat ω_{ij} as the intensity of yardstick competition, one can interpret ω_{ij} as the *district magnitude* that influences the relative difficulty of elections, and its reduction would lower the re-election probability and imply an intensification of yardstick competition.

$$\max_{g_{ij}} B_{ij} = T_{ij} - \tau g_{ij} + P_i T_{ij} \ \ (i = c, l, j = 1,2), \tag{7}$$

where the subscript i denotes either the upper-level government (c) or lower-level government (l), and j represents region 1 or 2.

The optimization condition is given by

$$\frac{\partial B_{ij}}{\partial g_{ij}} = -\tau + P_{g_{ij}} = \frac{\left(\sqrt{g_{ij}} + \sqrt{g_{\neq ij}}\right)\left(\sqrt{g_{i\neq j}} + \sqrt{g_{\neq i\neq j}}\right)^2 T\omega_{ij}}{\sqrt{g_{ij}}\left(\left(\sqrt{g_{i\neq j}} + \sqrt{g_{\neq i\neq j}}\right)^2 + \left(\sqrt{g_{ij}} + \sqrt{g_{\neq ij}}\right)^2 \omega_{ij}\right)^2} - \tau = 0. \tag{8}$$

Thus, the reaction function can be calculated as

$$\frac{dg_{i\neq j}}{dg_{ij}} = -\frac{(\sqrt{g_{\neq i\neq j}} + \sqrt{g_{i\neq j}})\sqrt{g_{i\neq j}}(-\sqrt{g_{\neq ij}}(\sqrt{g_{\neq i\neq j}} + \sqrt{g_{i\neq j}})^2 - (\sqrt{g_{i\neq j}} + \sqrt{g_{ij}})^2(\sqrt{g_{\neq ij}} + 4\sqrt{g_{ij}})\omega_{ij})}{-2(\sqrt{g_{i\neq j}} + \sqrt{g_{ij}})g_{ij}(\sqrt{g_{\neq i\neq j}} + \sqrt{g_{i\neq j}})^2 + 2(\sqrt{g_{i\neq j}} + \sqrt{g_{ij}})^3 g_{ij}\omega_{ij}} \tag{9}$$

$$\frac{dg_{\neq i\neq j}}{dg_{ij}} = -\frac{g_{\neq ij}\left(\sqrt{g_{\neq i\neq j}} + \sqrt{g_{i\neq j}}\right)^2 + \sqrt{g_{\neq ij}}\left(\sqrt{g_{\neq ij}} + \sqrt{g_{ij}}\right)^2\left(\sqrt{g_{\neq ij}} + 4\sqrt{g_{ij}}\right)\omega_{ij}}{-g_{ij}\left(\sqrt{g_{\neq i\neq j}} + \sqrt{g_{i\neq j}}\right)^2 + 3\left(\sqrt{g_{\neq ij}} + \sqrt{g_{ij}}\right)^2 g_{ij}\omega_{ij}}.$$

The sign of this reaction function is ambiguous. Because the upper-level government's politician and lower-level government's politician are distinguished by $\omega_{cj} > 1$ and $\omega_{lj} > 1$, respectively, their behaviors differ. However, the two regions are identical. Therefore, assuming a symmetric Nash equilibrium exists, because we can differentiate around the equilibrium, we obtain the following reaction functions under $g_{c\neq j} = g_{cj}$ and $g_{l\neq j} = g_{lj}$:

$$\begin{aligned}
\frac{dg_{l\neq j}}{dg_{lj}} &= \frac{4\sqrt{g_{lj}}\omega_{lj} + \sqrt{g_{cj}}\left(1 + \omega_{lj}\right)}{2\sqrt{g_{li}}\left(\omega_{lj} - 1\right)} > 0 \\
\frac{dg_{cj}}{dg_{lj}} &= -\frac{g_{cj}\left(\omega_{lj} + 1\right) + 4\sqrt{g_{cj}}\sqrt{g_{lj}}\omega_{lj}}{g_{li}\left(3\omega_{lj} - 1\right)} < 0 \\
\frac{dg_{c\neq j}}{dg_{cj}} &= \frac{4\sqrt{g_{cj}}\omega_{cj} + \sqrt{g_{lj}}\left(1 + \omega_{cj}\right)}{2\sqrt{g_{ci}}\left(\omega_{cj} - 1\right)} > 0 \\
\frac{dg_{lj}}{dg_{cj}} &= -\frac{g_{li}\left(\omega_{cj} + 1\right) + 4\sqrt{g_{lj}}\sqrt{g_{cj}}\omega_{cj}}{g_{ci}\left(3\omega_{cj} - 1\right)} < 0
\end{aligned} \tag{10}$$

$$\frac{dg_{cj}}{dg_{l\neq j}} = \frac{2g_{cj}(\omega_{lj}-1)}{g_{lj}+4\sqrt{g_{cj}}\sqrt{g_{lj}}\omega_{cj}+g_{lj}\omega_{cj}} > 0$$
$$\frac{dg_{lj}}{dg_{c\neq j}} = \frac{2g_{lj}(\omega_{lj}-1)}{g_{cj}+4\sqrt{g_{cj}}\sqrt{g_{lj}}\omega_{lj}+g_{cj}\omega_{lj}} > 0.$$

Thus, we have found that upper-level and lower-level governments that engage in yardstick competition have a strategic complements relationship, $\frac{dg_{l\neq j}}{dg_{lj}} > 0$ and $\frac{dg_{c\neq j}}{dg_{cj}} > 0$, whereas the vertical relationship between governments that experience a vertical political externality is one of strategic substitutes, $\frac{dg_{cj}}{dg_{lj}} < 0$ and $\frac{dg_{lj}}{dg_{cj}} < 0$.

Furthermore, the effect of the parameter ω_{lj} on the reaction function is

$$\frac{dg_{lj}}{d\omega_{lj}} = -\frac{2(\sqrt{g_{cj}}+\sqrt{g_{lj}})g_{lj}(\omega_{lj}-1)}{\omega_{lj}\left(4\sqrt{g_{lj}}\omega_{lj}+\sqrt{g_{cj}}(1+\omega_{lj})\right)} < 0. \tag{11}$$

This means that an increase of ω_{lj} consequently has the effect of decreasing public good expenditure.

Next, we analyze the expenditure for public good production for each government level and its properties in a symmetric Nash equilibrium. Assuming the existence of a symmetric Nash equilibrium and noting that $g_{c\neq i} = g_{ci}$, $g_{l\neq i} = g_{li}$, and $T_{ij} = \tau$, the equilibrium expenditures for public good production are as follows:

$$g_c^{PED} = \frac{{\omega_c}^2(1+\omega_l)^2}{(1+\omega_c)^2(\omega_c+\omega_l+\omega_c\omega_l(4+\omega_c+\omega_l))} < 1 \tag{12}$$
$$g_l^{PED} = \frac{(1+\omega_c)^2{\omega_l}^2}{(1+\omega_l)^2(\omega_c+\omega_l+\omega_c\omega_l(4+\omega_c+\omega_l))} < 1.$$

From Equation (12), the total public good level is

$$G^{PED} = \frac{1}{4}\left(\frac{\omega_c}{(1+\omega_c)^2}+\frac{\omega_l}{(1+\omega_l)^2}\right). \tag{13}$$

Thus, it is clear that, at the Nash equilibrium, $G^{PED} < G^{FB}$ (thus, G^{PED} is lower than the socially optimal public good level).

3.2. Effect of Yardstick Competition under Symmetric Vertical Political Externality

In this section, we compare the level of the public good to analyze the effects of symmetric vertical political externality, that is, a situation in which yardstick competition is engaged by both upper and lower-level governments.

We analyze the effect of yardstick competition on both upper and lower levels of government. Assuming $\omega_c = \omega_l = \omega > 1$, the public good supply of the upper and lower-level governments are equal ($g_c^{PED} = g_l^{PED} = \frac{\omega}{2(1+\omega)^2}$.) Consequently, the total public good supply is $G^{PED} = \frac{\omega}{2(1+\omega)^2}$. The effect of a weakened yardstick competition, that is, an increase in ω, results in

$$\left.\frac{dg_c^{PED}}{d\omega}\right|_{\omega_c=\omega_l} = \left.\frac{dg_l^{PED}}{d\omega}\right|_{\omega_c=\omega_l} = -\frac{\omega-1}{2(1+\omega)^3} < 0$$

$$\left.\frac{dG^{PED}}{d\omega}\right|_{\omega_c=\omega_l} = -\frac{(\omega-1)}{2(1+\omega)^3} < 0. \tag{14}$$

This implies that promoting yardstick competition (a decrease in ω) increases the public good supply by both the upper and lower-level governments, thereby increasing the public good supply level. We call this the "disciplinary effect."

Proposition 1

When yardstick competition operates at both levels, strengthening it (decreasing ω) raises public good provision at each level, which moves the total provision closer to the socially efficient level.

3.3 Effect of Yardstick Competition under Asymmetric Vertical Political Externality

Next, we examine the effect of yardstick competition for either the upper or lower-level government (i.e., a decrease in either ω_c or ω_l) on the public good expenditure levels of both governments and the total public good supply level at the Nash equilibrium. We analyze the effect of a change in ω_l while holding ω_c constant[11]. For comparison with Equation (14), initially, assuming the degree of yardstick competition is the same between the upper and lower-level governments, we consider the effect of yardstick competition for the lower-level government; that is, starting from an initial state where $\omega = \omega_c = \omega_l$, we examine the effect of d $d\omega_l$. Hence, under $\omega_l \neq \omega_c = \omega > 1$,

[11] The effect of a change in ω_c while holding ω_l constant has the same sign; only the subscripts (c and l) change.

$$\frac{dg_c^{PED}}{d\omega_l} = \frac{\omega^2(\omega^2-1)}{\left(2\omega+2\omega^2(4+2\omega)\right)^2} > 0, \text{and}$$
$$\frac{dg_l^{PED}}{d\omega_l} = -\frac{(\omega-1)\omega\left(\omega+\omega\left(2+\omega(6+3\omega)\right)\right)}{(1+\omega)\left(2\omega+\omega^2(4+2\omega)\right)^2} < 0. \quad (15)$$

This indicates that, although yardstick competition increases the public goods supply by the lower-level government$\left(-\frac{dg_l^{PED}}{d\omega_l} > 0\right)$, the presence of an asymmetric vertical political externality leads the upper-level government to shift the cost burden of the public good onto the lower-level government, thereby reducing its own public good expenditure $\left(-\frac{dg_c^{PED}}{d\omega_l} < 0\right)$.

Proposition 2

Intensifying yardstick competition among governments at one level raises that level's contribution to public good provision. However, because of asymmetric vertical political externality, this disciplinary effect is partially offset: the competing government level strategically reduces its provision, thereby attenuating the overall efficiency gain.

This result highlights the tension between horizontal electoral discipline and vertical fiscal free-riding, which implies that intergovernmental competition does not fully eliminate inefficiencies in joint provision.

To isolate the relative influence of these opposing forces on public good provision, we set $\omega_c = \omega_l$ and examine the marginal effect of yardstick competition, $\frac{dG^{PED}}{d\omega_l}\Big|_{\omega_c=\omega_l}$, at the lower level of government, captured by $d\omega_l$ starting from $\omega_c = \omega_l$. The result is

$$\frac{dG^{PED}}{d\omega_l}\Big|_{\omega_c=\omega_l} = -\left(\frac{\omega_l-1}{4\,(1+\omega_l)^3}\right) < 0, \quad (16)$$

which indicates that promoting yardstick competition among governments at one level increases the public good supply level $\left(-\frac{dG^{PED}}{d\omega_l} > 0\right)$. As illustrated in Figure 1, this implies that as the degree of yardstick competition weakens (i.e., ω_i increases), the public good supply level decreases.

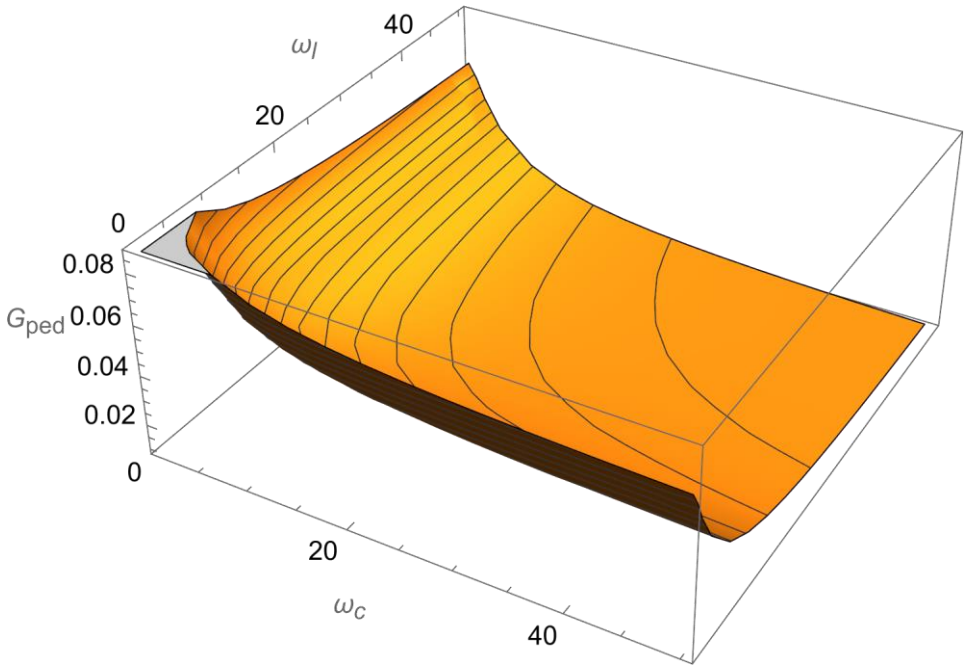

Figure 1. Relationship between public goods supply levels and yardstick competition

Finally, we examine the influence on public good supply in the scenario in which both ω_l and ω_c vary simultaneously. A strengthening (or loosening) of electoral discipline in the lower-level government, driven by a decrease (or increase) in ω_l, simultaneously induces a loosening (or strengthening) of discipline in the upper-level government. This reaction occurs because of the strategic substitutability between the two levels of government regarding public good provision. If an increase (or decrease) in ω_c is then added to this strategic reaction, the loosening (or strengthening) of the upper-level government's discipline is compounded by the strengthening (or loosening) of the lower-level government's discipline. Consequently, compared with the baseline supply level under the symmetric case ($\omega_l = \omega_c = \omega$), the net change in public good provision – whether it increases or decreases – is not uniquely determined; it is expected to depend on the relative levels of ω_l and ω_c. In the following analysis based on Taylor series expansions and simple numerical examples, we compare the public good supply levels under both the symmetric equilibrium ($\omega_l = \omega_c = \omega$) and asymmetric equilibrium ($\omega \neq \omega_c \neq \omega_l$).

First, we perform a Taylor expansion of Equation (13) around the neighborhood of the symmetric equilibrium ($\omega_c = \omega_l = \omega$) to examine the influence of introducing asymmetry (setting $\omega_c = \omega + \Delta$, $\omega_l = \omega - \Delta$) on the level of public good supply. Given that the first-order terms vanish because of symmetry, we obtain[12]

[12] See the Appendix for the details.

$$\Delta G \equiv G(\omega+\Delta,\omega-\Delta)-G(\omega,\omega) \approx \frac{1}{4}\cdot A''(\omega)\cdot\Delta^2 = \frac{(\omega-2)}{2(1+\omega)^4}\Delta^2. \quad (17)$$

This implies that the sign of ΔG depends on sign $(A''(\omega))$, which corresponds to $=$ sign $(\omega-2)$. Specifically, we find that $\Delta G>0$ when $\omega>2$ (the region of weak discipline), whereas $\Delta G<0$ when $\omega<2$ (the region of strong discipline). These results suggest that when ω_i is large (which indicates weak discipline), the yardstick competition effect dominates, which leads to an increase in public good provision. Conversely, when ω_i is small (which indicates strong discipline), the free-riding effect caused by vertical political externality dominates. Based on these theoretical findings, we verify the influence on the public good supply level using numerical examples that assume a transition from two symmetric equilibria ($\omega=10$ and $\omega=2$) to an asymmetric equilibrium where $\omega_l>\omega_c$.

Case 1 starts from an initial symmetric equilibrium of $\omega_c=\omega_l=10$ and shifts to an asymmetric state of $(\omega_c,\omega_l)=(11,9)$. Case 2 starts from an initial symmetric equilibrium of $\omega_c=\omega_l=2$ and shifts to an asymmetric state of $(\omega_c,\omega_l)=(3,1)$.

We observe the changes in g_c^{PED}, g_l^{PED}, and the total supply G^{PED} that results from these transitions. According to the results, in Case 1, G^{PED} under asymmetry exceeds the level under the symmetric benchmark $(\omega_c=\omega_l=10)$. By contrast, in Case 2, G^{PED} falls below the level of the symmetric benchmark $(\omega_c=\omega_l=2)$.

Case 1 (Initial Symmetric Equilibrium $\omega_c=\omega_l=10$)

	Symmetric $(\omega_c=\omega_l=10)$	Asymmetric $(\omega_c=11,\omega_l=9)$	Difference Asymmetric-Symmetric
G^{PED}	0.041322	0.041597	0.000275
g_c^{PED}	0.041322	0.035070	−0.006252
g_l^{PED}	0.041322	0.048681	0.007359

Case 2 (Initial Symmetric Equilibrium $\omega_c=\omega_l=2$)

	Symmetric $(\omega_c=\omega_l=2)$	Asymmetric $(\omega_c=3,\omega_l=1)$	Difference Asymmetric-Symmetric
G^{PED}	0.111111	0.109375	−0.001736
g_c^{PED}	0.111111	0.080357	−0.030754
g_l^{PED}	0.111111	0.142857	0.031746

The numerical examples above offer the following suggestions regarding whether the symmetric or asymmetric equilibrium approaches the efficient provision of public goods: Under conditions where yardstick competition is not functioning sufficiently (i.e., ω is large, as in Case 1), the increase in

public goods that results from the intensified yardstick competition in one government outweighs the decrease caused by the worsening vertical political externality in the other. Consequently, the total quantity of public goods exceeds that of the symmetric equilibrium. Conversely, under conditions where yardstick competition is already prevalent (i.e., ω is small, as in Case 2), the positive effect of yardstick competition is limited. In this scenario, the distortion caused by the vertical political externality dominates, which causes the total supply to fall below that of the symmetric equilibrium.

These results imply that when asymmetric vertical political externality exists between upper and lower governments, and yardstick competition is sufficiently prevalent, the efficacy of yardstick competition in achieving efficient public good provision is limited. Thus, suppressing vertical political externality becomes more effective.

4. Conclusion

The purpose of this paper was to clarify the effect of yardstick competition on public good supply levels when the responsibility for providing (and financing) a public good is shared by upper-level and lower-level governments. Specifically, we analyzed whether the inefficiency in public good supply caused by PED could be mitigated by yardstick competition.

Our analysis yields three main findings. First, yardstick competition always improves allocative efficiency by increasing the level of jointly supplied public goods and moving it closer to the social optimum under symmetric vertical political externality. Second, even under conditions of perfect yardstick competition, this improvement is attenuated by distortions arising from asymmetric vertical political externality—what Joanis (2014) terms "shared accountability." Third, when asymmetric vertical political externality exists between two-tier governments and yardstick competition is sufficiently prevalent, the efficacy of yardstick competition to achieve the efficient provision of public goods is limited and suppressing vertical political externality is more effective.

The degree to which inefficiency caused by shared accountability can be improved is clearly influenced by the intensity of yardstick competition, represented by the parameter ω_{ij}. Our interpretation is that changes in ω_{ij} can be achieved not only by improving the quality and quantity of policy information transmitted to voters but also by making the *district magnitude* of both governments more symmetric, in a manner closer to the lesser side.

Finally, we highlight two remaining challenges for future research. First, although our analysis follows that of Joanis (2014), we assume that the behavior of upper and lower-level governments is symmetric. This assumption is reasonable when comparing a prefecture to a municipality, but it is less so when comparing the national government to a prefecture. In the latter case, the upper-level government (the national government) may either apply public expenditure uniformly or, in pursuit of a "minimum winning coalition," allocate a large amount of public expenditure to specific lower-level governments to secure a majority of seats (Hindriks and Lockwood, 2009; Zudenkova, 2018). Our

model, which assumes multiple upper and lower-level governments, and politicians maximizing votes as representatives of their respective districts, could be extended to a model with a single upper-level government and multiple lower-level governments. In such a model, the lower-level governments could potentially intensify yardstick competition more than an upper-level government, which would alter the effects of electoral discipline.

The second remaining challenge is to analyze the influence of the yardstick bias between upper and lower-level governments. Farah (2016) and Di Liddo and Giuranno (2016) analyzed yardstick competition between asymmetric lower-level governments and concluded that it leads to greater resource allocation distortions (i.e., yardstick bias) compared with symmetric governments. In the two-tiered government structure of our model, the asymmetry of information between politicians and voters regarding public good supply is expected to be greater than in a single-tier model with only lower-level governments. This could increase politicians' incentive to extract political rent. Therefore, analyzing how the theoretical consequences of Farah (2016) and Di Liddo and Giuranno (2016) are modified by the yardstick bias that could result from asymmetric re-election probabilities between upper and lower-level governments is an important topic.

Appendix

In this Appendix, we detail the calculations used to obtain Equation (17). From Equation (13), given that $G^{PED} = \frac{1}{4}\left(\frac{\omega_c}{(1+\omega_c)^2} + \frac{\omega_l}{(1+\omega_l)^2}\right)$, if we define $A(\omega_i) \equiv \frac{\omega_i}{(1+\omega_i)^2}$, we can write $G^{PED} = \frac{1}{4}\big(A(\omega_c) + A(\omega_l)\big)$.

Next, we expand the smooth function A around ω. Because

$A(\omega \pm \Delta) = A(\omega) \pm A'(\omega)\Delta + \frac{1}{2}A''(\omega)\Delta^2 \pm \frac{1}{6}A^{(3)}(\omega)\Delta^3 + \frac{1}{24}A^{(4)}(\omega)\Delta^4 + \cdots,$

taking the sum causes the odd-order terms (those with $\pm$) to vanish, which results in

$A(\omega + \Delta) + A(\omega - \Delta) = 2A(\omega) + A''(\omega)\Delta^2 + \frac{1}{12}A^{(4)}(\omega)\Delta^4 +.$

Substituting this back into G and taking the difference $\Delta G \equiv G(\omega + \Delta, \omega - \Delta) - G(\omega, \omega)$, we obtain

$$\Delta G = \frac{1}{4}[A''(\omega)\Delta^2 + O(\Delta^4)] \approx \frac{1}{4}A''(\omega)\Delta^2.$$

The first-order terms vanish because we consider a symmetric perturbation ($+\Delta$ and $-\Delta$). As a result, the curvature (second derivative) becomes the first determining factor.

Calculations of derivatives:

First derivative:

$$A'(\omega) = (1+\omega)^{-2} + \omega(-2)(1+\omega)^{-3} = \frac{(1+\omega) - 2\omega}{(1+\omega)^3} = \frac{1-\omega}{(1+\omega)^3}.$$

Second derivative:

$$A''(\omega) = \frac{d}{d\omega}[(1-\omega)(1+\omega)^{-3}] = -(1+\omega)^{-3} + (1-\omega)\cdot(-3)(1+\omega)^{-4} = \frac{2(\omega-2)}{(1+\omega)^4}.$$

From the above form, the sign of the curvature is determined by

$$\text{sign}\big(A''(\omega)\big) = \text{sign}(\omega - 2).$$

This indicates the following: If $\omega > 2$ (weak discipline region), $A'' > 0$. Introducing asymmetry (Δ) leads to $\Delta G > 0$ (total supply increases). If $\omega < 2$ (strong discipline region), $A'' < 0$. Introducing asymmetry leads to $\Delta G < 0$ (total supply decreases). At $\omega = 2$, $A'' = 0$. The second-order net effect vanishes and the sign depends on higher-order terms (which are negligible)[13].

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[13] Note that the threshold value of $\omega = 2$ is derived from the inflection point of the function $A(\omega) = \omega/(1+\omega)^2$, which is determined by the specific shape of the contest function, the CES production function parameters, and the simultaneous election settings adopted in this paper.

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