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Matching with Minimal Quota: Case Study of a Student-Supervisor Assignment in a Japanese University

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Matching with Minimal Quota: Case Study of a Student-Supervisor Assignment in a Japanese University[#]

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Abstract

This study presents a case of student-supervisor matching in a Japanese university. We report on the recent reform in the matching mechanism between students and supervisors in the Japanese university. A mechanism based on the deferred acceptance (DA) was adopted in this reform. In this mechanism, both students and supervisors are classified as one of the types, depending on their affiliations. Then, supervisors set type-specific maximal and minimal quotas. For fulfilling minimal quotas, maximal quotas are dynamically adjusted. It is proved that the mechanism may not satisfy strategy-proofness and feasibility, but it eliminates justified envy among the same "type" of students. Moreover, if the sum of ranks of the student and supervisor in the final assignment is viewed as a measure of welfare, there is no domination relationship between this mechanism and the DA mechanism.

JEL classification: C78, C93, D78, I20

Keywords: school choice, strategy-proofness, distributional constraint, market design

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1. Introduction

In this study, we report on the recent reform in the matching mechanism between students and supervisors in a Japanese university.

In the Japanese university, for writing a graduation thesis, third-year undergraduate students designate their supervisors from among the university faculty members according to their preferences at the near end of the school year. Since each supervisor has limited capacity, a matching mechanism must be designed.

The most popular matching mechanism used so far in many Japanese universities was the *Boston* mechanism. In this mechanism, all the students apply to their first-choice supervisors, and each supervisor accepts applicants based on his/her own priority ordering of students. Once a supervisor's quota is filled, the remaining applicants are rejected. The accepted students' assignment is *final* at this point. Students rejected in this step apply to their next choice supervisors, and the iterating continues till all the students are assigned.

The other popular matching mechanism is *Priority Matching*. In this mechanism, students submit their preferences and priority orderings of supervisors, the sum of a student's ranking for a supervisor and the supervisor's ranking for the student is calculated, then the student-supervisor pairs are formed in ascending order of the sum (with a tie-breaking rule).

It is well known that neither of these mechanisms is *strategy-proof,* that is, manipulating students' preferences may be beneficial to them. To overcome this problem, a simple alternative may be the *Serial Dictatorship* (SD) mechanism. In this mechanism, given a specified order (based on factors like a GPA score), students apply to their first-choice supervisors, and each supervisor accepts them up to his/her quota. Essentially, SD is strategy-proof. Unfortunately, a matching outcome produced by SD as well as the other two mechanisms mentioned above are not *stable*, that is, a student-supervisor pair may have *justified envy* for another student-supervisor combination in the resulting matching outcome.

Accordingly, adopting the *Deferred Acceptance* (DA) mechanism (Gale and Shapley, 1962) is a natural consequence to overcome these problems. In fact, DA is strategy-proof, and a matching outcome produced by DA is always stable. Even though increasing number of DA applications in resident matching and school choice are reported, adoption of DA in a Japanese university is still uncommon.

In 2015, the education committee of Future University Hakodate, Japan, decided to use DA in the assignment process of undergraduate students, who write graduation thesis, to supervisors. One of the authors (Kawagoe) of the present study was a member of the committee. Based on his explanation of the working and desirable properties of DA, the committee soon understood and accepted DA, but the members asked several substantial qualifications to DA. The most challenging one is about imposing *minimal quota* in the assignment process, that is, every supervisor must be assigned a positive number of students in the resulting matching outcome. This is problematic because of the "rural hospitals theorem" in the matching theory. In the context of the present study, this theorem simply states that a supervisor who is not assigned any student in a *stable* matching is never assigned any student in any other *stable* matchings. Thus, for satisfying minimal quota, one must give up the stability requirement.

However, a non-negligible number of faculty members believed that imposing minimal quota was necessary. The reasons cited included that if a supervisor is not assigned a certain number of students, it may not be feasible for the supervisor to manage the on-going research projects. The other reason was a request for equal share of educational burden among the supervisors, that is, it was deemed that allowing a supervisor who was not assigned any student was unfair.

Due to such arguments, the committee decided to impose minimal quota. Note that imposing minimal quota generally leads to instable matching because a student's assignment in stable matching, for example, must be modified for fulfilling minimal quota.

In the literature, several studies propose a mechanism to implement minimal quota by discarding one of the components in the definition of stability, that is, no justified envy and non-wastefulness (Fragiadakis et al. 2015; Fragiadakis and Troyan, 2017; Tomoeda, 2018). By examining these mechanisms, the committee decided to choose the mechanism proposed by Fragiadakis and Troyan (2017) as the basis for the matching mechanism.

Another concern the committee discussed was about handling students' preferences for a supervisor whose affiliation is different from that of students. In Future University Hakodate, students are segregated into four major courses in the second year.[1] Thereafter, students choose their supervisors from among their course faculty members. However, a few students wanted to choose supervisors whose affiliation is different from their own, and such requests are respected.

Before 2015, as there were only a few students, they were assigned supervisors before other students' assignment process began. However, this was problematic. If

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[1] In Future University Hakodate, there are two departments with two sub-departments each. The sub-department is called "course" in the university.

students feel safe to be assigned to a supervisor whose affiliation is different from their own (as they need not compete with other students over supervisors whose affiliation is the same), they can misrepresent preferences as if the supervisor is their first-choice. Or, if a sufficient number of students with different affiliations have been accepted, a supervisor is reluctant to accept more students who share the same affiliation. In this case, students who have the same affiliation may experience justified envy against students whose affiliation is different. Thus, the previous treatment for students whose affiliation is different may violate strategy-proofness and stability.

This problem is very similar with the matching problem under *affirmative action* policy. There are several research papers concerned about the matching mechanism with affirmative action policy (e.g., Abdulkadiroglu, 2003, 2005; Abdulkadiroglu and Sönmez, 2003; Kojima, 2012; Matsubae, 2011; Hafalir et al., 2013; Kawagoe et al., 2017). The committee decided to choose the DA-based mechanism proposed by Kojima (2012) and Mastubae (2011) for handling students in different courses. Before the assignment process begins, each supervisor must declare quotas for both students whose affiliation is the same and for students with a different affiliation. It is assumed that all students whose affiliation is the same is acceptable to supervisors, but students whose affiliation is different may not be. In the context of affirmative action, the former students are considered as majority and the latter students as minority.

Thus, the mechanism proposed by the committee was a mixture of minimal quota and affirmative action policy. In this mechanism, both students and supervisors are classified as one of the types, depending on their affiliations. Then, supervisors set maximum and minimum type-specific quotas. For fulfilling minimal quotas, maximal quotas were dynamically adjusted. Unfortunately, it has been proved that the mechanism did not satisfy strategy-proofness and feasibility but eliminated justified envy among students with the same type.

The mechanism was implemented in 2016, and totally 254 students and 67 supervisors participated. All the students were matched, and minimal quotas were fulfilled for every supervisor, even though, in theory, the mechanism did not satisfy feasibility. The mechanism was not strategy-proof, but most students seemed to submit their true preferences. About 90% of the students were assigned to their fifth or better supervisors. For supervisors, about 70% of the students who were matched with them were their fifth choice or better. As for the sum of ranks of the student and supervisor in student-supervisor pairs in the resulting matching, about 80% were smaller than or equal to ten. Thus, the matching outcome was quite satisfactory.

The remainder of the study is structured as follows. Section 2 provides a brief overview of the institution of student-supervisor matching adopted in a Japanese university, based on DA with type-specific quotas and affirmative action. Section 3 presents the formal model of student-supervisor matching with affirmative action via type-specific quotas. Section 4 presents the results for the 2016 student-supervisor matching. Section 5 concludes. All the propositions and proofs are included in Appendix A. The raw data in our student-supervisor assignment problem are available in an online Appendix.

2. Overview of the institution of student-supervisor matching

Future University Hakodate is in Hokkaido, Japan. It is essentially a computer science department, which is divided into four major courses; Complex Systems, Intelligent Systems, Information Systems, and Information Design courses.

Undergraduate students are assigned to one of these courses during the second year. Later, before the fourth year of education begins, the process of matching students with supervisors is conducted so that students prepare their graduation thesis.

Supervisors also belong to one of the courses. Students are advised to choose supervisors who belong to the same course, but they can also choose supervisors from other courses. Students can also choose supervisors who belong to the Communication Media Laboratory (CML).

The Maximal quota for each supervisor in each course is determined by dividing the total number of students in the course by the total number of supervisors in the same course, so that students are not unmatched. Basically, the maximal quota determined through this method is four or five students. If supervisors would like, they can accept two additional students in the same course. As for students in different courses, supervisors must declare the maximal quota to them. However, if they do not want to accept any of them, it can be set equal to zero.

The Minimal quota is determined by a uniform consent among supervisors in the same course, and each supervisor in the same course has basically the same amount of minimal quota. Minimal quota used was one or two in the majority of courses, but one course chose three. As supervisors in the CML have no obligation to accept any student, the minimal quota for them is set equal to zero.

Before starting the matching process, the chief of the educational committee in the university explains about the matching process and the basic properties of the DA mechanism for students, including the fact that truth-telling is a dominant strategy.

Later, for two weeks, students interview with supervisors whom they would like to apply. After that, students submit their **preferences** for supervisors through paper forms. They also rank every supervisor in the same course, for avoiding no match. Of course, they can also rank supervisors in different courses, but the number is restricted to two.

Then, each supervisor is informed via an electronic file delivery system about students' preferences and additional information such as GPA, number of compulsory courses, and the total number of courses that students have already secured credits. The supervisors can only see the preferences of the students who rank themselves. Based on such information, supervisors submit their **priority orderings** to students via the electronic file delivery system. Supervisors must rank all the students who belong to the same course, but they can eliminate students who are in different courses, if they do not want to accept them.

Subsequently, based on students' preferences, supervisors' priority orderings, and type-specific maximal and minimal quotas, a matching outcome is determined. The algorithm used is based on the **Deferred Acceptance (DA)** mechanism with modifications. We call it the **DAMin** mechanism, which is explained in the next section.

3. Model

We present a formal model of a student-supervisor matching with type-specific maximum and minimum quotas described in Section 2. As the algorithm used is a DA-based mechanism, we first describe DA with no distributional constraints for simplicity.

3.1 Student-supervisor problem in general

The basic setups for a student-supervisor problem are as follows:

(1) A non-empty finite set of students $S = \{s_1, s_2, \dots, s_n\}$;

(2) A non-empty finite set of supervisors $T = \{t_1, t_2, \dots, t_m\}$;

(3) Students' preference profile is $P = \{P_{s_1}, P_{s_2}, \dots, P_{s_n}\}$, where P_{s_i} is a preference relation over T for the student s_i. We assume in this study that preferences are strict for all the students. $t_k P_{s_i} t_l$ means that student s_i prefers t_k to t_l. Let R_{s_i} denote the weak preference relationship induced by P_{s_i}, that is, $t_k P_{s_i} t_l$ or $t_k = t_l$ if and only if $t_k R_{s_i} t_l$. We also assume that every supervisor is acceptable to every student and there is no constraint on the size of the submitted preference.[2]

[2] Calsamiglia et al. (2010) conducted an experiment to compare school choice problems with and without the constraint on the size of the submitted preference list and found

(4) Supervisors' priority profile is $\succsim = (\succsim_{t_1}, \succsim_{t_2}, \dots, \succsim_{t_m})$, where $\succsim_{t_k}$ is the priority ordering over S for supervisor t_k.[3] The priority ordering is also assumed to be strict for all supervisors. Denote $\succ_{c_k}$ as supervisor t_k's strict priority ordering; thus, $s_i \succ_{t_k} s_j$ if and only if $s_i \succsim_{t_k} s_j$ but not $s_j \succsim_{t_k} s_i$.

(5) Each supervisor $t_k \in T$ has a non-negative integer q_{t_k}, which is the capacity of the supervisor t_k (i.e., total number of students she can accept). We call it **total capacity**. Let $q = \{q_{t_k}\}_{k=1}^{m}$ be the vector of total capacity.

Then, matching μ involves mapping from the set $S \cup T$ to the set of all subsets of $S \cup T$ such that

(M1) $|\mu(s_i)| = 1$ for each student s_i, and $\mu(s_i) = s_i$ if $s_i \notin \mu(t_k)$ for any supervisor c_k;
(M2) For each $s_i \in S$ and $c_k \in C$, $\mu(s_i) = t_k$ if and only if $s_i \in \mu(t_k)$; and
(M3) $|\mu(t_k)| \leq q_{t_k}$ and $\mu(t_k) \subseteq S$ for each supervisor t_k.

(M1) and (M2) mean that all the students are matched with at most one supervisor or with themselves. (M3) implies that all the supervisors are matched with up to the number of students allowed by their total capacity.

Mechanism φ is a mapping that produces a matching for any preference profile. For determining a matching μ, DA mechanism (Gale and Shapley, 1962) is used. The mechanism runs as follows.

Assignment process of DA mechanism:

Step 1: Every student applies to her first-choice supervisor. For each supervisor t_k, q_{t_k}applicants who have highest priority for t_k are *tentatively* accepted by t_k, and the others are rejected.

Step $k \geq 2$: Applicants who were rejected at step $k-1$ apply to their next-choice supervisors. For each school t_k, q_{t_k} students who have the highest-priority for t_k among the new applicants and those accepted by step $k-1$ are *tentatively* accepted, and the rest are rejected.

that the proportion of (truncated) truth-telling is significantly higher in the unconstrained than in the constrained case. As a result, efficiency is significantly reduced, and stability is low in the constrained compared to the unconstrained case.

[3] This means that every student is acceptable to every supervisor.

Terminal condition: If either every student is accepted or no more supervisors remain in the submitted preferences for unmatched students, the process terminates.

The mechanism stops at a finite number of steps, and the resulting matching μ is unique. Moreover, the matching μ is *stable* and *student optimal*.

Stability: A matching μ is **stable** if the following apply:

(S1) $\mu(s_i)P_{s_i}s_i$ for each student $s_i \in$ S, and

(S2) if $t_k P_{s_i}\mu(s_i)$, then $|\mu(t_k)| = q_{t_k}$ and $s_j \succ_{t_k} s_i$ for any student $s_j \in \mu(t_k)$.

(S1) means the condition of **individual rationality** that all the students prefer matching with a supervisor than matching with themselves. (S2) means that no pair (t_k, s_i) can be a **blocking pair** for matching μ; thus, if student s_i prefers supervisor t_k to the outcome obtained under matching μ, then the total capacity of supervisor t_k is already full, and supervisor t_k does not give student s_i higher priority than any other student s_j she accepts under μ. If a matching outcome is not stable, one student feels **justified envy** against another student.

Justified envy: For a matching μ, s_i feels *justified envy* against another student s_j if $\mu(s_j)\ P_{s_i}\mu(s_i)$, $s_i \succ_{\mu(s_j)} s_j$, and there exists another matching ν such that

$$\nu(s_i) = \mu(s_j), \nu(s_j) \neq \mu(s_j)\ and\ \nu(s_l) = \mu(s_l)\ for\ all\ s_{l\neq i,j}.$$

The definition of student optimality is as follows.

Student-optimal stable matching: A stable matching that every student weakly prefers to any stable matching.

Mechanism φ is a *student-optimal stable mechanism* (SOSM) if it produces student-optimal stable matching for any preference profile.

Theorem 1 (Theorems 1, 2 in Gale and Shapley 1962): Given $(P, \gtrsim)$, the DA mechanism is SOSM.

Further, the DA mechanism is *strategy-proof*: stating a true preference is the dominant strategy for students.

Strategy-proofness: Let $\varphi(P)$ be a matching induced by a matching mechanism φ under a true preference profile P, and $\varphi_{s_i}(P)$ be student s_i's matching outcome in $\varphi(P)$. For any student s_i's preference P'_{s_i} and any profile of other students' preferences other than s_i, $\hat{P}_{-s_i}$, if

$$\varphi_{s_i}(P_{s_i}, \hat{P}_{-s_i}) R_{s_i} \varphi_{s_i}(P'_{s_i}, \hat{P}_{-s_i})$$

holds, φ is strategy-proof.

Theorem 2 (Theorem 9 in Dubins and Freedman, 1981; Theorem 5 in Roth, 1982b): the DA mechanism is strategy-proof.

However, the resulting matching with DA is not always *Pareto-efficient* (Abdulkadiroglu, 2003).[4]

Pareto-efficiency: A matching is *Pareto-efficient* if it is not Pareto-dominated by any other matching.

A matching μ *Pareto-dominates* another matching ν if $\mu(s_i) R_{s_i} \nu(s_i)$ for every student $s_i \in S$ and $\mu(s_j) P_{s_j} \nu(s_j)$ for at least one $s_j \in S$. Thus, a matching μ Pareto-dominates another matching ν if every student prefers the supervisor assigned under μ to the supervisor assigned under ν, and at least one student strictly prefers the outcome obtained under μ.

3.2 Student-supervisor problem with type-specific maximal and minimal quotas. We then consider a student-supervisor problem with type-specific maximal and minimal quotas. This is also called the controlled school choice problem in the literature and studied by Ehlers et al. (2014) and Fragiadakis and Troyan (2017).

To consider this problem, we add the following settings to the general setting described in the previous subsection.

[4] Note that the outcome of the DA mechanism is not necessarily efficient in the context of school choice. Ergin (2002) shows that the outcome of the DA mechanism is Pareto efficient if and only if the school priorities satisfy a certain acyclicity condition. Ehlers and Erdil (2010) generalize the result in the case where school priorities are coarse. This can be interpreted as a negative result for the efficiency of the DA mechanism, since school priorities are not likely to satisfy the acyclicity conditions of Ergin (2002) and Ehlers and Erdil (2010) in applications.

$\Theta = \{\theta_1, \dots, \theta_r\}$ is the finite set for types of students, and each student belongs to at most one of the following types. We interpret "type" as a course that a student belongs to. The function $\tau: S \to \Theta$ assigns one of the types for each student and denotes that S_θ is the set of students of type $\theta \in \Theta$, and T_θ is the set of supervisors of type $\theta \in \Theta$. We assume that types are publicly observable (i.e., types cannot be misreported).

For all types $\theta \in \Theta$ and supervisor $t \in T_\theta$, in addition to total quota q_t, each supervisor has a type-specific maximal quota $U_{t,\theta}$ and a type-specific minimal quota $L_{t,\theta}$. Let $U = (U_{t,\theta})_{t \in T, \theta \in \Theta}$ be the vector of maximal quota and $L = \left(L_{t,\theta}\right)_{t \in T, \theta \in \Theta}$ be the vector of minimal quota. Moreover, denote $U_{t,-\theta}$ as the maximum number of students other than type θ assigned to supervisor $t \in T_\theta$, and $L_{t,-\theta}$ as the minimum number of students other than type θ assigned to supervisor $t \in T_\theta$. We assume $0 \leq L_{t,\theta} \leq U_{t,\theta} \leq q_t$ for all $(t\ (\in T_\theta), \theta)$. We also assume that $U_{t,-\theta} = q_t$ and $L_{t,-\theta} = 0$. Thus, we consider a school choice problem where minimal quota constraint is binding when student and supervisor types are the same.

Denote $\mathcal{M}$ as the set of matchings. For any $\mu \in \mathcal{M}$, let $\mu_\theta(t)$ be the set of students of type θ assigned to a supervisor $t \in T_\theta$ under matching μ.

Feasibility: A matching μ is feasible if $L_{t,\theta} \leq |\mu_\theta(t)| \leq U_{t,\theta}$ for all $(t\ (\in T_\theta), \theta)$ and $|\mu(t)| \leq q_t$. In other words, feasible matching satisfies type-specific minimal and maximal quotas for any type as well as the total capacity for any supervisor.

Denote $\mathcal{M}_f \subset \mathcal{M}$ as the set of feasible matchings. We assume in the study that $\mathcal{M}_f \neq \emptyset$; this is the (obviously necessary) requirement that the distributional constraints such as maximal and minimal quotas are consistent with the number of students of each type actually present in the market.

The definition of justified envy defined in subsection 3.1 should be modified accordingly with the introduction of maximal and minimal quotas.

Justified envy: For a matching μ, a student $s_i \in \mu(t_k)$ *justifiably envies student* $s_j \in \mu(t_l)$ if (i) $t_l P_{s_i} t_k$, (ii) $s_i \succ_{t_l} s_j$, and (iii) there exists an alternative matching $\nu \in \mathcal{M}_f$ such that $\nu(s_i) = t_l, \nu(s_j) \neq t_l$, and $\nu(s_h) = \mu(s_h)$ for all $h \neq i, j$.

In other words, student s_i justifiably envies another student s_j if (i) she prefers supervisor t_l to whom student s_j is assigned to t_k, (ii) has a higher rank than s_j on the priority ordering of supervisor t_l, (iii) and s_i and s_j can be reassigned without violating any distributional constraints (and without altering the allocation of any other student). If no student justifiably envies any other student, then the matching **eliminates justified envy**.

Then, denote $DA^{(U')}(\cdot)$ as the DA mechanism with maximal quota vector U'. A DA mechanism with minimal quota (henceforth, **DAMin**) is defined as follows. Note that the maximal quota for each supervisor would be reduced at each period if the matching outcomes are not feasible.

We also use the following notations in describing DAMin: the mapping $Rank(\cdot)$ maps a supervisor t assigned to a student to the supervisor's ranking number $\{1, \dots, n\}$ for the student on her priority, $\bar{T} \coloneqq \{t' \in T : |\mu_\theta(t')| > L_{t',\theta}, for\ any\ \theta\}$, and for any $t' \in \bar{T}, \overline{Rank}(t') \coloneqq \max_{s \in \mu(t')} Rank(t')$.

Assignment process of DAMin mechanism:

Step 1: Starting with maximal quotas vector $U^1 = \left(U^1_{t_1}, U^1_{t_2}, \cdots, U^1_{t_m}\right)$, determine a matching outcome with $DA^{(U^1)}(P)$ (as defined in subsection 3.1) with U^1. If the matching outcome μ is feasible, the algorithm is terminated, and the matching outcome is finalized. If not, proceed to next Step.

Step $k \geq 2$: If a feasible matching is not obtained in Step $k-1, \exists\ t \in T_\theta\ s.t. |\mu_\theta(t)| < L_{t,\theta}$, the rankings of student rated lowest are compared among supervisors whose minimal quotas are fulfilled. Thus, we choose supervisor t'such that $\overline{Rank}(t')$ is maximum in $t' \in \bar{T}$, and the maximal quota for supervisor t' is adjusted to $U^{k-1}_{t'} - 1$. Note that if there are two or more supervisors satisfying the above condition, one of them would be randomly chosen. With this updated maximal quota vector U^k, run $DA^{(U^k)}(P)$ again. If the resulting matching μ is feasible, terminate the algorithm and the matching outcome is finalized. If not, proceed to Step $k+1$.

Terminal condition: If (i) either every student is accepted, or (ii) no more acceptable supervisors remain in the submitted preferences for unmatched students, or (iii) if for every supervisor, the number of students of the same type who is assigned is more than his/her minimal quota; $L_{t,\theta} \leq |\mu_\theta(t)|$, or (iv) $U_{t,\theta} = L_{t,\theta}$ for all $(t(\in T_\theta), \theta)$ (by updating maximal quota vector, the maximal quota for any supervisor will be less than his/her minimal quota), the process terminates. The mechanism stops at a finite number of steps, and the resulting matching μ is unique.

The following example shows the working of DAMin.

Example 1.

There are five students $S = \{s_1, s_2, s_3, s_4, s_5\}$ and there are two types $\Theta = \{\theta_1, \theta_2\}$. So, the students are divided as $S_{\theta_1} = \{s_2, s_3\}$ and $S_{\theta_2} = \{s_1, s_4, s_5\}$. There are three supervisors $T = \{t_1, t_2, t_3\}$, and they are divided as $T_{\theta_1} = \{t_1\}$ and $T_{\theta_2} = \{t_2, t_3\}$. Total quotas, and type-specific minimal and maximal quotas are as follows.

$$\begin{aligned}(q, L, U) \quad = \quad & \{(q_{t_1}, L_{t_1,\theta_1}, U^1_{t_1,\theta_1}) = (3, 1, 2),\\ & (q_{t_2}, L_{t_2,\theta_1}, U^1_{t_2,\theta_1}) = (1, 1, 1),\\ & (q_{t_3}, L_{t_3,\theta_2}, U^1_{t_3,\theta_2})\} = (2, 1, 2)\}.\end{aligned}$$

True preference profiles for each student are as follows.[5]

$$\begin{aligned}&P_{s_1}: t_3 t_1 t_2,\\ &P_{s_2}: t_1 t_3 t_2,\\ &P_{s_3}: t_1 t_2 t_3,\\ &P_{s_4}: t_1 t_2 t_3,\\ &P_{s_5}: t_3 t_1 t_2.\end{aligned}$$

Priority orderings for each supervisor are as follows.[6]

$$\begin{aligned}&\succ_{t_1}: s_2 s_4 s_3 s_1 s_5 ,\\ &\succ_{t_2}: s_1 s_2 s_3 s_4 s_5,\\ &\succ_{t_3}: s_5 s_2 s_3 s_4 s_1.\end{aligned}$$

Assume that every student submits her true preference. Then, all the students are assigned to their first choice, but this matching is not feasible. In fact, supervisor t_2 has not satisfied her minimal quota. Then, since $5 = \overline{Rank}(t_3) > \overline{Rank}(t_1) = 3$, the maximal quota for supervisor t_3 is reduced to $U^2_{t_3}$=1 and the DA runs again with this updated quota vector. As a result, student s_1 is rejected by his first-choice supervisor t_3. Then, he applies to his second-choice supervisor t_1 and is again rejected; finally, he is accepted by his third-choice supervisor t_2. As every student is assigned, DAMin ends and the resulting matching μ is as follows.

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[5] The notational convention is that supervisors are listed in the order of students' preferences and supervisors who are not on the preference list are unacceptable: for instance, for student s_1, supervisor t_3 is preferred to supervisor t_1, and supervisor t_1 is preferred to supervisor t_2. Henceforth, the same notation is used in the study.

[6] The notational convention here is that students are listed in the order of supervisors' priorities. Henceforth, the same notation is used in the study.

$$\mu = \begin{pmatrix} t_1 & t_2 & t_3 \\ s_2, s_3, s_4 & s_1 & s_5 \end{pmatrix}.$$

Remark 1:

DAMin starts with the standard DA, and then checks if, given the submitted preferences, the resulting matching fulfills the feasibility constraint, especially the minimal quota. If feasibility is satisfied, the algorithm produces the same matching outcome as the standard DA. If not, one of the supervisors' maximal quotas is reduced by one. This does not necessarily reduce the number of assigned students to the supervisor (i.e., when the number of assigned students is strictly less than the maximal quota, reduction of maximal quota does not change anything.). However, repeating this reduction process finally removes a student from the supervisor's quota. If the rejected student then applies to his next-choice supervisor and the supervisor accepts him, she may reject her lowest priority student who was assigned in the previous step. In this way, the rejection chain starts and continues until no student is rejected, according to the DA algorithm. Thus, by lowering the maximal quota in sequence, we gradually fulfill the minimal quota for every supervisor.

Remark 2:

Fragiadakis and Troyan (2017) proposed that the method for reducing maximal quota *exogenously* determined one (for example, a randomly chosen sequence of the supervisors). One of the authors (Kawagoe) of the present study, strongly recommended it to the university's education committee, but the chief of the committee finally decided to choose the method for the *endogenous* one, that is, among the supervisors whose number of assigned students are strictly greater than the minimal quota in the previous step, the supervisor who is assigned *the lowest ranked student* with respect to his/her submitted priority order is chosen. By changing the reduction sequence from exogenous to endogenous, some good properties of DAMin, for example, strategy-proofness, is lost.

The reason underlying the chief's decision is the fact that with an *exogenously* chosen sequence, a student was not removed from a supervisor whose worst student's rank was lower (in fact, 81st) but from another supervisor whose worst student's rank was relatively higher (in fact, 6th). With an *endogenously* chosen sequence, a supervisor was in a better position by eliminating the lowest ranked student because the student was unacceptable to her and the student could have a chance to be assigned to his next-choice supervisor, changing the reduction sequence from exogenous to endogenous may not be so detrimental. In fact, the assumption that all students are acceptable may not hold. In the Japanese university, it was enforced that supervisors must accept any student if seats are available. The sum of ranks of the student and supervisor in the final assignment with the *endogenously* chosen sequence was slightly higher than the with the *exogenously* chosen sequence, as well as with the original DA.

Remark 3: DAMin mechanism is SOSM.

Eliminates justified envy among students with same type
Thus, if DAMin satisfies the desired properties described in subsection 3.1, it guarantees that the final resulting matching eliminates justified envy among students with the same type.

Proposition 1. *DAMin eliminates justified envy among students with same type.*

Feasibility
DAMin may produce infeasible matching outcomes because with DAMin any student can apply to supervisor t who is different type θ and whose maximal quota $U_{t,-\theta}$ is not reduced.

Proposition 2. *For any preference profile*, *DAMin may not be a feasible matching.*

Remark 4:
The problem of infeasibility may occur not only under the DA-based mechanism such as DAMin but also under any other kind of mechanism, if type-specific minimal quotas are imposed, as some supervisors of type θ cannot fulfill their minimal quotas when sufficient number of students are assigned to supervisors whose types are other than θ.

Strategy-proofness
If the method of reducing maximal quotas is *exogenously* determined, as Fragiadakis and Troyan (2017) show, DAMin is strategy-proof. However, if it is *endogenously* determined, DAMin may not be strategy-proof, as explained in Remark 2.

For example, consider a situation where student *s* applied to a popular supervisor t as his first choice. Even though maximal quota of supervisor t is still vacant; if there is another supervisor t' whose minimal quota is not filled, the maximal quota of supervisor t may be reduced because she is the supervisor who is assigned the worst student s among supervisors who fulfill minimal quotas. As a result, if student s may be assigned to a supervisor worse than t. Anticipating this, student s may hesitate to state her true preference.

Proposition 3. *For any P, DAMin is not strategy-proof.*

Efficiency

First, we state that there is no Pareto dominance relationship between DAMin and the original DA. As DAMin includes DA as a special case (e.g., when the final matching outcome is determined in Step 1 with DAMin), no ordinal Pareto domination relationship holds between them.

Proposition 4. DAMin is not necessarily Pareto dominated by DA.

Then, as stated in Remark 2, in the actual matching of student-supervisor data in 2016, the sum of ranks of the student and supervisor in the final assignment with *endogenously* chosen sequence was rather slightly higher than with the *exogenously* chosen sequence and original DA.

When we consider the sum of ranks of the student and supervisor as a measure of welfare, DAMin is cardinally more efficient than DA by the sum of ranks of the student and supervisor. We formally define this type of efficiency as follows.

Cardinal efficiency: A matching outcome μ is cardinally more efficient than μ' by the sum of ranks of the student and supervisor if

$$\sum_{s\in S} Rank\left(\mu(s)\right) + \sum_{t\in T} \sum_{s'\in\mu(t)} Rank(t) < \sum_{s\in S} Rank\left(\mu'(s)\right) + \sum_{t\in T} \sum_{s'\in\mu'(t)} Rank(t),$$

where the mapping $Rank(\cdot)$ maps a student s assigned to a supervisor based on the student's ranking number $\{1, \ldots, m\}$ for the supervisor on her preference.

Cardinal domination: If a matching μ with a mechanism φ is cardinally more efficient than μ' with another mechanism ψ, φ cardinally dominates ψ.

Then, we have the following proposition.

Proposition 5. DAMin is not necessarily cardinally dominated by DA

However, the following proposition shows that under a certain priority structure, DAMin is cardinally dominated by DA. To prove this, we use the following definition of an essentially homogeneous priority structure introduced by Kojima (2013).

Essentially homogeneous (Kojima, 2013)
A priority structure $(\{P_t, q_t\}_{t\in T})$ is *essentially homogeneous* if there exist no $t, t' \in T$ and $s, s' \in S$ such that (1) $s\ P_t\ s'$ and $s'\ P_t\ s$, and (2) there exist sets of students $S_t\ S_{t'} \subset S \setminus \{s, s'\}$ such that $|S_t| = q_t - 1, |S_{t'}| = q_{t'} - 1,\ S_t = \{s'' \in S : s''\ P_t\ s'\}$, and $S_{t'} = \{ s'' \in S: s''\ P_{t'}\ s\}$.

Proposition 6. If a school's priority structures are essentially homogeneous, then DAMin is cardinally dominated by DA.

The concept of an essentially homogenous priority structure seems to be similar with the *acyclic* condition introduced by Ergin (2002), or Kesten (2006).[7] Thus, if a priority structure has a *cycle*, the above negative result for DAMin may not hold. The following example shows that this conjecture may be right.

Example 3.
We show the above statement with the following example. There are five students $S = \{s_1, s_2, s_3, s_4, s_5\}$ and there are two types $\Theta = \{\theta_1, \theta_2\}$. So, the students are divided as $S_{\theta_1} = \{s_3, s_4, s_5\}$ and $S_{\theta_2} = \{s_1, s_2\}$. There are three supervisors $T = \{t_1,\ t_2,\ t_3\}$, and they are divided as $T_{\theta_1} = \{t_1, t_2\}$ and $T_{\theta_2} = \{t_3\}$. The total quotas, type-specific

[7] Ergin (2002): A *strong cycle* is a set $t, t' \in T$ and $s_i, s_j, s_k \in S$ such that (1) $s_i\ P_t\ s_j\ P_a\ s_k, s_k\ P_t'\ s_i,$ and (2) there exist (possibly empty) disjoint sets of students $S_t, S_{t'} \subset S \setminus \{s_i, s_j, s_k\}$ such that $S_t \subset U_t(s_j) = \{s \in S : s\ P_t\ s_j\}, |S_t| = q_t - 1, S_{t'} \subset U_{t'}(s_i), |S_{t'}| = q_{t'} - 1$. A profile of supervisor priorities and capacities is *weakly acyclic* if no strong cycle exists.
Kesten (2006): A *cycle* is a set $t, t' \in T$ and $S_i, s_j, s_j \in S$ such that (1) $s_i\ P_t\ s_j\ P_t\ s_k, s_k\ P_{t'} s_i, s_j$, and (2) there is a (possibly empty) set $S_t \subset S \setminus \{s_i, s_j, s_k\} such\ that\ S_t \subset U_t\ (s_i) \cup (U_t\ (s_j) \setminus U_{t'}(s_k))\ and\ |S_t| = q_t - 1.$ A profile of school priorities and capacities is acyclic if no cycle exists. Note that if a priority structure is a satisfied cycle, then the priority structure becomes a strong cycle.

minimal and maximal quotas are as follows.

$$
\begin{aligned}
(q, L, U) \quad = \quad & \{(q_{t_1}, L_{t_1,\theta_1}, U_{t_1,\theta_1}) = (2, 1, 2), \\
& (q_{t_2}, L_{t_2,\theta_1}, U_{t_2,\theta_1}) = (3, 1, 3), \\
& (q_{t_3}, L_{t_3,\theta_2}, U_{t_3,\theta_2}) = (2, 1, 2)\}.
\end{aligned}
$$

Under true preference profiles for each student

$$
\begin{aligned}
& P_{s_1}: t_3 t_1 t_2, \\
& P_{s_2}: t_3 t_1\ t_2, \\
& P_{s_3}: t_2 t_1 t_3, \\
& P_{s_4}: t_2 t_1 t_3, \\
& P_{s_5}: t_2 t_1 t_3,
\end{aligned}
$$

and priority orderings for each supervisor

$$
\begin{aligned}
& \succ_{t_1}: s_3 s_1 s_2 s_4 s_5\ , \\
& \succ_{t_2}: s_4 s_5 s_3 s_2 s_1, \\
& \succ_{t_3}: s_1 s_2 s_3 s_4 s_5,
\end{aligned}
$$

the resulting final matching with DA is as follows. Note that the priority structures have *cycle* structure.

$$
\mu = \begin{pmatrix} t_1 & t_2 & t_3 \\ \emptyset & s_3, s_4, s_5 & s_1, s_2 \end{pmatrix}.
$$

The sum of students' and supervisors' ranks is 14. The result of DAMin is μ^{Min}:

$$
\mu^{Min} = \begin{pmatrix} t_1 & t_2 & t_3 \\ s_3 & s_4, s_5 & s_1, s_2 \end{pmatrix}.
$$

The sum of ranks of the student and supervisor is 12.

This example shows that if a priority structure has a *cycle*, DAMin cardinally dominates DA. However, this property does not hold for any matching market as the following example shows

Example 4.

There are five students $S = \{s_1, s_2, s_3, s_4, s_5\}$ and there are two types $\Theta = \{\theta_1, \theta_2\}$. So, the students are divided as $S_{\theta_1} = \{s_3, s_4, s_5\}$ and $S_{\theta_2} = \{s_1, s_2\}$. There are three supervisors $T = \{t_1,\ t_2,\ t_3\}$, and they are divided as $T_{\theta_1} = \{t_1, t_2\}$ and $T_{\theta_2} = \{t_3\}$. Total quotas, type-specific minimal and maximal quotas are as follows.

$$
\begin{aligned}
(q, L, U) \quad = \quad & \{(q_{t_1}, L_{t_1,\theta_1}, U_{t_1,\theta_1}) = (2, 1, 2), \\
& (q_{t_2}, L_{t_2,\theta_1}, U_{t_2,\theta_1}) = (3, 1, 3), \\
& (q_{t_3}, L_{t_3,\theta_2}, U_{t_3,\theta_2}) = (2, 1, 2)\}.
\end{aligned}
$$

Under true preference, the profiles for each student are

$$P_{s_1}: t_3 t_1 t_2,$$
$$P_{s_2}: t_3 t_1\ t_2,$$
$$P_{s_3}: t_2 t_3 t_1,$$
$$P_{s_4}: t_2 t_1 t_3,$$
$$P_{s_5}: t_2 t_1 t_3,$$

and priority orderings for each supervisor are

$$\succ_{t_1}: s_4 s_1 s_2 s_3 s_5\ ,$$
$$\succ_{t_2}: s_4 s_5 s_3 s_2 s_1,$$
$$\succ_{t_3}: s_1 s_2 s_3 s_4 s_5,$$

the resulting final matching with DA is as follows. Note that the priority structure has a *cycle.*

$$\mu = \begin{pmatrix} t_1 & t_2 & t_3 \\ \emptyset & s_3, s_4, s_5 & s_1, s_2 \end{pmatrix}.$$

The sum of students' and supervisors' rank is 14. The result of DAMin is μ^{Min}:

$$\mu^{Min} = \begin{pmatrix} t_1 & t_2 & t_3 \\ s_3 & s_4, s_5 & s_1, s_2 \end{pmatrix}.$$

The sum of students' and supervisors' rank is 17.

Then, DAMin with *endogenous* sequence can Pareto-dominate *exogenous* sequence in general? Our conjecture is that there is no Pareto-dominated relationship among these mechanisms. The following Proposition 7 supports our conjecture.

Proposition 7. There is no Pareto-dominated relationship in DAMin with *endogenous* sequence, and the one with *exogenous* sequence for any matching market.

Then, DAMin with *endogenous* sequence cardinally dominates one with *exogenous* sequence? We show that from the following proposition, there is no cardinal domination relationship between both types of DAMin.

Proposition 8. There is no cardinal domination relationship between both types of DAMin for any matching market.

These findings suggest that the only limitation of DAMin with *endogenous* sequence is lack of strategy-proofness, when we compare it with the one with *exogenous* sequence.

However, the number of experimental findings show that (e.g., Kawagoe et al., 2018) strategy-proofness is not satisfied even for the DA mechanism. Therefore, lack of strategy-proofness may not be a major problem in practice.

Comparison with related mechanisms

Fragiadakis et al. (2015) also study a model of school choice with minimal quotas (hard floor constraints). The model they proposed is restricted to the case that each school has an aggregate floor constraint, that is, they do not allow a school to have separate floors for different types of students. Unlike the model proposed by Fragiadakis et al. (2015) and Fragiadakis and Troyan (2017), DAMin allows a supervisor to have separate floors for different types of students.

The extended-seat DA (ESDA) mechanism proposed by Fragiadakis et al. (2015) fills an aggregate floor constraint by separating each school's total quotas into two parts, one part corresponds to minimal quotas and the other the remaining seats. Subsequently, students are assigned to each school by filling minimum quota as early as possible unless any justified envy causes any delay. Specifically, when students apply to their first-choice schools, the schools accept them up to their *minimal quotas* based on their priority orderings. Then, students who are rejected by their first-choice schools apply for the *remaining seats* (total quotas minus minimal quotas) at their first-choice schools. If they are again rejected, then they apply to their second-choice schools, and so on. The ESDA fulfills each school's seats *from the bottom*, while the model proposed by Fragiadakis and Troyan (2017) and DAMin reduce each school's seats *from the top* to fulfill minimal quotas. Fragiadakis et al. (2015) proved that ESDA is strategy-proof and satisfies a modified sense of stability (Fragiadakis et al., (2015) Theorem 3).

In the DAMin, maximal quota is *reduced* one by one. This causes unwarranted and unfair feelings for supervisors who were removed students once assigned in previous steps in the algorithm. On the other hand, ESDA fulfills minimal quotas, unless justified envy is caused. In this case, as students are not *removed* but *added up* for each supervisor, unfair feelings do not seem to occur. In fact, the example used in Proposition 2 is feasible under ESDA:

$$\mu^{ESDA} = \begin{pmatrix} t_1 & t_2 & t_3 \\ s_2, s_3 & s_1, s_4 & s_5 \end{pmatrix}.$$

There is no cardinal domination between ESDA and DAMin. In the following proposition, DAMin cardinally dominates ESDA.

Proposition 9. There is cardinally no dominating relationship between DAMin and

ESDA for any matching market.

Tomoeda (2018) also studies the model of a school choice with minimal quotas. He proposes a mechanism called by Deferred Acceptance Mechanism with Precedence Lists (DAPL). He also considers a dynamic DA based mechanism. In his model, for fulfilling minimal quotas, the *rankings* over students of the same type are dynamically changed, while *maximal quotas* are changed in DAMin. The DAPL is also strategy-proof and satisfies a modified sense of stability (Tomoeda (2018) Proposition 2). The proposed model is like our model, but it does not seem applicable to our situation because interpretations of types in Tomoeda's (2018) model and ours are different. Tomoeda (2018) interprets a type as student's characteristics, but we interpret a type as a student's *as well as supervisor's* affiliation. In other words, in Tomoeda (2018), a type is defined only for students.

The inherent weakness of DAMin is the lack of strategy-proofness when a reduction sequence is *endogenously* determined. However, some studies show that mechanisms with some dynamic adjustment processes do not satisfy strategy-proofness. For example, Haeringer and Iehlé (2016), who study a dynamic DA-based mechanism such that students can resubmit their preferences for obtaining a better match in the later stages of admission to a French college, prove that the mechanism is not strategy-proof. Okumura (2017) studies a certain kind of school choice problem for resolving shortages in childcare in nursery schools in Japan, where different quotas are set to different age groups in each nursery school, and these quotas are dynamically adjusted in the school for resolving co-existence of excess demand and supply for different age groups. The proposed mechanism satisfies a modified sense of stability but is not strategy-proof.

However, if we put DAMin in the context of a large economy environment, it would be possible to show that it is strategy-proof.[8]

4. Empirical data

The DAMin mechanism was implemented in 2016 at Future University Hakodate with 254 students and 67 supervisors. Totally, there are four courses in the university, and each student belongs to at the most one course. In the following analyses, we refer to

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[8] For example, Kojima and Pathak (2009) study a large economy environment and the same technique can be used to show strategy-proofness for DAMin in a large economy.

these courses as A, B, C, and D.[9] Supervisors belong to either one of the four courses or the Communication Media Laboratory (CML).

Students were recommended to apply to supervisors who belonged to the same course, but they could apply to any supervisor in any course. In fact, 21.2% students (54 out of 254) were assigned to supervisors in different courses. Due to this, although minimal quota was fulfilled for all the supervisors, the maximal quota for students in the same course was not fulfilled for 7.5% supervisors (5 out of 67).

The maximal quotas for students in the same course were equal among supervisors in the same course. The maximal quotas were set to ensure that if every student in the same course applied to supervisors in the same course, they could not be unmatched. Therefore, the maximal quotas in each course were set equal to the number of students in the course divided by the number of supervisors in the same course (if the calculated number contained decimal points, it was rounded up to the closest integer).

The minimal quotas for students in the same course were basically equal among supervisors belonging to the same course. However, it was different for the four courses, thereby reflecting the educational objective of each course. Supervisors had to set strictly positive minimal quotas for students who belonged to the same course, but they could set minimal quotas equal to zero for students who belonged to different courses. For supervisors who belonged to CML, maximal quotas were four and they could set minimal quotas equal to zero for students in any course.

The number of students and supervisors, maximal and minimal quotas for students in the same course and in each course are summarized in Table 1.

	A	**B**	**C**	**D**	**CML**	**Total**
# of students	62	62	86	44	0	254
# of supervisors	13	12	19	13	10	67
Maximal quota	6	6	4	4	4	---
Minimal quota	2	2	2	3	0	---

Table 1. # of students, # of supervisors, and maximal and minimal quotas for students in the same course and in each course.

Students had to rank all the supervisors who belonged to the same course in their submitted preferences, and supervisors also had to rank all the students who belonged to the same course in their submitted priority orderings. When maximal quotas

[9] A, B, C, and D correspond to Complex Systems, Intelligent Systems, Information Systems, and Information Design courses, respectively.

were set, this did not allow any student to be left unmatched. Students did not have to rank supervisors in different courses, and supervisors also did not have to rank students in different courses.

Given the preferences, priority orderings, maximal and minimal quotas, the matching outcome was determined by DAMin. No student was left unmatched, but eight supervisors could not fulfill their minimal quota at the first step of the algorithm in DAMin. As a result, the quota adjusting process started and, finally, every supervisor fulfilled her minimal quota at the 47th step.

The maximal ranking of a supervisor, as ranked by a student was 23 and minimal ranking was 14, depending on the number of supervisors in the course and a student's preference for supervisors in different courses.

Assignments in the final matching outcome

In the resulting matching, 71.7% students were matched with their first-choice supervisor. Totally, 90% students were matched with their fifth or better supervisor. In the worst-case scenario, a student was matched with his nineteenth choice. In the next worst case, a student was matched with his fourteenth choice.

Rank	**1st**	**2nd**	**3rd**	**4th**	**5th**	**6th**	**7th**	**8th**	**9th**	**10th**
Freq.	182	22	13	9	5	6	2	4	4	0
%	71.7	8.7	5.1	3.5	2.0	2.4	0.8	1.6	1.6	0.0
Cum. %	71.7	80.3	85.4	89.0	90.0	93.3	94.1	95.7	97.2	97.2

Table 2. Matching results for students

The number of students who were assigned to their tenth choice, its relative percentages, and cumulative percentages in the population are shown in Table 2.

Except for supervisors in the CML, the maximal number of students a supervisor ranked was 96 and minimal number was 47, depending on the number of students in the course and the supervisor's preference for students in different courses. Up to four students applied to supervisors in the CML.

In the resulting matching, 21.7% students who were matched with supervisors were their first-choice students. Totally 90.2% students whom supervisors were matched with were their 13th or better students. In the worst-case scenario, a supervisor was matched with his 81st choice. In next worst case, a supervisor was matched with his 73rd choice.

One may think that the matching outcome for supervisors are relatively worse than students. However, note that supervisors had to accept up to their maximal quota.

Even in the best case, each supervisor had to accept four or more students, if the maximal quota was fulfilled. Then, as 60.2% (73.6%) of students whom the supervisors were matched with were up to their fourth (sixth) choice, relative performance of the mechanism for supervisors is not worse than students (also remember that DAMin is a student optimal.).

The number of supervisors who were assigned up to their tenth choice, its relative percentages, and cumulative percentages in the population are shown in Table 3.

Rank	**1st**	**2nd**	**3rd**	**4th**	**5th**	**6th**	**7th**	**8th**	**9th**	**10th**
Freq.	55	42	30	26	22	12	12	11	5	4
%	21.7	16.5	11.8	10.2	8.7	4.7	4.7	4.3	2.0	1.6
Cum. %	21.7	38.2	50.0	60.2	68.9	73.6	78.3	82.7	84.6	86.2

Table 3. Matching results for supervisors

Welfare

The sums of a student's rank for supervisor and a supervisor's rank for a student in the student-supervisor pairs are shown in Table 4. If both a student and supervisor is matched with their first choice, the sum of ranks is 2. Such cases occurred in 20.9% matching cases. As already pointed out, each supervisor had to accept four or more students, if the maximal quota is fulfilled, the sum of ranks should be 5 to 8. Then, 74% of the matching observed is such cases.

Rank	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11-**
Freq.	53	38	33	25	20	13	6	8	6	52
%	20.9	15.0	13.0	9.8	7.9	5.1	2.4	3.1	2.4	20.5
Cum. %	20.9	35.8	48.8	58.7	66.5	71.7	74.0	77.2	79.5	100.0

Table 4. Distribution of the sum of ranks for students and supervisors

Example 2 in Section 3 shows that if we consider the sum of ranks of the student and supervisor in the final assignment as a measure of welfare, DAMin can dominate DA. It is also the case in this matching. In fact, the sum of ranks of the student and supervisor in the final assignment with the original DA is 2460, and the sum with DAMin with endogenous sequence is 2398. Thus, in this regard, DAMin improves welfare more than DA.

Strategy-proofness

In this student-supervisor matching, it is not possible to determine if students submitted their true-preferences. Therefore, we asked students to respond via an online questionnaire after the matching outcome was reported. About 45.3% students (115 out of 254) responded to the questionnaire.

Nearly 87.8% students (101 out of 115) answered that they are aware that this matching process is based on the DA mechanism, 57.4% students (58 out of 101) read books or searched the Internet about the DA mechanism, 65.5% students (38 out of 58) answered that they understood the DA mechanism, its working, and properties.

Nearly 54.8% students (63 out of 115) answered that **strategy-proofness** was attractive, and 50.4% students (58 out of 115) responded that eliminating justified envy is desirable.

About 41.7% students (48 out of 115) said that they had submitted true preferences in full length, that is, ranked all the supervisors according to their true preferences. Moreover, 55.7% students (64 out of 115) submitted true preferences for supervisors whom they had ranked relatively higher. Only 2.6% students (3 out of 115) did not submit their true preferences intentionally. In fact, they avoided their first-choice supervisors. Probably they avoided popular supervisors and misrepresented their second choice as their first choice.

Other strategic problems

In this matching process, before submitting their preferences, students had the opportunity to interview with supervisors for two weeks. For all students, their preferences for supervisors are usually incomplete information. This is partly because they had never attended the lecture some supervisors taught. Thus, during this interview period, recognizing each supervisor’s personality and educational and research objective, students formed their “true” preferences. In this sense, **preferences are endogenous** (see Antler, 2015).

Nearly 97.4% students (112 out of 115) interviewed with at least one supervisor. Students who interviewed with five or more supervisors were only 8.0% (9 out of 112). About 75.0% students (84 out of 112) interviewed with two to four supervisors, but 17.0% students (19 out of 112) interviewed with only one supervisor. Though students were forced to submit their preferences for all supervisors in the same course to avoid being unmatched, this result suggests that non-negligible number of supervisors were unacceptable to students. In other words, students’ preference was virtually **truncated**, that is, supervisors whom they had ranked as the first or a relatively high choice were only reliable. It is well known that truncated preferences may cause undesirable outcomes. Of course, as some students and supervisors knew each other, it may be that students needed not **search** for other supervisors for achieving better matching during the interview period.

Another concern is that students' preference may be affected by supervisors' **persuasion** during this interview period. In fact, we realized through informal talks with students that some supervisors made a **credible threat** to students that if they did not rank the supervisor as their first-choice, they would not accept them at all. As fulfilling maximal quota was a major concern for some supervisors (gathering enough number of staff members in the laboratory was necessary to run the ongoing research project), they had made such threats. These threats were really "credible" because supervisors could submit their priority orderings *after* knowing students' had submitted their preferences.[10] This may be the reason that a large number of students were matched with their first choice.

Nonetheless, it is not clear if the truncated preference and/or credible threat distorted the matching outcome. As a matter of fact, 80.0% students (92 out of 115) answered that they had decided on submitted rankings according to their own preferences. About 15.7% students (18 out of 115) worried about the competition for popular supervisors, and 13.9% students (16 out of 115) answered that whether or not known senior students have already been assigned to and/or mutual friends also applied to the same supervisor was another factor of their choice, as in **resident matching with couples** (Kojima and Kamada, 2013). Only 6.1% students (7 out of 115) randomly decided.

These data seem to suggest that most students submitted their true preferences.

5. Discussion and conclusion

We have reported about a recent reform in the matching mechanism between students and supervisors in a Japanese university. The proposed mechanism, DAMin, can eliminate justified envy among students with the same type, but does not satisfy

[10] The chief of education committee and one of the authors (Kawagoe) of this study strongly recommend that supervisors should submit their priority orderings *without knowing* students' preferences for avoiding any strategic effect. However, a non-negligible number of supervisors claimed that they could not rank all students in the same course without knowing their preferences. In a personal communication with Fuhito Kojima, we were informed that he had ever heard of Alvin Roth also facing a similar situation when he consulted the matching process for freshmen in a certain university.

strategy-proofness and feasibility. As for strategy-proofness, experimental evidence from laboratory experiments (Chen and Sönmez, 2006; Featherstone and Niederle, 2008; Pais and Pintér, 2008; Kawagoe et al., 2018) and field data (e.g., Echenique, Wilson and Yariv, 2013; Chen and Kesten, 2017) suggest that a non-negligible number of students mispresent their preferences. Thus, it is still debatable if lack of strategy-proofness is detrimental.

Although the fact that DAMin does not satisfy feasibility may be critical, in actual matching outcomes in 2016, feasibility constraint was satisfied. This result may be explained fully by extending our model to a large economic environment.[11]

During our discussion with the university education committee, a member raised a question if submitting **cardinal preferences** may be better than ordinal preferences to measure the strength of the preferences. The school choice problem with cardinal preferences studied so far is in the context of resolving the problem caused by tie-breaking.[12] What causes matching process with type-specific quotas by allowing cardinal preferences is not clear at this moment.

Another concern relates to the **lattice structure** of stable matching. As DAMin is based on student-proposing DA, the resulting matching is student-optimal. However, a non-negligible number of supervisors seemed to feel uneasy about this. One of the reasons why *endogenous* reduction sequence was adopted in DAMin was to improve supervisors' welfare, as discussed in Remark 2 in subsection 3.2. If reflecting a supervisor's welfare as well as student's is desirable, **median matching** or fractional matching may be the candidate to achieve, although they are not strategy-proof. However, we consider that reflecting a supervisor's welfare is an important objective at a matching mechanism between students and supervisors because as supervisors are long-run players, they have more concern for the matching outcome than students. We look forward to future research answering these questions.

[11] For example, see Kojima and Pathak (2009).

[12] For example, see Abdulkadiroglu, Che and Yasuda (2015) .

Appendix A.

Proposition 1. *DAMin eliminates justified envy among same type students.*

Proof. Without loss of generality, consider DA with U^k $(k = 1, \dots, r)$. Denote the matching produced by DA with U^k as μ. Assume that a student s_i envies another student s_j, who is the same type as his: $\mu(s_j)\, P_{s_i}\, \mu(s_i)$ and $\tau(s_i) = \tau(s_j) = \theta$. Let step r be the step in the DA algorithm at which student s_i is rejected from $\mu(s_j)$. In step r, s_i is rejected because type θ specific minimal quota is filled with $L_{\mu(s_j),\theta}$ students of type θ ranked higher than s_i according to $\succ_{\mu(s_j)}$, and the remaining seats are also filled with $q^k_{\mu(s_j)} - \sum_{\theta \in \Theta} L_{\mu(s_j),\theta}$ students of any type ranked higher than $s_i$according to $\succ_{\mu(s_j)}$. In future steps, a student accepted in step r can be rejected from type θ specific minimal quota only if a higher ranked student of type θ applies, and the same is true for students in the remaining seats. Thus, at the end of the algorithm, all students assigned to $\mu(s_j)$ either in type θ specific minimal quota or in the remaining seats must be ranked higher than s_i. Since $\tau(s_j) = \theta$ as well, this implies that $s_j \succ_{\mu(s_j)} s_i$, that is, s_i does not have any justified envy against s_j.

Q.E.D.

Proposition 2. *For any preference profile*, *DAMin may not be a feasible matching.*

Proof. We prove the statement with the following example. There are five students $S = \{s_1, s_2, s_3, s_4, s_5\}$ and there are two types $\Theta = \{\theta_1, \theta_2\}$. So, the students are divided as $S_{\theta_1} = \{s_1, s_2\}$ and $S_{\theta_2} = \{s_3, s_4, s_5\}$. There are three supervisors $T = \{t_1,\ t_2, t_3\}$, and they are divided as $T_{\theta_1} = \{t_1, t_2\}$ and $C_{\theta_2} = \{t_3\}$. Total quotas, and type-specific minimal and maximal quotas are as follows.

$$
\begin{aligned}
(q, L, U) \quad = \quad & \{(q_{t_1}, L_{t_1,\theta_1}, U_{t_1,\theta_1}) = (2, 1, 1), \\
& (q_{t_2}, L_{t_2,\theta_1}, U_{t_2,\theta_1}) = (2, 1, 2), \\
& (q_{t_3}, L_{t_3,\theta_2}, U_{t_3,\theta_2}) = (1, 1, 1)\}.
\end{aligned}
$$

Under true preference profiles for each student

$$
\begin{aligned}
&P_{s_1}: t_2 t_1 t_3, \\
&P_{s_2}: t_2 t_1\ t_3, \\
&P_{s_3}: t_1 t_2 t_3, \\
&P_{s_4}: t_1 t_2 t_3,
\end{aligned}
$$

$$P_{s_5}: t_3 t_1 t_2,$$

and priority orderings for each supervisor

$$\succ_{t_1}: s_3 s_4 s_2 s_1 s_5\ ,$$
$$\succ_{t_2}: s_1 s_2 s_3 s_4 s_5,$$
$$\succ_{t_3}: s_2 s_5 s_3 s_4 s_1,$$

the resulting in final matching with DAMin as follows.

$$\mu = \begin{pmatrix} t_1 & t_2 & t_3 \\ s_3, s_4 & s_1, s_2 & s_5 \end{pmatrix}.$$

However, μ is not feasible because no type θ_1 student is assigned to type θ_1 supervisor c_1.

Q.E.D.

Proposition 3. *For any P, DAMin is not strategy-proof.*

Proof. We prove the statement with the following example. There are five students $S = \{s_1, s_2, s_3, s_4, s_5\}$ and there are two types $\Theta = \{\theta_1, \theta_2\}$. Thus, the students are divided as $S_{\theta_1} = \{s_1, s_2, s_3, s_4\}$ and $S_{\theta_2} = \{s_5\}$. There are three supervisors $T = \{t_1,\ t_2,\ t_3\}$, and they are divided as $T_{\theta_1} = \{t_1, t_2\}$ and $T_{\theta_2} = \{t_3\}$. Total quotas, and type-specific minimal and maximal quotas are as follows.

$$\begin{aligned}(q, L, U) \quad = \quad & \{(q_{t_1}, L_{t_1,\theta_1}, U_{t_1,\theta_1}) = (2, 1, 2), \\ & (q_{t_2}, L_{t_2,\theta_1}, U_{t_2,\theta_1}) = (1, 1, 1), \\ & (q_{t_3}, L_{t_3,\theta_2}, U_{t_3,\theta_2}) = (2, 1, 2)\}.\end{aligned}$$

Under true preference profiles for each student

$$P_{s_1}: t_1 t_2 t_3,$$
$$P_{s_2}: t_1 t_2\ t_3,$$
$$P_{s_3}: t_3 t_1 t_2,$$
$$P_{s_4}: t_1 t_3 t_2,$$
$$P_{s_5}: t_3 t_1 t_2,$$

and priority orderings for each supervisor

$$\succ_{t_1}: s_2 s_1 s_3 s_4 s_5\ ,$$
$$\succ_{t_2}: s_1 s_3 s_2 s_4 s_5,$$
$$\succ_{t_3}: s_4 s_5 s_3 s_2 s_1,$$

the resulting final matching with DAMin is as follows.

$$\mu(P) = \begin{pmatrix} t_1 & t_2 & t_3 & s_3 \\ s_2 & s_1 & s_4, s_5 & s_3 \end{pmatrix}.$$

Consider that student s_3 states the following preference instead (any other student reveals true preference.).

$$P'_{s_3}: t_2\ t_1\ t_3.$$

Then, the resulting final matching with DAMin is as follows.

$$\mu(P_{-s_3}, P'_{s_3}) = \begin{pmatrix} t_1 & t_2 & t_3 \\ s_1, s_2 & s_3 & s_4, s_5 \end{pmatrix}.$$

In this matching outcome, student s_3 is strictly better off than in $\mu(P)$. Thus, student s_3 has an incentive to misrepresent her true preference.

Q.E.D.

Proposition 4. DAMin is not necessarily Pareto dominated by DA.

We prove the statement with the following example. There are five students $S = \{s_1, s_2, s_3, s_4\}$ and there are two types $\Theta = \{\theta_1, \theta_2\}$. So, the students are divided as $S_{\theta_1} = \{s_1, s_2, s_3\}$ and $S_{\theta_2} = \{s_4\}$. There are three supervisors $T = \{t_1, t_2, t_3\}$, and they are divided as $T_{\theta_1} = \{t_1, t_2\}$ and $T_{\theta_2} = \{t_3\}$. Total quotas, and type-specific minimal and maximal quotas are as follows.

$$\begin{aligned}(q, L, U) \quad = \quad & \{(q_{t_1}, L_{t_1,\theta_1}, U_{t_1,\theta_1}) = (2,1,2), \\ & (q_{t_2}, L_{t_2,\theta_1}, U_{t_2,\theta_1}) = (1,1,1), \\ & (q_{t_3}, L_{t_3,\theta_2}, U_{t_3,\theta_2}) = (2,1,2)\}.\end{aligned}$$

Under true preference profiles for each student

$$\begin{aligned} & P_{s_1}: t_1 t_2 t_3, \\ & P_{s_2}: t_1 t_2\ t_3, \\ & P_{s_3}: t_2 t_1 t_3, \\ & P_{s_4}: t_3 t_1 t_2, \end{aligned}$$

and priority orderings for each supervisor

$$\begin{aligned} & \succ_{t_1}: s_2 s_1 s_3 s_4\ , \\ & \succ_{t_2}: s_1 s_3 s_2 s_4, \\ & \succ_{t_3}: s_4 s_3 s_2 s_1, \end{aligned}$$

the resulting final matching with DAMin is as follows.

$$\mu^{DAMin}(P) = \begin{pmatrix} t_1 & t_2 & t_3 \\ s_1, s_2 & s_3 & s_4 \end{pmatrix}.$$

the resulting final matching with the original DA is as follows.

$$\mu^{DA}(P) = \begin{pmatrix} t_1 & t_2 & t_3 \\ s_1, s_2 & s_3 & s_4 \end{pmatrix}.$$

Thus, both matching outcomes are the same. Therefore, we conclude that DAMin is not

Pareto dominated by the original DA. *Q.E.D.*

Proposition 5. The DAMin is not necessarily cardinally dominated by DA.

Proof. We prove this statement using an example. There are three students $S = \{s_1, s_2, s_3\}$ and there is one type $\Theta = \{\theta_1\}$; thus, assuming that every student is the same type, $S_{\theta_1} = \{s_1, s_2, s_3\}$. There are two supervisors $T = \{t_1, t_2\}$, and their types are the same as students, $T_{\theta_1} = \{t_1, t_2\}$. Total quotas, type-specific minimal and maximal quotas are as follows.

$$(q, L, U) = \{(q_{t_1}, L_{t_1,\theta_1}, U_{t_1,\theta_1}) = (3, 1, 3), \\ (q_{t_2}, L_{t_2,\theta_1}, U_{t_2,\theta_1}) = (2, 1, 2)\}.$$

Under preference profiles for each student

$$P_{s_1}: t_1 t_2,$$
$$P_{s_2}: t_1 t_2,$$
$$P_{s_3}: t_1 t_2,$$

and priority orderings for each supervisor;

$$\succ_{t_1}: s_1 s_2 s_3,$$
$$\succ_{t_2}: s_3 s_1 s_2,$$

the resulting final matching with the original DA is as follows.

$$\mu(P) = \begin{pmatrix} t_1 & t_2 \\ s_1, s_2, s_3 & \emptyset \end{pmatrix}$$

The sum of ranks of the student and supervisor at final assignment in this case is 9. The resulting final matching with DAMin either with *endogenous* or *exogenous* sequence is as follows.

$$\mu'(P) = \begin{pmatrix} t_1 & t_2 \\ s_1, s_2 & s_3 \end{pmatrix}$$

The sum of ranks of the student and supervisor at final assignment in this case is 8.

Q.E.D.

Proposition 6. If a school's priority structures are essentially homogeneous, then DAMin is cardinally dominated by DA.

Proof. Let α be the sum of ranks of the student and supervisor under the original DA matching outcome. If the matching outcome satisfied minimum quotas for all supervisors, then the sum of ranks of the student and supervisor under the DAMin matching outcome would be α.

Next, we consider a case in which the matching outcome does not satisfy minimum quotas for some supervisors. We divide α into two: $\alpha = \alpha^S + \alpha^T$. Let α^S be the sum of ranks of student, and α^T be the sum of ranks of supervisor. By the DA procedure, α^S becomes necessarily larger than the sum of ranks of student α'^S. We prove α^T becomes necessarily bigger than the sum of ranks of supervisor. α'^T. To run the proof, we assume that $\alpha'^T < \alpha^T$. Thus, some supervisor is matched with better students under the DAMin than under the DA. Let the student and the supervisor be s_1 and t_1, respectively. By the DA procedure, s_1 is matched with a better supervisor under the DA than under the DAMin. Supervisor t_1 has two cases: in **case 1**, t_1 is not satisfied with the minimum quota; in **case 2**, t_1 is matched with a student worse than s_1. By assuming the priority structure, both cases also become $\alpha'^T = \alpha^T$. This is a contradiction. Thus, we consider the following relation:

$$\alpha = \alpha^S + \alpha^T < \alpha'^S + \alpha'^T = \alpha'.$$

Q.E.D.

Proposition 7. There is no Pareto domination relationship between DAMin with an *endogenous* sequence and the one with an *exogenous* sequence for any matching market.

Proof. We prove the statement by the following example.
There are five students $S = \{s_1, s_2, s_3, s_4, s_5\}$ and there are two types $\Theta = \{\theta_1, \theta_2\}$. So, the students are divided as $S_{\theta_1} = \{s_1, s_2, s_3\}$ and $S_{\theta_2} = \{s_4, s_5\}$. There are three supervisors $T = \{t_1, t_2, t_3\}$, and they are divided as $T_{\theta_1} = \{t_1, t_2\}$ and $T_{\theta_2} = \{t_3\}$. The total quotas and type-specific minimal and maximal quotas are as follows.

$$\begin{aligned}(q, L, U) \quad = \quad & \{(q_{t_1}, L_{t_1,\theta_1}, U_{t_1,\theta_1}) = (2, 1, 2),\\ & (q_{t_2}, L_{t_2,\theta_1}, U_{t_2,\theta_1}) = (2, 1, 1),\\ & (q_{t_3}, L_{t_3,\theta_2}, U_{t_3,\theta_2}) = (2, 2, 2)\}.\end{aligned}$$

Under true preference profiles for each student

$$\begin{aligned}&P_{s_1}: t_1 t_2 t_3,\\ &P_{s_2}: t_2 t_1\ t_3,\\ &P_{s_3}: t_1 t_1 t_1,\\ &P_{s_4}: t_3 t_1 t_2,\\ &P_{s_5}: t_3 t_1 t_2,\end{aligned}$$

and the priority orderings for each supervisor

$$\succ_{t_1}: s_2 s_1 s_3 s_4 s_5\ ,$$
$$\succ_{t_2}: s_2 s_3 s_1 s_4 s_5,$$
$$\succ_{t_3}: s_4 s_5 s_3 s_2 s_1,$$

the resulting final matching with DAMin with an *endogenous* reduction sequence is as follows.

$$\mu(P) = \begin{pmatrix} t_1 & t_2 & t_3 \\ s_1 s_3 & s_2 & s_4, s_5 \end{pmatrix}.$$

For DAMin with an *exogenous* reduction sequence, suppose that the sequence is given as $\{U^1 = (2,1,2), U^2 = (1,1,1)\}$. Then, the resulting final matching is as follows.

$$\mu(P) = \begin{pmatrix} t_1 & t_2 & t_3 \\ s_1 s_3 & s_2 & s_4, s_5 \end{pmatrix}.$$

Both matchings are the same. Thus, there is no Pareto domination among both types of DAMin.

Q.E.D.

Proposition 8. There is no cardinal domination relationship between both types of DAMin.

Proof. We show the statement with the following example. There are six students $S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$ and there are two types $\Theta = \{\theta_1, \theta_2\}$. So, the students are divided as $S_{\theta_1} = \{s_1, s_2, s_3, s_6\}$ and $S_{\theta_2} = \{s_4, s_5\}$. There are three supervisors $T = \{t_1,\ t_2,\ t_3\}$, and they are divided as $T_{\theta_1} = \{t_1\}$ and $T_{\theta_2} = \{t_2, t_3\}$. Total quotas, and type-specific minimal and maximal quotas are as follows.

$$\begin{aligned}(q, L, U) \quad = \quad & \{(q_{t_1}, L_{t_1,\theta_1}, U_{t_1,\theta_1}) = (4,1,3), \\ & (q_{t_2}, L_{t_2,\theta_1}, U_{t_2,\theta_1}) = (2,1,2), \\ & (q_{t_3}, L_{t_3,\theta_2}, U_{t_3,\theta_2}) = (2,1,2)\}.\end{aligned}$$

Under true preference profiles for each student

$$P_{s_1}: t_1 t_2 t_3,$$
$$P_{s_2}: t_1 t_3 t_2,$$
$$P_{s_3}: t_1 t_2 t_3,$$
$$P_{s_4}: t_3 t_2 t_1,$$
$$P_{s_5}: t_3 t_2 t_1,$$
$$P_{s_6}: t_1 t_2 t_3,$$

and priority orderings for each supervisor

$$\succ_{t_1}: s_6 s_1 s_3 s_2 s_4 s_5\ ,$$
$$\succ_{t_2}: s_4 s_3 s_2 s_1\ s_6 s_5,$$
$$\succ_{t_3}: s_4 s_2 s_5 s_1 s_3 s_6,$$

the resulting final matching with DAMin with *endogenous* sequence μ^{en} is as follows. Note that the priority structures have a cyclic structure.

$$\mu^{en} = \begin{pmatrix} t_1 & t_2 & t_3 \\ s_1, s_3\ , s_6 & s_5 & s_2, s_4 \end{pmatrix}.$$

The sum of ranks of the student and supervisor is 18.

For DAMin with an *exogenous* reduction sequence, suppose that the sequence is given as $\{U^1 = (3,2,2), U^2 = (3,2,1)\ \}$. Then, the resulting final matching with DAMin with an *endogenous* sequence μ^{ex}:

$$\mu^{ex} = \begin{pmatrix} t_1 & t_2 & t_3 \\ s_1, s_2, s_3,\ s_6 & s_5 & s_4 \end{pmatrix}.$$

The sum of ranks of the student and supervisor is 24.

On the other hand, the following example shows that the sum of ranks of the student and supervisor under DAMin with an *exogenous* one is lower than those under DAMin with an *endogenous* sequence.

There are six students $S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$ and there is one type $\Theta = \{\theta_1\}$. So, the students are divided as $S_{\theta_1} = \{s_1, s_2, s_3, s_4, s_5, s_6\}$. There are three supervisors $T = \{t_1,\ t_2,\ t_3\}$, and they are divided as $T_{\theta_1} = \{t_1, t_2, t_3\}$. The total quotas and type-specific minimal and maximal quotas are as follows.

$$\begin{aligned}(q, L, U) \quad = \quad & \{(q_{t_1}, L_{t_1,\theta_1}, U_{t_1,\theta_1}) = (3,1,3), \\ & (q_{t_2}, L_{t_2,\theta_1}, U_{t_2,\theta_1}) = (2,1,2), \\ & (q_{t_3}, L_{t_3,\theta_1}, U_{t_3,\theta_1}) = (3,1,3)\}.\end{aligned}$$

Under true preference profiles for each student

$$P_{s_1}: t_1 t_2 t_3,$$
$$P_{s_2}: t_3 t_1 t_2,$$
$$P_{s_3}: t_1 t_2 t_3,$$
$$P_{s_4}: t_3 t_2 t_1,$$
$$P_{s_5}: t_3 t_2 t_1,$$

$$P_{s_6}: t_1 t_2 t_3,$$

and priority orderings for each supervisor

$$\succ_{t_1}: s_6 s_1 s_4 s_2 s_3 s_5\ ,$$
$$\succ_{t_2}: s_5 s_4 s_2 s_1\ s_6 s_3,$$
$$\succ_{t_3}: s_4 s_2 s_5 s_1 s_3 s_6,$$

the resulting final matching with DAMin with an *endogenous* sequence μ^{en} is as follows. Note that the priority structures have a cyclic structure.

$$\mu^{en} = \begin{pmatrix} t_1 & t_2 & t_3 \\ s_1, s_6 & s_3 & s_2, s_4, s_5 \end{pmatrix}.$$

The sum of ranks of the student and supervisor is 23. For DAMin with an *exogenous* reduction sequence, suppose that the sequence is given as $\{U^1 = (3, 2, 3), U^2 = (3, 2, 2)\ \}$. Then, the resulting final matching with DAMin with an *endogenous* sequence μ^{ex}:

$$\mu^{ex} = \begin{pmatrix} t_1 & t_2 & t_3 \\ s_1, s_3,\ s_6 & s_5 & s_2, s_4 \end{pmatrix}.$$

The sum of students' and supervisors' rank is 19.

Q.E.D.

Proposition 9. There are no cardinally dominant relationships between DAMin and ESDA for any matching market.

Proof. We show the statement with the following example. There are five students $S = \{s_1, s_2, s_3, s_4, s_5\}$ and there is one type $\Theta = \{\theta_1\}$. So, the students are divided as $S_{\theta_1} = \{s_1, s_2, s_3, s_4, s_5\}$. There are three supervisors $T = \{t_1,\ t_2,\ t_3\}$, and they are divided as $T_{\theta_1} = \{t_1, t_2, t_3\}$. Total quotas, type-specific minimal and maximal quotas are as follows.

$$\begin{aligned}(q, L, U) \quad = \quad & \{(q_{t_1}, L_{t_1,\theta_1}, U_{t_1,\theta_1}) = (3, 1, 3), \\ & (q_{t_2}, L_{t_2,\theta_1}, U_{t_2,\theta_1}) = (2, 1, 2), \\ & (q_{t_3}, L_{t_3,\theta_1}, U_{t_3,\theta_1}) = (2, 1, 2)\}.\end{aligned}$$

Under true preference profiles for each student

$$P_{s_1}: t_1 t_2 t_3,$$
$$P_{s_2}: t_1 t_2 t_3,$$
$$P_{s_3}: t_1 t_2 t_3,$$

$$P_{s_4}: t_2 t_3 t_1,$$
$$P_{s_5}: t_2 t_1 t_3,$$

and priority orderings for each supervisor

$$\succ_{t_1}: s_1 s_2 s_3 s_4 s_5,$$
$$\succ_{t_2}: s_5 s_4 s_3 s_2 s_1,$$
$$\succ_{t_3}: s_1 s_2 s_3 s_4 s_5,$$

the resulting final matching with DAMin μ^{Min} is as follows:

$$\mu^{Min} = \begin{pmatrix} t_1 & t_2 & t_3 \\ s_1, s_2 & s_4, s_5 & s_3 \end{pmatrix}.$$

The sum of ranks of the student and supervisor is 16. The resulting final matching with ESDA, μ^{ESDA} is as follows:

$$\mu^{ESDA} = \begin{pmatrix} t_1 & t_2 & t_3 \\ s_1, s_2, s_3 & s_5 & s_4 \end{pmatrix}.$$

The sum of ranks of the student and supervisor is 17.

On the other hand, the following example is that ESDA cardinally dominates DAMin. There are five students $S = \{s_1, s_2, s_3, s_4, s_5\}$ and there is one type $\Theta = \{\theta_1\}$. So, the students are divided as $S_{\theta_1} = \{s_1, s_2, s_3, s_4, s_5\}$. There are three supervisors $T = \{t_1,\ t_2,\ t_3\}$, and they are divided as $T_{\theta_1} = \{t_1, t_2, t_3\}$. The total quotas and type-specific minimal and maximal quotas are as follows:

$$\begin{aligned}(q, L, U) \quad = \quad & \{(q_{t_1}, L_{t_1,\theta_1}, U_{t_1,\theta_1}) = (3, 1, 3), \\ & (q_{t_2}, L_{t_2,\theta_1}, U_{t_2,\theta_1}) = (2, 1, 2), \\ & (q_{t_3}, L_{t_3,\theta_1}, U_{t_3,\theta_1}) = (2, 1, 2)\}.\end{aligned}$$

Under true preference profiles for each student

$$P_{s_1}: t_1 t_2 t_3,$$
$$P_{s_2}: t_1 t_2 t_3,$$
$$P_{s_3}: t_1 t_2 t_3,$$
$$P_{s_4}: t_2 t_3 t_1,$$
$$P_{s_5}: t_2 t_1 t_3,$$

and priority orderings for each supervisor

$$\succ_{t_1}: s_1 s_2 s_3 s_4 s_5,$$
$$\succ_{t_2}: s_5 s_4 s_3 s_2 s_1,$$
$$\succ_{t_3}: s_4 s_1 s_2 s_3 s_5,$$

the resulting final matching with DAMin μ^{Min} is as follows:

$$\mu^{Min} = \begin{pmatrix} t_1 & t_2 & t_3 \\ s_1, s_2 & s_4, s_5 & s_3 \end{pmatrix}.$$

The sum of ranks of the student and supervisor is 17. The resulting final matching with ESDA, μ^{ESDA} is as follows:

$$\mu^{ESDA} = \begin{pmatrix} t_1 & t_2 & t_3 \\ s_1, s_2, s_3 & s_5 & s_4 \end{pmatrix}.$$

The sum of ranks of the student and supervisor is 14.

Q.E.D.

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