Discussion Paper No.328

Controlling Non-point Source Pollution in Cournot Oligopolies with Hyperbolic Demand

Akio Matsumoto
Chuo University

Ferenc Szidarovszky
Corvinus University

December 2019

INSTITUTE OF ECONOMIC RESEARCH
Chuo University
Tokyo, Japan

Controlling Non-point Source Pollution in Cournot Oligopolies with Hyperbolic Demand*

Akio Matsumoto[†] Ferenc Szidarovszky[‡]

Abstract

In a case of non-point source pollution, the regulator is unable to monitor the individual emission levels of the firms, having access only to the total industry emission level. An emission standard is defined by the regulator and if the industry emission level exceeds this standard, then the firms are uniformly penalized, otherwise uniformly awarded. This environmental regulation policy is added to a hyperbolic n-firm oligopoly. The equilibrium output and emission levels for each firm and for the industry are determined and conditions are given under which the regulation policy can control emission levels.

Keywords: Nonpoint source pollution, Ambient charge, Hyperbolic price function, Duopoly and triopoly, Complex dynamics

*The first author highly acknowledges the financial supports from the Japan Society for the Promotion of Science (Grant-in-Aid for Scientific Research (C) 18K0163) and from Chuo University (Grant for special research)

†Department of Economics, Chuo University, 742-1, Higashi-Nakano, Hachioji, Tokyo, 192-0393, Japan. akiom@tamacc.chuo-u.ac.jp

‡Depaartment of Mathematics, Corvinus University, Budapest, Fövám tér 8, 1093, Hungary. szidarka@gmail.com

1 Introduction

Non point source (NPS) pollution is produced by multiple diffuse sources and in sharp contrast to point source pollution generated from a single source. Today's main pollutions such as water pollution contaminating water bodies that include river, lake, ocean and groundwater, air pollution that may cause harm to humans as well as other living organisms and ocean plastic pollution that recently comes to focus are NPS. Its key feature is, due to multiple sources, non-observability of individual contributions to the total concentration. Consequently, the traditional environmental policy implementation such as emission charge in form of Piguvian tax, marketable emission permit, direct control on emission level are not applicable. Segerson (1988) suggests ambient based policies involving the use of rewards and penalties to control NPS pollution. More precisely, the regulator selects an emission standard and if the industry emission level is higher than this standard, then the firms are uniformly penalized, otherwise, uniformly awarded. This type of ambient charges are included in oligopoly models in which it strictly depends on the actions of other firms whether a firm receives penalty or award.[1]

Much of the literature is studying how to address NPS pollution is game theoretic. This is because it strictly depends on the actions of other agents whether an agent receives penalty or award. Ganguli and Raju (2012) construct a Bertrand duopoly and numerically demonstrate an increase of the ambient charge could increase the total concentration, which they call a "perverse" effect. Ishikawa et al. (2019) extend their duopoly model to an n-firm model and show that the ambient charge is effective in controlling NPS pollution in duopoly and triopoly models whereas the sign of the effect depends on the number of the firms and the degree of substitutability in the model with $n \geq 4$. Raju and Ganguli (2013) examine the ambient charge effect in a Cournot duopoly with product differentiation and numerically show its effectiveness under a two-stage game. Sato (2017) analytically exhibits that a higher ambient charge reduces the total emission in a Cournot market without product differentiation. Matsumoto et al. (2017) construct an n-firm Cournot model and reexamine the result in a dynamic framework when the products are homogenous. Those results are obtained in a special circumstance in which the demand functions are linear. In this study, a linear demand is replaced with a hyperbolic demand and the special results derived in hyperbolic duopolies and triopolies of Matsumoto et al. (2019) is generalized to n-firm hyperbolic models. Conditions are derived under which the ambient charges can control the emission levels of the firms as well as the industry emission.

This paper is organized as follows. In Section 2 the basic model is outlined, and in Section 3 conditions are derived for the possibility of controlling emission

[1] Large number of works have been devoted to the different variants of the classical Cournot (1838) model including oligopolies with and without product differentiation, multi-product models, labor managed oligopolies, market share and contest games among others. Okuguchi (1976) offers a comprehensive summary of the earlier results up to the mid 1970's, their multiproduct generalizations are introduced in Okuguchi and Szidarovszky (1999).

levels of the firms and the industry total emission. Concluding remarks and further research directions are presented in Section 4.

2 Basic Model

Consider an n-firm oligopoly without product differentiation. Output of firm i is x_i and let Q be the total demand. The price function is hyperbolic,

$$p = \frac{1}{Q} \tag{1}$$

where the total demand is supposed to be equal to the industry output, $\sum_{i=1}^{n} x_i$. Let c_i be marginal cost and e_i the emission technology coefficient of firm i, implying that $e_i x_i$ is the emission level. If $\bar{E}$ is the emission standard and θ the penalty or reward fraction specified by the regulator,[2] then the payoff of firm i is

$$\pi_i = \frac{x_i}{\sum_{j=1}^{n} x_j} - c_i x_i - \theta \left(\sum_{j=1}^{n} e_j x_j - \bar{E} \right). \tag{2}$$

Firm i maximizes its payoff with given value of the outputs of the other firms. Solving the first-order condition

$$\frac{\sum_{j=1}^{n} x_j - x_i}{\left(\sum_{j=1}^{n} x_j \right)^2} - c_i - \theta e_i = 0 \tag{3}$$

yields the best reply of firm i as

$$x_i = \sqrt{\frac{y_i}{\bar{c}_i}} - y_i \tag{4}$$

where y_i and $\bar{c}_i$ are the output of the rest of the industry and the extended marginal cost, which are defined as

$$y_i = \sum_{j \neq i}^{n} x_j \text{ and } \bar{c}_i = c_i + \theta e_i.$$

If $Q = \sum_{j=1}^{n} x_j$, then from equation (4)

$$Q^2 = \frac{y_i}{\bar{c}_i} = \frac{Q - x_i}{\bar{c}_i}. \tag{5}$$

Adding equation

$$\bar{c}_i Q^2 = y_i$$

[2] $\bar{E}$ and m are the strategic variables of the regulator and should be determined so as to maximize some form of a welfare function. In the present paper these are assumed to be given exogeneously, however their optimal determination will be considered in our future studies.

for all i with $\bar{C} = \sum_{j=1}^{n} \bar{c}_j$ yields

$$\bar{C}Q^2 = (n-1)\,Q$$

then

$$Q = \frac{n-1}{\bar{C}}. \tag{6}$$

From equation (5)

$$x_i = Q - \bar{c}_i Q^2.$$

Substituting equation (6) presents the Cournot level of firm i's production

$$x_i^c = \frac{(n-1)\left[\bar{C} - (n-1)\bar{c}_i\right]}{\bar{C}^2}. \tag{7}$$

The second factor of the numerator should be positive to have positive production at the Nash equilibrium. It can be written as

$$\begin{aligned}\bar{C} - (n-1)\bar{c}_i \quad &= \left(\sum\nolimits_{j=1} c_j - (n-1)c_i\right) + \theta\left(\sum\nolimits_{j=1} e_j - (n-1)e_i\right) \\ &= n\left\{\left(\bar{c} - c_i + \frac{1}{n}c_i\right) + \theta\left(\bar{e} - e_i + \frac{1}{n}e_i\right)\right\}\end{aligned}$$

where $\bar{c}$ and $\bar{e}$ are the average production cost and the average abatement technology defined as

$$\bar{c} = \frac{\sum_{j=1} c_j}{n} \text{ and } \bar{e} = \frac{\sum_{j=1} e_j}{n}.$$

If firm i has more efficient production cost and abatement technology than the industrial averages, then its optimal production is positive. Further, it can be said that positive production is possible in case of $(\bar{c} > c_i,\ \bar{e} < e_i)$ or $(\bar{c} < c_i,\ \bar{e} > e_i)$ or $(\bar{c} < c_i,\ \bar{e} < e_i)$ if the differences are small enough. We make the two assumptions.

Assumption 1. $\sum_{j=1}^{n} c_j - (n-1)c_i \geq 0$ for $i = 1, 2, ..., n$.

Assumption 2. $\sum_{j=1}^{n} h_j - (n-1)h_i \geq 0$ for $i = 1, 2, ..., n$, where

$$e_i = h_i e_1 \text{ with } h_i \geq 0 \text{ for } i = 2, 3, ...n,\ e_1 > 0 \text{ and } h_1 = 1.$$

Notice that at least one of the values $\sum_{j=1}^{n} c_j - (n-1)c_i$ is positive which can be seen by adding these expressions for all i. Same holds for expressions $\sum_{j=1}^{n} h_j - (n-1)h_i$. Assumption 1 concerns the marginal production costs and makes the marginal production costs not be much different from each other. Needless to say, it is satisfied if $c_i = c \geq 0$ for all i. Assumption ensures that the abatement technologies of the firms are close to each other. The Cournot outputs can be rewritten as

$$x_i^c = \frac{(n-1)\,\theta e_1}{\bar{C}^2}\left(a_i + \sum_{j=1}^{n} h_j - (n-1)h_i\right) \tag{8}$$

with

$$a_i = \frac{\sum_{j=1}^n c_j - (n-1)c_i}{\theta e_1}.$$

The following is clear:

Theorem 1 *Given Assumptions 1 and 2, the optimal production level for each firm is positive.*

3 Ambient Charge Effect

We now consider the effect caused by changing the value of m on the total level of emission. The total level of emission at Cournot point is defined as

$$E^C = \sum_{i=1}^n e_i x_i^c$$

Using equation (8), the right hand side is rewritten as

$$\frac{n-1}{\bar{C}^2}\left\{\left[\sum_{i=1}^n e_i\left(\sum_{j=1}^n c_j - (n-1)c_i\right)\right] + \theta\left[\sum_{i=1}^n e_i \cdot \sum_{j=1}^n e_j - (n-1)\sum_{i=1}^n e_i^2\right]\right\}.$$

The first square bracketed term of the last line can be summarized as

$$\sum_{i=1}^n e_i\left[\sum_{j=1}^n c_j - (n-1)c_i\right] \quad = e_1 S_1$$

where

$$S_1 = \sum_{i=1}^n h_i\left[\sum_{j\neq i}^n c_j - (n-2)c_i\right] > 0. \tag{9}$$

The inequality is due to Assumption 1. The second square bracketed term can be rewritten as

$$\sum_{i=1}^n e_i \cdot \sum_{j=1}^n e_j - (n-1)\sum_{i=1}^n e_i^2 \quad = e_1^2 S_2$$

where

$$S_2 = \sum_{i=1}^n h_i\left(\sum_{j=1}^n h_j - (n-1)h_i\right) \tag{10}$$

which is positive due to Assumption 2. With the new parameters, S_1 and S_2, the total emission level is written as

$$E^C = \frac{(n-1)\,e_1}{\bar{C}^2}\left(S_1 + \theta e_1 S_2\right) > 0.$$

Differentiating the last equation of E^c with respect to θ gives

$$\frac{\partial E^C}{\partial \theta} = \frac{(n-1)\,e_1}{\bar{C}^3}\left[e_1 S_2 \bar{C} - 2\left(S_1 + \theta e_1 S_2\right)\sum_{i=1}^n e_i\right].$$

Since the second term in the square brackets is always positive, we have the following result,

Lemma 1 *If* $e_1 S_2 \bar{C} - 2(S_1 + \theta e_1 S_2)\sum_{i=1}^{n} e_i \leq 0$, *then* $\dfrac{\partial E^C}{\partial \theta} < 0$.

From Lemma 1, the following two results can be derived.

Corollary 1 *If* $\sum_{i=1}^{n} c_i \leq \theta \sum_{i=1}^{n} e_i$, *then* $\dfrac{\partial E^C}{\partial \theta} < 0$.

Proof. Notice that simple calculation shows that

$$\frac{\partial E^C}{\partial \theta} = \frac{(n-1)\,e_1}{\bar{C}^3}\left[e_1 S_2 \sum_{i=1}^{n}(c_i - \theta e_i) - 2S_1 \sum_{i=1}^{n} e_i\right]$$

which implies the assertion. ■

We now summarize the results obtained.

Theorem 2 *Under Assumptions 1,2 and* $\sum_{i=1}^{n} c_i \leq \theta \sum_{i=1}^{n} e_i$, *the ambient charge is effective in controlling the NPS pollution,*

$$\frac{\partial E^C}{\partial \theta} < 0.$$

We now turn attention to the individual level of emission value of firm i, although the regulator is unable to observe this effect. Differentiating equation (7) with respect to θ gives

$$\frac{\partial x_i^c}{\partial \theta} = \frac{n-1}{\bar{C}^3}\left[\left(\sum_{j=1}^{n} e_j - (n-1)e_i\right)\bar{C} - 2\left[\bar{C} - (n-1)\bar{c}_i\right]\sum_{j=1}^{n} e_j\right]. \tag{11}$$

Lemma 2 *If* $\left(\sum_{j=1}^{n} e_j - (n-1)e_i\right)\bar{C} - 2\left[\bar{C} - (n-1)\bar{c}_i\right]\sum_{j=1}^{n} e_j < 0$, *then* $\dfrac{\partial x_i^c}{\partial \theta} < 0$.

The bracketed term in (11) can be rewritten as

$$\sum_{j=1}^{n} e_j\left(-\bar{C} + (n-1)\,\bar{c}_i\right) + (n-1)\bar{c}_i \sum_{j=1}^{n} e_j - (n-1)e_i\bar{C}.$$

The first term is nonpositive, since

$$\begin{aligned} & \bar{C} + (n-1)\,\bar{c}_i \\ = \ & \sum_{j=1}^{n}(c_j + \theta e_j) - (n-1)\,(c_i + \theta e_i) \\ = \ & \left(\sum_{j=1}^{n} c_j - (n-1)c_i\right) + \theta e_j \left(\sum_{j=1}^{n} h_j - (n-1)h_i\right) \geq 0 \end{aligned}$$

by Assumptions 1 and 2. The second term hs the form

$$\begin{aligned}
&(n-1)\left(c_i+\theta e_i\right)\sum_{j=1}^{n} e_j-(n-1)e_i\left(\sum_{j=1}^{n} c_j+\theta\sum_{j=1}^{n} e_j\right)\\
= \quad &(n-1)\left[c_i\sum_{j=1}^{n} e_j-e_i\sum_{j=1}^{n} c_j\right].
\end{aligned}$$

So we have the following result.

Theorem 3 *Under Assumptions 1 and 2, the ambient charge can control the individual level of ouput (or emition) of firm i if*

$$\alpha_i=c_i\sum_{j=1}^{n} e_j-e_i\sum_{j=1}^{n} c_j<0 \tag{12}$$

Notice that

$$\sum_{j=1}^{n}\alpha_j=\sum_{j=1}^{n} c_j\sum_{j=1}^{n} e_j-\sum_{i=1}^{n} c_i\sum_{i=1}^{n} e_i=0$$

showing that (12) cannot hold for all firms. Relation (12) means that the ration of c_i and e_i is below the ratio of their average.

4 Concluding Remarks

This paper examined the effect of ambient charges in n-firm hyperbolic oligopolies assuming non-point source (NPS) pollution. Conditions were first derived which guaranteed non-negative equilibrium output levels for the firms. Then the effect of ambient charges on the emission levels of the individual firms as well as on the total emission level of the industry was examined giving conditions under which regulation is able to control emission. In this model linear production cost functions and simple hyperbolic price function were assumed. In our future research projects more general function types and different kinds of oligopoly models will be investigated.

References

Cournot, A. Researches on the mathematical principles of the theory of wealth, 1838.

Ganguli, S., and Raju, S., Perverse environmental effects of ambient charges in a Bertrand duopoly, *Journal of Environmental Economics and Policy*, 1, 289-296, 2012.

Ishikawa, T., Matsumoto, A., and Szidarovszky, F., Regulation of non-point source pollution under n-firm Bertrand competition, *Environmental Economics and Policy Studies*, doi.org/10.1007/s100-18-019-00243-9, 2019.

Matsumoto, A., Nakayama, K., and Szidarovszky, F., The ambient charge in hyperbolic duopoly and triopoly: static and dynamic analysis, in Nakayama, K., Miyata, Y. (ed) *Theoretical and Empirical Analysis in Environmental Economics*, 3-24, Springer-Verlag, 2019.

Matsumoto, A., Szidarovszky, F., and Yabuta, M., Environmental effects of ambient charge in Cournot oligopoly, *Journal of Environmental Economics and Policy, 7, 41-56,* doi: 10.1080/21606844.2017.1347527, 2017.

Okuguchi, K., *Expectations and stability in oligopoly models*, Berlin, Springer, 1976.

Okuguchi, K., and Szidarovszky, F. *The theory of oligopoly with multi-product firms* (2nd ed.), Berlin, Springer, 1999.

Raju, S., and S. Ganguli, Strategic firm interaction, returns to scale, environmental regulation and ambient charges in a Cournot duopoly, *Technology and Investment*, 4, 113-122, 2013.

Segerson, K., Uncertainty and incentives for non-point pollution control, *Journal of Environmental Economics and Management*, 15, 87-98, 1988.