

EFFECTS OF INTEGRATION, TYING AND UPSTREAM MARKET STRUCTURES

(Revised Edition)

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Abstract

This paper shows that vertical integration enables an upstream monopolist to exploit leverage in a market and to exclude a rival from a market, but firms other than a monopolist can not foreclose a rival from a market. It will be shown when integration is permissible and when it is not. Moreover, the model is applicable to the analysis of tying and shows the reason why tying MS Windows to Internet Explorer browser drives rival products out of the Japanese market and why introduction of Face Screen in Europe has reduced the market share of Internet Explorer browser.

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1 Introduction

Market foreclosure has been a controversial issue in U.S. antitrust law and industrial organization. Vertical integration is one of typical means to achieve market foreclosure. There is the fear that vertical integration makes a market inefficient by serving to reduce competition through market foreclosure. Thus, it is crucially important and controversial not only for the firm's management strategies, but also for courts and regulators to determine whether the merger is competition-neutral or not. On the other hand, if it is competitive-neutral or improves market efficiencies, the integration is likely to lower market prices through efficiency gains of the merger.

Thus, there have been two strands of thought about effects of the vertical integration; one is that it is competitively neutral or promotes market efficiencies through its efficiency gains; for example, see Bork (1978) and Chen (2001). Especially, Chen (2001) makes an important contribution on the subject; he develops an equilibrium theory of vertical integration and shows that the merger lowers downstream prices and that it can even if market foreclosure arises. The other stand is that it has anticompetitive effect on a market; for example, see Salinger (1988,1991), Ordover, Saloner and Salop (1990), Hart and Tirole (1990), Ma (1997), Riordan (1998), Choi and Yi (2000), Church and Gandale (2000). However, Chen and Riordan (2007) show that the vertical integration and the exclusive contract lead to market foreclosure. If the integration has anticompetitive effect, market price is enhanced because it confers more monopolistic power on the integrated firm.

In this paper, the upstream market structures and the productivity of downstream firms which have been ignored will play crucial roles in our analysis of vertical integration. We shall argue that vertical integration benefits consumers and that the foreclosure effects of the integration depend crucially upon upstream market structures; an upstream monopolist can exploit leverage through the integration in a second market, but an upstream duopolist can not. It is interesting to note that our model can apply to the analysis of tying and that tying by a monopolist can exclude a rival from a market. Moreover, the model can explain why "keiretsu" emerges in games in which there are multiple upstream firms.

The model presented in this paper will mainly examine effects of the vertical integration in vertically related markets. We model these markets as a two-stage game: the first stage corresponds to the upstream market and the second stage to the downstream market. Given the total demand for their outputs, the downstream firms compete to supply it. Once equilibrium outputs of downstream firms are determined, demand for inputs of the upstream firms is derived

from equilibrium outputs of the downstream firms. With known total demand for inputs by the downstream firms, the upstream firms compete to supply it. Note that the present model does not consider incentives for the merger.

The characteristics of the integration effect are different from others because the integration has two distinct kinds of effects. One is cost-reducing effect (hereafter, cost effect) which reduces the price because the integrated firm can get access to inputs (or throughputs) at the marginal costs. The other is efficiency effect, which is due to the fact that the downstream division of the integrated firm may have superior or inferior production facilities relative to a rival. For example, if the downstream division holds superior facilities, efficiency gains have a tendency to lower market price, but inferior efficiency has an opposite effect. Chen (2001) also identifies two effects of the integration: the efficiency and the collusive effects. The former is our cost effect and the latter, which is different from our efficiency effect, comes solely from the fact that downstream products are differentiated.

The model will examine two distinct kinds of games. The first is called the upstream monopoly game (hereafter, monopoly game) in which there is an upstream monopolist and two downstream firms. The other is called the upstream duopoly game (hereafter, duopoly game) in which there are two firms both operating in the upstream and the downstream market. It will be shown that an upstream monopolist can foreclose a rival firm by the vertical integration *per se* even if there are no efficiency gains through the integration. In other words, the cost effect is so strong in the monopoly game that a rival has to exit from a market even if the efficiency effect does not work against the rival.

On the other hand, when we consider the duopoly game, the integrated firm can not exclude a rival from the downstream market even if an efficient upstream duopolist merges with an efficient downstream firm; the cost effect is moderate enough for a competitor to stay in the downstream market even if the efficiency effect works to the advantage of the integrated firm. Although the vertical integration enables the upstream monopolist to exploit leverage too effectively for the rival to stay in the downstream market, the duopolist fails. Thus, the strength of the cost effect depends upon upstream market structure. These results are in stark contrast with those in Chen (2001) and Chen and Riordan (2007) in which the presence of an upstream rival does not make any difference to their analysis.

Moreover, our model shows that there are no trade-offs between competitiveness and market efficiency in both games. Even if the merger is competitively neutral in the sense that it does not result in foreclosing rival firms, market efficiencies are enhanced because post-merger equilibrium

price goes down.¹ Finally, it is of some interest to note that the merged duopoly maximizes profits by not supplying inputs to the upstream market.² This may correspond to “keiretu” in Japanese industries.

As pointed out by Scherer (1980), the analysis of the vertical integration can be applied to that of tied sales because the tied sales is a special type of the vertical merger in vertically related markets. It will be shown that tying a product of an *upstream monopolist* with products of a downstream firm excludes a downstream rival from the market.³ Considering a real case, the tying of MS Windows to Internet Explorer browser (henceforth, IE) has enabled Microsoft, which is a *de facto* upstream monopolist of computer OS, to foreclose Netscape’s Navigator web browser. Tied sales have been allowed in Japan until now, and hence IE has been able to enjoy an almost 100 percent market share. Recently, the introduction of a Choice Screen in Europe has caused the market share of IE to shrink, and other major browsers to enjoy gains of market shares in 2009.⁴ However, it will be shown that tying a product of one of *upstream duopolists* with products of a downstream firm does not foreclose a rival from the market. For example, tying a Toyota car with an air-conditioner, which is produced by its subcontractor, has not excluded rival firms from this Japanese auto market. Although the details of these examples differ from our model, they illustrate the empirical relevance of our arguments.

Our paper is organized as follows. Section 2 sets forth the effects of the integration by an upstream monopolist. The integration by the monopolist causes market foreclosure even if the monopolist has the same production facilities. This in turn enables us to show that tied sales result in excluding un-tied products. This explains why Microsoft excludes Netscape Navigator web browser and the IE has enjoyed a 100% share in the Japanese market. Section 3 explores the duopoly game and shows that the integration does not exclude a rival firm from the market. It will be shown that the upstream division does not supply inputs to the upstream market. This may explain “keiretu” . We end with our conclusions and a discussion of limitations of the model in Section 4.

¹These observations are consistent with what Hortacsu and Syverson (2007) observed in their empirical analysis of cements and ready-mixed industries in the US.

²Salinger (1988) also reached the same conclusions under some assumptions.

³A contribution to the leverage theory of tying was made in Whinston (1990). His model, however, does not enable us to draw up appropriate guidelines for antitrust and regulatory rules.

⁴For detail, see Net Applications’ Annual Industry Report.

2 The basic Ingredients

Consider a vertically related market in which two firms 1 and 2 supply a consumer good. The consumer good is produced by using inputs which are provided by two firms, firms 3 and 4. The former market, supplied by firms 1 and 2, is called the downstream market and the latter the upstream market. This game is called a duopoly game and is a benchmark game against which other games are evaluated. The demand for the consumer good is given by

$$P = a - x = a - (x_1 + x_2), \quad (1)$$

where P represents the price of consumer good, x_i the quantity of output by firm i , $i = 1, 2$, and a a fixed constant.

To simplify our analysis, assume that α_i units of inputs are translated into a unit of output (or a consumer good) by firm i , $i = 1, 2$. It follows that

$$x_i = \frac{1}{\alpha_i} y_i, \quad i = 1, 2, \quad (2)$$

where y_i is the quantity of inputs for firm i . This is the production function of firms 1 and 2. To proceed with our analysis, it will be assumed that productivity of firm 2 is higher than that of firm 1, and that the difference between them is not very large. Formally, this assumption can be expressed as

Assumption 1. $1 < \alpha = \alpha_1/\alpha_2 \leq 2$,

Together with this, it is also assumed that

$$c_i = \alpha_i p, \quad i = 1, 2, \quad (3)$$

where c_i stands for the unit cost of the consumer good produced by firms 1 and 2, and p for the price of inputs. This means that firm i using α_i units of inputs produces one unit of a consumer good at a cost $\alpha_i p$.

To simplify our analysis and to ensure positive outputs of firms in various types of games, assume that the marginal costs (β_3 and β_4) of firms 3 and 4 are positive and that

$$0 < \beta_3 \leq \beta_4. \quad (4)$$

To proceed with our analysis, the following assumption is made:

Assumption 2. $a \geq 4\alpha_1\beta_4$.

Several interesting integrated market structures emerge depending upon the number of independent firms in upstream and downstream markets. In what follows, consider first a game in which there is a monopolist in the upstream market and two independent downstream firms. Assume that outputs, which is henceforth called inputs, produced by the monopolist, is an essential input without which downstream firms can not produce outputs. Words “inputs” and “outputs” are named from the viewpoint of downstream firms. Thus, this game is the monopoly game, which is depicted in Fig. 1. When the upstream monopolist merges with one of the two downstream firms, one independent firm is left in the downstream market. We will now analyze when the independent firm can be viable and when it exits.

Second, consider a one-sided vertical integration in a duopoly game in which there are two independent firms in the upstream and also in the downstream market. One of the upstream firms merges with one of the downstream firms, and the other firms remain independent. The problem examined will be whether the independent firms can stay in the market or not in the post-merger game. Comparison of the pre- and the post-merger game enables us to explore effects of vertical integration in vertically related markets. It is interesting to know whether the integration in these games forecloses an independent firm in the downstream market or not. These comparisons reveal the differences in the effects of the integration in different market structures. These games will be examined in the next section.

In this section, we study first the monopoly game. The upstream monopolist (or firm 3) merges with downstream firm 1 and downstream firm 2 remains independent; see Fig. 1. Note also that the focus is mainly on the effects of vertical integration and that the motivations for the integration are not studied in this paper.

Note also that firms 1 and 3 form the integrated firm and the marginal cost of inputs for the downstream division of the integrated firm is β_3 . Thus, the profits of the two firms, integrated firm I and independent firm 2, are given by

$$\begin{aligned}\pi_I &= \pi_d + \pi_u = (P - \alpha_1\beta_3)x_I + \alpha_2(p - \beta_3)x_2 = (a - \alpha_1\beta_3 - (x_I + x_2))x_I + \alpha_2(p - \beta_3)x_2, \\ \pi_2 &= (P - \alpha_2p)x_2 = (a - \alpha_2p - (x_I + x_2))x_2,\end{aligned}$$

where π_d and π_u stand for profits in the downstream and upstream markets by the integrated monopolist. The first order conditions for the maximization of profits are

$$\frac{\partial \pi_I}{\partial x_I} = \frac{\partial \pi_d}{\partial x_I} = a - \alpha_1\beta_3 - 2x_I - x_2 = 0,$$

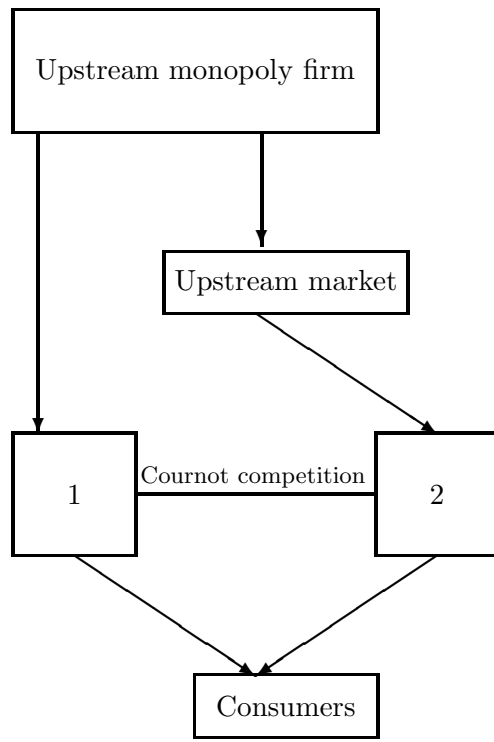


Figure 1: The upstream monopoly game

$$\frac{\partial \pi_2}{\partial x_2} = a - \alpha_2 p - x_I - 2x_2 = 0,$$

where x_I stands for output of the monopolist and x_2 for that of the independent competitor. The Nash equilibrium outputs of these firms are expressed as

$$x_I^* = \frac{a - 2\alpha_1\beta_3 + \alpha_2 p}{3}, \quad (5)$$

$$x_2^* = \frac{a + \alpha_1\beta_3 - 2\alpha_2 p}{3}. \quad (6)$$

Firm 2 has to purchase inputs from the integrated firm and its demand is given by $\alpha_2 x_2^*$. Demand for inputs of firm 2 is

$$\alpha_2 x_2^* = Y = \frac{\alpha_2(a + \alpha_1\beta_3 - 2\alpha_2 p)}{3}.$$

Solving this equation for p , we have the inverse demand p for inputs:

$$p = \frac{-3Y + \alpha_2(a + \alpha_1\beta_3)}{2\alpha_2^2}.$$

Firm I faces demand for its products derived above. Its profit is

$$\begin{aligned} \pi_I &= \pi_d + \pi_u = (P - \alpha_1\beta_3)x_I^* + (p - \beta_3)Y = (P - \alpha_1\beta_3)x_I^* + \left(\frac{-3Y + \alpha_2(a + \alpha_1\beta_3)}{2\alpha_2^2} - \beta_3\right)Y \\ &= \frac{-5Y^2 + 4Y(\alpha_1 - \alpha_2)\alpha_2\beta_3 + \alpha_2^2(a - \alpha_1\beta_3)^2}{4\alpha_2^2}. \end{aligned}$$

It follows that the condition for optimality is

$$\frac{d\pi_I}{dY} = -\frac{5Y}{2\alpha_2^2} + \left(-1 + \frac{\alpha_1}{\alpha_2}\right)\beta_3 = 0.$$

The monopolist supplies inputs to firm 2, which are given by

$$Y^* = \frac{2(\alpha_1 - \alpha_2)\alpha_2\beta_3}{5}.$$

Noting the inverse demand for input, price charged for this input by the supplier is

$$p_{bm}^* = \frac{-3Y^* + \alpha_2(a + \alpha_1\beta_3)}{2\alpha_2^2} = \frac{5a - \alpha_1\beta_3 + 6\alpha_2\beta_3}{10\alpha_2}. \quad (7)$$

It would be of some interest to check whether the equilibrium price in the upstream market is higher than the provider's marginal cost β_3 . Subtracting β_3 from p_{bm}^* , we have

$$p_{bm}^* - \beta_3 = \frac{5a - \alpha_1\beta_3 + 6\alpha_2\beta_3}{10\alpha_2} - \beta_3 = \frac{5a - \alpha_1\beta_3 - 4\alpha_2\beta_3}{10\alpha_2} > 0,$$

which will be positive under our Assumptions 1 and 2. Thus, the integrated monopolist has cost advantage over its competitor in the downstream market.

Thus, we can now summarize our analysis above as:

Lemma 1. *Equilibrium prices in the upstream and downstream markets where there is an upstream monopolist are given by*

$$\begin{aligned} p_{bm}^* &= \frac{5a - \alpha_1\beta_3 + 6\alpha_2\beta_3}{10\alpha_2}, \\ P_{BM} &= \frac{5a + \beta_3(3\alpha_1 + 2\alpha_2)}{10}, \end{aligned}$$

where P_{BM} is higher than $\alpha_2 p_{bm}^*$. Moreover, output of the independent firm is given by

$$x_2^* = \frac{2(\alpha_1 - \alpha_2)\beta_3}{5} > 0.$$

Proof. Substituting eq. (7) into eq. (5) and eq. (6) yields,

$$\begin{aligned} x_I^* &= \frac{a - 2\alpha_1\beta_3 + \alpha_2 p_{bm}^*}{3} = \frac{a - 2\alpha_1\beta_3 + \alpha_2 \frac{5a - \alpha_1\beta_3 + 6\alpha_2\beta_3}{10\alpha_2}}{3} \\ &= \frac{5a - 7\alpha_1\beta_3 + 2\alpha_2\beta_3}{10}, \end{aligned}$$

and

$$x_2^* = \frac{a + \alpha_1\beta_3 - 2\alpha_2 p_{bm}^*}{3} = \frac{a + \alpha_1\beta_3 - 2\alpha_2 \frac{5a - \alpha_1\beta_3 + 6\alpha_2\beta_3}{10\alpha_2}}{3} = \frac{2(\alpha_1 - \alpha_2)\beta_3}{5} > 0,$$

where the inequality comes from Assumption 1 and x_2^* is equal to zero at $\alpha = \alpha_1/\alpha_2 = 1$. Then, it follows from eq. (1), x_I^* and x_2^* that we have

$$P_{BM} = a - x_I^* - x_2^* = \frac{5a + \beta_3(3\alpha_1 + 2\alpha_2)}{10}. \quad (8)$$

As noted before, firm I has cost advantage over its downstream competitor. However, it was stated in Assumption 1 that the competitor, firm 2, has more efficient technology. Then, consider whether this cost advantage is enough for firm I to drive firm 2 out of the market. Using P_{BM} and p_{bm}^* derived above, we get:

$$P_{BM} - \alpha_2 p_{bm}^* = \frac{2\beta_3(\alpha_1 - \alpha_2)}{5} > 0.$$

where the sign comes from Assumption 1 and the equality holds at $\alpha = 1$. Thus, both the integrated firm and the independent firm can make positive profits and they can be viable. \square

Noting these, the model shows that the vertical merger has two distinct effects on market price: cost effect and efficiency effect. The former comes from the fact that the merged firm can get access to inputs at lower costs, which in turn causes lower price in a downstream market. This is the cost effect, which is due to the cancellation of the double marginalization. The efficiency effect is due to the fact that the merger causes market prices to go down if the downstream division is more productive than firm 1 and causes the prices to go up otherwise. This effect works for the merged firm in the former case ($\alpha \leq 1$). If $\alpha > 1$ (the merged firm has less efficient production technologies relative to firm 1), the efficiency effect works against the merged firm.

Lemma 1 states that although the integrated firm has cost advantages over the independent, the latter can be viable after integration in the monopoly game if the latter has superior production technologies. Consequently, the output of the independent firm is positive and the firm can make positive profits as long as the independent has more efficient technologies. It follows from *Lemma 1* that the downstream rival can not be foreclosed from a market if the efficiency effect works against the merged firm, but that the competitor can be excluded from a market if the efficiency effect is inactive or works for the integrated firm.

Our conclusions are mainly dependent on Assumptions 1 and 2. If either of these two assumptions fails, market structure changes into pure monopoly. If Assumption 1 does not hold, there does not exist competitors in the downstream market. It is interesting to note that the integrated firm can foreclose the rival from the market, even if its downstream division has the same productivity as the rival does.

If $\alpha_1 = \alpha_2 = 1$, one unit of essential inputs produces one unit of outputs. For example, MS Windows and IE enable us to enjoy internet services, and the combination of MS Windows and Netscape Navigator provides us with the same services. Thus, when $\alpha_1 = \alpha_2 = 1$, the integration causes the monopoly game to change into a game of tying by products of an upstream monopolist and firm 1. Generally, tied sales emerge when $\alpha_1 = \alpha_2$. In fact, Scherer (1980) pointed out that ties have similar effects as vertical integration by a firm to exercise power over price of the tied market. Following Scherer (1980) and the arguments above, the analysis of tie-in sales can be made in our model in which $\alpha_1 = \alpha_2$. In our model, it will be assumed away that the upstream monopolist has incentives to engage in tying. The focus here is on what effect tying has.

Proposition 1. *Tying enables the integrated upstream monopolist to foreclose a competitor from a downstream market.*

Proof. Noting that tying is equivalent to the condition that $\alpha_1 = \alpha_2$, our proof goes as follows.

In the proof of *Lemma 1*, the output of firm 2 is given by

$$x_2^* = \frac{2(\alpha_1 - \alpha_2)\beta_3}{5}.$$

In the present model, tie-in sales are considered as $\alpha_1 = \alpha_2$. This in turn causes x_2^* to become zero. □

These results can reveal some of the important aspects of tying by an upstream monopolist such as MS. From the viewpoint of our model, companies offering application software which enables us to enjoy internet service are sure to be foreclosed. In fact, Netscape's Navigator web browser was foreclosed in the U.S. and Ichitaro has been foreclosed by MS Word in Japan. As mentioned in the Introduction, the presence of a Choice Screen that displays a list of 12 different web browsers has resulted in the shrinkage of MS IE dominance in 2009. More freedom of choice of browsers has caused MS IE to lose market share. These observations are consistent with our *Proposition 1*.

3 One-sided Vertical Integration in Duopoly Game

There is a long history of whether the vertical integration matters because it may cause market foreclosure of rival firms. Two controversial issues have been examined by those concerned with antitrust proceedings and with regulation. One is whether the vertical integration is anticompetitive or not, and the other is whether it promotes market efficiencies. Effects of one-sided vertical integration are examined in this section.

To simplify our analysis, in what follows, consider the duopoly game. In particular, it will be considered whether tie-in sales result in foreclosing a competitor from a market. The point to note is that there is no upstream monopolist in the duopoly game. When we examine effects of vertical integration in the monopoly and the duopoly games, the effects of vertical integration and tie-in in a market will be examined in different market structures.

In the duopoly game, there are two competitors in each of the two markets as shown in Fig. 2. This game is analyzed as a two-stage game with perfect and complete information. In the first stage, upstream firms supply their products to downstream firms. Demand for upstream products is derived from outputs by downstream firms. In the second stage, downstream firms sell their products given the demand function eq. (1) of their product. To solve our two-stage game, we rely on backward induction. It follows that firms in a downstream market play a simultaneous-move (or a static) game given the demand function eq. (1), and maximize their respective profits.

Using eqs. (2) and (3), profits of firm i are given by

$$\pi_i = (P - c_i)x_i = (a - \alpha_i p - (x_1 + x_2))x_i, \quad i = 1, 2,$$

where fixed costs are assumed away because they will not play any role in the analysis to follow. The objective of each firm is to maximize π_i , which requires

$$\frac{\partial \pi_1}{\partial x_1} = a - \alpha_1 p - 2x_1 - x_2 = 0$$

and

$$\frac{\partial \pi_2}{\partial x_2} = a - \alpha_2 p - x_1 - 2x_2 = 0.$$

Solving these equations for equilibrium outputs yields

$$\begin{aligned} x_1^* &= \frac{a - (2\alpha_1 - \alpha_2)p}{3}, \\ x_2^* &= \frac{a + (\alpha_1 - 2\alpha_2)p}{3}, \end{aligned}$$

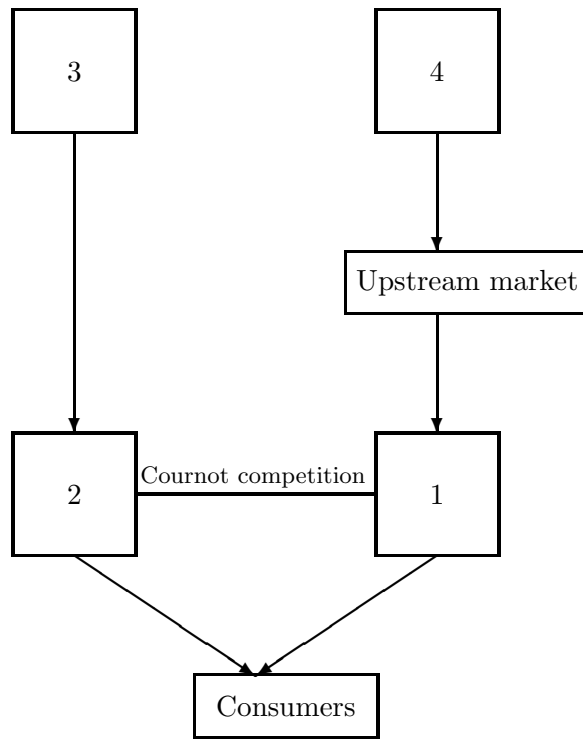


Figure 2: The duopoly game

where the outputs of both firms are assumed positive. Total equilibrium output X^* is

$$X^* = x_1^* + x_2^* = \frac{2a - (\alpha_1 + \alpha_2)p}{3}.$$

Demand for inputs is derived from outputs in the downstream market. In view of the production function of firms in this market, total equilibrium demand Y for inputs is given by

$$Y = \alpha_1 x_1^* + \alpha_2 x_2^* = \frac{a(\alpha_1 + \alpha_2) + 2p(\alpha_1 \alpha_2 - \alpha_1^2 - \alpha_2^2)}{3}.$$

Solving this for p yields inverse demand for inputs, which is given by

$$p = \frac{3Y - a(\alpha_1 + \alpha_2)}{2(\alpha_1 \alpha_2 - \alpha_1^2 - \alpha_2^2)}. \quad (9)$$

Given the demand for inputs, the two firms play a simultaneous-move game in the upstream market. Under these assumptions, upstream firms supply inputs to an upstream market. Then,

$$Y = y_3 + y_4.$$

It follows from this and eq. (9) that profit of firm 3 is

$$\pi_3 = (p - \beta_3)y_3 = \left(\frac{3Y - a(\alpha_1 + \alpha_2)}{2(\alpha_1 \alpha_2 - \alpha_1^2 - \alpha_2^2)} - \beta_3 \right) y_3 = \left(\frac{3(y_3 + y_4) - a(\alpha_1 + \alpha_2)}{2(\alpha_1 \alpha_2 - \alpha_1^2 - \alpha_2^2)} - \beta_3 \right) y_3.$$

where β_3 is the constant marginal cost of firm 3 and y_i is the inputs of firm i , $i = 3, 4$. Differentiating profits with respect to y_3 yields

$$\frac{\partial \pi_3}{\partial y_3} = \frac{6y_3 + 3y_4 - a(\alpha_1 + \alpha_2)}{2(\alpha_1\alpha_2 - \alpha_1^2 - \alpha_2^2)} - \beta_3 = 0.$$

Similarly, we have

$$\pi_4 = (p - \beta_4)y_4 = \left(\frac{3Y - a(\alpha_1 + \alpha_2)}{2(\alpha_1\alpha_2 - \alpha_1^2 - \alpha_2^2)} - \beta_4 \right) y_4 = \left(\frac{3(y_3 + y_4) - a(\alpha_1 + \alpha_2)}{2(\alpha_1\alpha_2 - \alpha_1^2 - \alpha_2^2)} - \beta_4 \right) y_4,$$

where β_4 is the constant marginal cost of firm 4. Differentiating π_4 with respect to y_4 and equating the derivative to zero gives

$$\frac{\partial \pi_4}{\partial y_4} = \frac{3y_3 + 6y_4 - a(\alpha_1 + \alpha_2)}{2(\alpha_1\alpha_2 - \alpha_1^2 - \alpha_2^2)} - \beta_4 = 0.$$

Solving these equations for y_3 and y_4 , we get the equilibrium values of inputs supplied by upstream firms:

$$\begin{aligned} y_3^* &= \frac{a(\alpha_1 + \alpha_2) + 2(\beta_4 - 2\beta_3)(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}{9} \\ y_4^* &= \frac{a(\alpha_1 + \alpha_2) + 2(\beta_3 - 2\beta_4)(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}{9}, \end{aligned}$$

which are both positive in view of Assumption 1, eq. (4) and Assumption 2. In fact, it follows from Assumption 1, eq. (4) and Assumption 2 that the numerator of y_3^* is reduced to

$$\begin{aligned} & 2(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(\beta_4 - 2\beta_3) + a(\alpha_1 + \alpha_2) \\ & \geq 2(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(\beta_4 - 2\beta_3) + 4\alpha_1\beta_3(\alpha_1 + \alpha_2) \\ & > 2(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(-2\beta_3) + 4\alpha_1\beta_3(\alpha_1 + \alpha_2) \\ & = \beta_3(-4\alpha_1^2 + 4\alpha_1\alpha_2 - 4\alpha_2^2 + 4\alpha_1^2 + 4\alpha_1\alpha_2) \\ & = 4\alpha_2^2\beta_3(2\alpha - 1) > 0, \end{aligned}$$

where the inequality is due to Assumption 1.

The numerator of y_4^* is reduced to

$$\begin{aligned} & 2(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(\beta_3 - \beta_4 - \beta_4) + a(\alpha_1 + \alpha_2) \\ & > 2(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(-2\beta_4) + 4\alpha_1\beta_4(\alpha_1 + \alpha_2) \\ & = 4\beta_4(-\alpha_1^2 + \alpha_1\alpha_2 - \alpha_2^2 + \alpha_1^2 + \alpha_1\alpha_2) \\ & = 4\beta_4(2\alpha_1\alpha_2 - \alpha_2^2) \\ & = 4\alpha_2^2\beta_4(2\alpha - 1) > 0, \end{aligned}$$

where again the inequality is due to Assumption 1.

Then, the total demand for inputs is

$$Y^* = y_3^* + y_4^* = \frac{2(\alpha_1\alpha_2 - \alpha_1^2 - \alpha_2^2)(\beta_3 + \beta_4) + 2a(\alpha_1 + \alpha_2)}{9}.$$

The equilibrium price p_N^* of inputs is given by substituting Y^* into eq. (9), and it is

$$p_N^* = \frac{a(\alpha_1 + \alpha_2) + 2(\beta_3 + \beta_4)(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}{6(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}.$$

It is easy to show that price p_N^* is higher than β_4 . In fact,

$$p_N^* - \beta_4 = \frac{a(\alpha_1 + \alpha_2) + 2(\beta_3 - 2\beta_4)(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}{6(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)},$$

where the denominator is positive under Assumption 1 and the numerator is positive because it is the same as that of y_4^* in the duopoly game.

Next, consider if x_1^* is positive or not at $p = p_N^*$. Substituting p_N^* into x_1^* yields

$$\begin{aligned} x_1^* &= \frac{a - (2\alpha_1 - \alpha_2)p_N^*}{3} = \frac{a - (2\alpha_1 - \alpha_2)\frac{a(\alpha_1 + \alpha_2) + 2(\beta_3 + \beta_4)(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}{6(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}}{3} \\ &= \frac{6a(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2) - (2\alpha_1 - \alpha_2)(a(\alpha_1 + \alpha_2) + 2(\beta_3 + \beta_4)(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2))}{18(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}. \end{aligned}$$

To simplify our calculations, consider the following:

$$\begin{aligned} &6a(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2) - (2\alpha_1 - \alpha_2)(a(\alpha_1 + \alpha_2) + 2(\beta_3 + \beta_4)(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)) \\ &= a(4\alpha_1^2 - 7\alpha_1\alpha_2 + 7\alpha_2^2) - 2(2\alpha_1 - \alpha_2)(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(\beta_3 + \beta_4) \\ &> 4\alpha_1\beta_4(4\alpha_1^2 - 7\alpha_1\alpha_2 + 7\alpha_2^2) - 8\alpha_1(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)\beta_4 = 4\alpha_1\beta_4(2\alpha_1^2 - 5\alpha_1\alpha_2 + 5\alpha_2^2) \\ &= 4\alpha_1\beta_4(2\alpha^2 - 5\alpha + 5)\alpha_2^2 > 0, \end{aligned}$$

where the last inequality is due to Assumption 1. It then follows from these calculations that x_1^* is positive. Similar calculations will yield the result that x_2^* is positive.

In view of the demand function for products in the downstream market, it is easy to show that the equilibrium price P_N in the downstream market is

$$P_N = a - x_1^* - x_2^* = \frac{a + (\alpha_1 - \alpha_2)p_N^*}{3} = \frac{a(7\alpha_1^2 - 4\alpha_1\alpha_2 + 7\alpha_2^2) + 2(\alpha_1^3 + \alpha_2^3)(\beta_3 + \beta_4)}{18(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}, \quad (10)$$

which is larger than $\alpha_1 p_N^*$. In fact, the difference is reduced to

$$\begin{aligned} P_N - \alpha_1 p_N^* &= \frac{a(4\alpha_1^2 - 7\alpha_1\alpha_2 + 7\alpha_2^2) - 2(2\alpha_1^3 - 3\alpha_1^2\alpha_2 + 3\alpha_1\alpha_2^2 - \alpha_2^3)(\beta_3 + \beta_4)}{18(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\ &= \frac{a(4\alpha^2 - 7\alpha + 7) - 2\alpha_2(2\alpha^3 - 3\alpha^2 + 3\alpha - 1)(\beta_3 + \beta_4)}{18(\alpha^2 - \alpha + 1)}. \end{aligned}$$

It is easy to show that the denominator of the difference is positive, but the sign of the numerator is not obvious. However, it will be shown that it is also positive. From Assumptions 1, eq. (4) and Assumption 2, the numerator is given by

$$\begin{aligned}
& a(4\alpha^2 - 7\alpha + 7) - 2\alpha_2(2\alpha^3 - 3\alpha^2 + 3\alpha - 1)(\beta_3 + \beta_4) \\
\geq & 4\alpha_1\beta_4(4\alpha^2 - 7\alpha + 7) - 4\alpha_2\beta_4(2\alpha^3 - 3\alpha^2 + 3\alpha - 1) \\
\geq & 4\alpha_2\beta_4(4\alpha^2 - 7\alpha + 7) - 4\alpha_2\beta_4(2\alpha^3 - 3\alpha^2 + 3\alpha - 1) \\
= & 4\alpha_2\beta_4((4\alpha^2 - 7\alpha + 7) - (2\alpha^3 - 3\alpha^2 + 3\alpha - 1)) \\
= & 4\alpha_2\beta_4(-2\alpha^3 + 7\alpha^2 - 10\alpha + 8) > 0,
\end{aligned}$$

which is positive in view of Assumption 1. Then, P_N is larger than $\alpha_1 p_N^*$. It follows from these arguments that the independent firms can make positive profits and can stay in the up- and down-stream markets.

We can now establish:

Lemma 2. *The equilibrium prices in the upstream and downstream markets are given by*

$$\begin{aligned}
p_N^* &= \frac{a(\alpha_1 + \alpha_2) + 2(\beta_3 + \beta_4)(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}{6(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}, \\
P_N &= \frac{a(7\alpha_1^2 - 4\alpha_1\alpha_2 + 7\alpha_2^2) + 2(\alpha_1^3 + \alpha_2^3)(\beta_3 + \beta_4)}{18(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}.
\end{aligned}$$

Next consider the following duopoly game: If efficient firm 2 integrates with upstream firm 3 as shown in Fig.2, the downstream division of the integrated firm is superior in technologies to a downstream rival, and it can get access to inputs at lower costs than independent firm 4. This game is a benchmark against which effects of the integration are evaluated. In the previous section, effects of vertical integration were examined in the monopoly game. Now, we will focus on effects of the vertical integration on a market in the duopoly game. To examine the features of the integration, consider a game in which efficient firm 2 merges with the more productive firm 3. Thus, the integrated firm is the strongest both in costs and efficiency.

The integrated firm may be able to make profits by supplying outputs produced by using inputs not only from the upstream division, but also from an upstream market. Thus, profits of the integrated firm and the un-integrated firm are expressed as

$$\begin{aligned}
\pi_I &= (P - \alpha_2\beta_3)x_I + (P - \alpha_2p)x_{21} + (p - \beta_3)y_3 \\
&= (a - \alpha_2\beta_3 - x_I - x_{21} - x_1)x_I + (a - \alpha_2p - x_I - x_{21} - x_1)x_{21} + (p - \beta_3)y_3, \\
\pi_1 &= (P - \alpha_1p)x_1 = (a - \alpha_1p - x_I - x_{21} - x_1)x_1,
\end{aligned}$$

where x_I stands for output of the integrated firm produced by through-puts , x_{21} for output of the firm by using inputs purchased in the upstream market and p for input price.

The first order conditions for profit maximization of two firms yield

$$\begin{aligned}\frac{\partial \pi_I}{\partial x_I} &= a - \alpha_2 \beta_3 - 2x_I - x_{21} - x_1 - x_{21} = a - \alpha_2 \beta_3 - 2x_I - 2x_{21} - x_1 = 0, \\ \frac{\partial \pi_I}{\partial x_{21}} &= a - \alpha_2 p - x_I - 2x_{21} - x_1 - x_I = a - \alpha_2 p - 2x_I - 2x_{21} - x_1 = 0, \\ \frac{\partial \pi_1}{\partial x_1} &= a - \alpha_1 p - x_I - x_{21} - 2x_1 = 0.\end{aligned}$$

Generally, β_3 is not equal to p . This means that simultaneous equations above are unsolvable. Moreover, if β_3 is equal to p , the integrated firm can not decide how much inputs are to be purchased from the upstream market and how much is to be provided by its upstream division. This in turn means that demand for inputs is indeterminate and that vertical merger does not play a role. Thus, it is not possible to explore games in which the integrated firm purchases inputs in the upstream market. These games will not be considered in what follows.

Noting arguments above, the demand for inputs comes solely from firm 1. When the integrated firm and firm 1 compete in the downstream market, their profits are

$$\begin{aligned}\pi_I &= (a - \alpha_2 \beta_3 - x_I - x_1)x_I + (p - \beta_3)y_3, \\ \pi_1 &= (a - \alpha_1 p - x_I - x_1)x_1.\end{aligned}$$

The first order conditions for maximum profits are

$$\begin{aligned}\frac{\partial \pi_I}{\partial x_I} &= a - \alpha_2 \beta_3 - 2x_I - x_1 = 0, \\ \frac{\partial \pi_1}{\partial x_1} &= a - \alpha_1 p - x_I - 2x_1 = 0.\end{aligned}$$

Solving these equations for x_I and x_1 , we have

$$\begin{aligned}x_I^* &= \frac{a - 2\alpha_2 \beta_3 + \alpha_1 p}{3}, \\ x_1^* &= \frac{a + \alpha_2 \beta_3 - 2\alpha_1 p}{3},\end{aligned}$$

which are analogous to eqs. (5) and (6). As noted above, derived demand d for inputs by firm 1 is given by

$$d = \alpha_1 x_1^* = \alpha_1 \frac{a + \alpha_2 \beta_3 - 2\alpha_1 p}{3}.$$

Solving this for p yields inverse demand for the inputs, and is given by

$$p = \frac{(a + \alpha_2 \beta_3)\alpha_1 - 3d}{2\alpha_1^2}. \quad (11)$$

Given this demand for the inputs, the upstream division and independent firm 4 supply the inputs to the downstream firms, and their profits are

$$\begin{aligned}
\pi_I &= (a - \alpha_2\beta_3 - x_I^* - x_2^*)x_I^* + (p - \beta_3)y_3 = (a - \alpha_2\beta_3 - x_I^* - x_2^*)x_I^* + \left(\frac{(a + \alpha_2\beta_3)\alpha_1 - 3d}{2\alpha_1^2} - \beta_3\right)y_3 \\
&= (a - \alpha_2\beta_3 - x_I^* - x_2^*)x_I^* + \left(\frac{(a + \alpha_2\beta_3)\alpha_1 - 3(y_3 + y_4)}{2\alpha_1^2} - \beta_3\right)y_3, \\
\pi_4 &= (p - \beta_4)y_4 = \left(\frac{(a + \alpha_2\beta_3)\alpha_1 - 3d}{2\alpha_1^2} - \beta_4\right)y_4 = \left(\frac{(a + \alpha_2\beta_3)\alpha_1 - 3(y_3 + y_4)}{2\alpha_1^2} - \beta_4\right)y_4, \\
d &= y_3 + y_4.
\end{aligned}$$

Substituting x_I^* , x_2^* , and p derived above into π_I and π_4 , the first order condition for maximum profit yields

$$\frac{\partial \pi_I}{\partial y_3} = -\frac{5y_3 + 2(y_4 + \alpha_1(\alpha_1 - \alpha_2)\beta_3)}{2\alpha_1^2} \leq 0, \text{ for } y_3, y_4 \geq 0,$$

where the inequality is due to Assumption 1. It follows from the Kuhn-Tucker conditions that optimal supply \hat{y}_3 of inputs to firm 3 is equal to 0. Thus, the integrated firm alone produces throughputs and does not supply inputs to the upstream market. It follows from these arguments that firm 4 is the sole supplier of inputs, which are demanded by the downstream independent firm. Firm 4 maximizes its profits given the derived demand for inputs. Then, we have

$$\frac{\partial \pi_4}{\partial y_4} = \frac{-6y_4 + \alpha_1(a + \alpha_2\beta_3 - 2\alpha_1\beta_4)}{2\alpha_1^2} = 0,$$

where $(a + \alpha_2\beta_3 - 2\alpha_1\beta_4)$ is positive because of Assumption 2. Solving this equation for y_4 , it is given by

$$\hat{y}_4 = \frac{\alpha_1(a + \alpha_2\beta_3 - 2\alpha_1\beta_4)}{6} > 0,$$

where the inequality is due to Assumption 2. Substituting this into eq. (11) and noting that $d = y_4$, equilibrium price \hat{p} of inputs is determined and then substituting this \hat{p} into x_I^* , x_1^* yields equilibrium outputs \hat{x}_I , \hat{x}_1 of the two firms.

Now we can summarize our arguments above as:

Lemma 3. *If the most efficient firm 2 merges with the most productive upstream firm 3, the one-sided vertically merged firm can maximize profits by not supplying inputs to an upstream market. Equilibrium outputs, prices and inputs of firms in the post-merger game are given by*

$$\begin{aligned}
\hat{y}_3 &= 0, \\
\hat{y}_4 &= \frac{\alpha_1(a + \alpha_2\beta_3 - 2\alpha_1\beta_4)}{6},
\end{aligned}$$

$$\begin{aligned}
\hat{p} &= \frac{a + \alpha_2\beta_3 + 2\alpha_1\beta_4}{4\alpha_1}, \\
\hat{x}_I &= \frac{5a - 7\alpha_2\beta_3 + 2\alpha_1\beta_4}{12}, \\
\hat{x}_1 &= \frac{a + \alpha_2\beta_3 - 2\alpha_1\beta_4}{6}, \\
\hat{P} &= \frac{5a + 5\alpha_2\beta_3 + 2\alpha_1\beta_4}{12}.
\end{aligned}$$

Proof. In the post-merger game, firm 4 maximizes profits by supplying products for the given demand eq. (11) for inputs. Note also that $d = y_4$ because the upstream division does not supply inputs to the upstream market. Then, profits of firm 4 are given by

$$\pi_4 = (p - \beta_4)y_4 = \left(\frac{\alpha_1(a + \alpha_2\beta_3) - 3d}{2\alpha_1^2} - \beta_4 \right) y_4,$$

Solving the first order condition for y_4 yields

$$\hat{y}_4 = \frac{\alpha_1(a + \alpha_2\beta_3 - 2\alpha_1\beta_4)}{2\alpha_1^2}.$$

Substituting this into eq. (11), we have

$$\hat{p} = \frac{a + \alpha_2\beta_3 + 2\alpha_1\beta_4}{4\alpha_1}.$$

It is easy to check that $\hat{p} > \beta_4$ under Assumption 2. In fact,

$$\hat{p} - \beta_4 = \frac{a + \alpha_2\beta_3 + 2\alpha_1\beta_4}{4\alpha_1} - \beta_4 = \frac{a + \alpha_2\beta_3 - 2\alpha_1\beta_4}{4\alpha_1} > 0.$$

As \hat{y}_4 is positive, profits of firm 4 are positive and firm 4 can supply inputs to the upstream market. Substituting \hat{p} into eqs. (5) and (6) yields

$$\begin{aligned}
\hat{x}_I &= \frac{a - 2\alpha_2\beta_3 + \alpha_1\hat{p}}{3} = \frac{5a - 7\alpha_2\beta_3 + 2\alpha_1\beta_4}{12} > 0, \\
\hat{x}_1 &= \frac{a + \alpha_2\beta_3 - 2\alpha_1\hat{p}}{3} = \frac{a + \alpha_2\beta_3 - 2\alpha_1\beta_4}{6} > 0.
\end{aligned}$$

Finally, together with eq. (1), equilibrium price in the downstream market is

$$\hat{P} = a - \hat{x}_I - \hat{x}_2 = \frac{5a + 5\alpha_2\beta_3 + 2\alpha_1\beta_4}{12}.$$

It is easy to show that \hat{P} is larger than $\alpha_1\hat{p}$. In fact, per-units profits ($\hat{P} - \alpha_1\hat{p}$) are

$$\hat{P} - \alpha_1\hat{p} = \frac{5a + 5\alpha_2\beta_3 + 2\alpha_1\beta_4}{12} - \alpha_1 \frac{a + \alpha_2\beta_3 + 2\alpha_1\beta_4}{4\alpha_1} = \frac{a + \alpha_2\beta_3 - 2\alpha_1\beta_4}{6} > 0,$$

where the sign comes from Assumption 2. This means that firm 1 can reap positive profits and sell outputs in the downstream market because equilibrium output \hat{x}_1 of firm 1 is positive. \square

It is of some interest to note that the vertical integration does not exclude a rival firm from a market in spite of the fact that the integrated firm is the most competitive because the downstream division (firm 2) is more productive in view of Assumption 1 and the upstream division (firm 3) has advantages in costs in view of (4). Thus, these results are in a striking contrast with the effects of the vertical integration by the upstream monopolist, who can foreclose a rival from a market even if the downstream division does not have advantages in production technologies (see, for example, *Lemma 1*). In other words, even if the cost effect and the efficiency effect work for the integrated firm, a downstream rival can be viable in a duopoly game. On the other hand, the cost effect is so strong in the monopoly game that a rival has to exit from the market even if the efficiency effect is inactive. However, as was shown above, even if both the cost and efficiency effects do work for the duopolist, the integration does not enable the upstream duopolist to exclude a rival from the market. Thus, the strength of the cost effect depends upon upstream market structure.

Moreover, the firm can reap the maximum profits by not supplying inputs to a market because $\hat{y}_3 = 0$ in *Lemma 3*. As the downstream division does not purchase inputs in the upstream market, this result is similar to the Salinger (1988) model, where the integrated firms do not participate in an upstream market.⁵ This reminds us of “keiretu” in Japanese manufacturing industries. For example, Toyota has purchased inputs (or parts) solely from its subcontractors and so has Nissan. The subcontractors of Toyota have not supplied their products to Nissan and *vice versa*, though the situations have recently been changing. Our results above may be similar to these situations. The model explains situations of “keiretu”, in which parties to the contract are not allowed to trade with firms not concerned with the contract. If firm 1 integrates with upstream firm 3, it will be shown that the integrated firm may maximize profits by not supplying inputs to the upstream market.⁶

There are two important controversial questions on effects of the vertical merger on a market: is it anti-competitive and does it promote market efficiency? The present model can answer these questions. We first examine if it is anti-competitive or not. Our *Lemma 3* shows that the merger is not anti-competitive in the duopoly game because rival firms are not excluded by the merger. However, it follows from *Lemma 1* that the vertical merger enables the upstream monopolist to

⁵Chen and Riordan (2007) observed that vertical integration enables the integrated firm to preempt an upstream independent firm.

⁶ $\frac{\partial \pi_1}{\partial y_3} > 0$ for positive y_3, y_4 . If the best response functions of two firms intersect in the first quadrant, then $\hat{y}_3 > 0$ and the upstream division can maximize profits by supplying inputs to the upstream market. However, calculations similar to previous ones will show that $\hat{y}_3 = 0$.

drive a rival out of a downstream market when an integrated upstream monopoly has equally productive facilities (*i.e.*, $\alpha = 1$). Thus, the answer to the first issue depends crucially upon upstream market structure. However, Chen and Riordan (2007) concluded in *Prop. 2* that the answer is independent of upstream market structure.

Second, it is possible to examine whether the one-sided vertical merger improves market efficiency or not. If the vertical merger improves efficiency in a market, post-merger market prices decline. Usually, it has been considered that the merger confers more monopoly power to the integrated firm and enables the firm to control a market. Then, the merger results in aggravating market efficiency. However, it is not easy to show which is correct. We can be more precise than this:

Proposition 2. *One-sided vertical integration of firm 2 and 3 is neutral in competitiveness, but it does promote market efficiency.*

Proof. As shown in *Lemma 3* and the argument above, firm 4 can reap positive profits by supplying inputs in the upstream market. Moreover, it will be shown that firm 1 can be viable in the downstream market. Profits of firm 1 are given by $\hat{x}_1 \times (\hat{P} - \alpha_1 \hat{p})$, where \hat{x}_1 is positive in view of *Lemma 3*. Per-units profits $(\hat{P} - \alpha_1 \hat{p})$ were shown in *Lemma 3* to be positive.

Thus, the independent firm can be viable in the post-merger game. The one-sided merger is neutral from the viewpoint of the competitiveness of a market.

It follows from *Lemmas 2* and *3* that we can compare equilibrium prices in pre- and post-merger game:

$$\begin{aligned}
P_N - \hat{P} &= \frac{a(7\alpha_1^2 - 4\alpha_1\alpha_2 + 7\alpha_2^2) + 2(\alpha_1^3 + \alpha_2^3)(\beta_3 + \beta_4)}{18(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} - \frac{5a + 5\alpha_2\beta_3 + 2\alpha_1\beta_4}{12} \\
&= \frac{-(\alpha_1^2 - 7\alpha_1\alpha_2 + \alpha_2^2)a + (\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(-6\alpha_1\beta_4 - 15\alpha_2\beta_3 + 4(1 + \alpha)\alpha_2(\beta_3 + \beta_4))}{36(1 - \alpha + \alpha^2)} \\
&= \frac{-(\alpha_1^2 - 7\alpha_1\alpha_2 + \alpha_2^2)a + (\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(-6\alpha_1\beta_4 - 15\alpha_2\beta_3 + 4\alpha_2(\beta_3 + \beta_4) + 4\alpha_1(\beta_3 + \beta_4))}{36(1 - \alpha + \alpha^2)} \\
&= \frac{-(\alpha_1^2 - 7\alpha_1\alpha_2 + \alpha_2^2)a + (\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(-11\alpha_2\beta_3 + 4\alpha_1\beta_3 + 4\alpha_2\beta_4 - 2\alpha_1\beta_4)}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\
&\geq \frac{-(\alpha_1^2 - 7\alpha_1\alpha_2 + \alpha_2^2)a + (\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(4\alpha_1\beta_3 - 4\alpha_1\beta_3 - 7\alpha_2\beta_3 + 2\beta_4(2\alpha_2 - \alpha_1))}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\
&\geq \frac{-(\alpha_1^2 - 7\alpha_1\alpha_2 + \alpha_2^2)a + (\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(4\alpha_1\beta_3 - 4\alpha_1\beta_3 - 7\alpha_1\beta_4 + 2\beta_4(2\alpha_2 - \alpha_1))}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\
&\geq \frac{-4\alpha_1\beta_4(\alpha_1^2 - 7\alpha_1\alpha_2 + \alpha_2^2) - 7\alpha_2\beta_3(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\
&= \frac{\alpha_1\beta_4(28\alpha_1\alpha_2 - 4\alpha_1^2 - 4\alpha_2^2 - 7\alpha_1^2 + 7\alpha_1\alpha_2 - 7\alpha_2^2)}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}
\end{aligned}$$

$$\begin{aligned}
&= \frac{\alpha_1\beta_4(35\alpha_1\alpha_2 - 11\alpha_1^2 - 11\alpha_2^2)}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\
&= \frac{\alpha_1\beta_4(35\alpha - 11\alpha^2 - 11)}{36(1 - \alpha + \alpha^2)} > 0,
\end{aligned}$$

where the inequality comes from Assumptions 1 and 2. The calculations above show that equilibrium price is higher in the pre-merger game than in the post-merger game: $P_N > \hat{P}$. Equilibrium price goes down after the merger. This means that the merger promotes market efficiency. \square

It was shown in Yang and Kawashima (2008) that in the monopoly and the duopoly games, the vertical integration causes equilibrium downstream prices to go down. Together with this proposition, whether the upstream market is a monopoly or not, the answer to the second question is that the integration results in higher market efficiency.

Noting that tying is equivalent to $\alpha_1 = \alpha_2$ in this model, we can examine effects of tying by using our model. When α_1 is equal to α_2 , \hat{x}_2 , $(\hat{p} - \beta_4)$ and $(\hat{P} - \alpha_1\hat{p})$ are positive, and then firm 2 can reap positive profits. In fact, it follows from *Lemma 3* that for $\alpha_1 = \alpha_2$,

$$\begin{aligned}
\hat{x}_2 &= \frac{a + \alpha_2\beta_3 - 2\alpha_2\beta_4}{6} = \frac{a + \alpha_1(\beta_3 - \beta_4)}{6} > 0, \\
\hat{p} - \beta_4 &= \frac{a + \alpha_2\beta_3 + 2\alpha_1\beta_4}{4\alpha_1} - \beta_4 = \frac{a + \alpha_1\beta_3 - 2\alpha_1\beta_4}{4\alpha_1} > 0, \\
\hat{P} - \alpha_1\hat{p} &= \frac{a + \alpha_1\beta_3 - 2\alpha_1\beta_4}{6} > 0,
\end{aligned}$$

where inequalities are due to Assumption 2. Thus, tying is not anti-competitive. Our results are in a stark contrast with those for the merger in the monopoly game. In fact, as *Proposition 1* has shown, the merger enables the integrated upstream monopolist to drive a rival from the market whose downstream division has the same productivity as the independent firm. On the other hand, tied sales by firms in the duopoly game do not drive a rival from a market even if the merged firm has an efficient downstream division. Effects of the merger depend upon market structures in the upstream market.

Now we can establish:

Proposition 3. *Tied sales are anti-competitive in the monopoly game, but not in the duopoly game.*

Tie-in sales are a ubiquitous business practice. For example, in the auto industry a car is sold with an air-conditioner which is usually supplied by subcontracting firms. Thus, this may be regarded as one example of tying. This market is featured by the fact that there is no upstream

monopolist. As with the case of the Japanese auto industry, *Proposition 2* implies that tying a car with an air-conditioner does not foreclose a rival firm from a market. Although the details of the two examples are different from our theoretical model, they illustrate the empirical relevance of our arguments.

4 Conclusions

The present model has been applied to examine conditions under which the vertical integration *per se* causes market foreclosure and when it does not. The integration has two distinct effects on a market: (1) the cost effect which comes from the fact that the integration enables the integrated firm to get access to inputs at lower costs, and (2) the efficiency effect which comes from the fact that the integrated firm owns a downstream division. Both the cost and efficiency effects exert influences on market prices. If an upstream monopolist merges with an equally efficient downstream firm, the monopolist can exploit leverage in the downstream market and exclude the rival from the market. The cost effect (or leverage) is strong enough for the integrated firm to exclude the rival from a market. On the other hand, when the merged firm has advantages in costs and production relative to an independent firm in the duopoly game, market foreclosure does not take place because the cost effect (or leverage) of the duopolist is not strong and hence the integration does not exclude the independent from the market.

Our analysis can also shed some light on tied sales. Tied sales are a special form of the vertical integration. Tying by the upstream monopolist excludes a rival from the downstream market. In fact, MS is a *de facto* upstream monopolist in computer OS market and MS Windows has been sold with IE installed. Consequently, tying by MS has resulted in the foreclosure of Netscape's Navigator web browser. From the viewpoint of welfare, the vertical integration results in lower downstream prices even if the foreclosure occurs. Although it is anti-competitive because the number of rivals is reduced, it nevertheless promotes market efficiency by lowering prices. Thus, there is no trade-off between anti-competitiveness and market efficiency.

In a one-sided vertical integration game, firm 2 and 3 are merged and the upstream division does provide throughputs only to the downstream division. This is similar to "keiretu" of Japanese industries. This is a game in which an efficient upstream firm and an efficient downstream firm merge. Moreover, as mentioned in Footnote 5, our result above is independent of the pair of firms which merge. Our future work will explore games in which the upstream division of the merged firm supplies inputs to the upstream market.

There are several limitations to our model. For example, it was assumed that the upstream and the downstream divisions of the integrated firm have constant returns to scale. If the upstream division exhibits increasing returns to scale, it is probable that the division can maximize profits by supplying inputs to the upstream market. The model assumes a linear demand function. This enabled us to make our model tractable. However, when we try other demand

functions, it may be very difficult to solve the two-stage games even with complete and perfect information because it is necessary to derive an upstream demand from downstream equilibrium outputs. It is easy to solve for downstream equilibrium, but it is hard to solve for upstream equilibrium. Until now we have not found other demand functions which enable us to explicitly solve the two-stage games.

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