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COMPLEXITIES OF DYNAMIC FORESTRY MANAGEMENT POLICIES

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"A 'Public Domain,' once a velvet carpet of rich buffalo-grass and grama, now an illimitable waste of rattlesnake-bush and tumbleweed, too impoverished to be accepted as a gift by the states within which it lies. Why? Because the ecology of the Southwest happened to be set on a hair trigger."

- Aldo Leopold, "The Conservation Ethic," *Journal of Forestry*, 1933, vol. 33, pp. 636-637.

I. Introduction

Forestry management is a problem of increasing controversy and difficulty in many parts of the world. Whereas once the emphasis was simply on cutting trees and replanting them to maximize the present value of their lumber content, now forest managers face multiple demands from the public around the world. Forests are seen as the homes of diverse species that may be hunted or fished, or should be preserved because of the rarity or uniqueness. They provide positive externalities at the local level due to preventing flooding and soil erosion and at the global level because of their role in carbon sequestration. However, carbon sequestration may conflict with biodiversity preservation, and within the latter, preserving one species may imply not preserving another one. In less developed countries a variety of social issues arise in the management of forests from dealing with aboriginal inhabitants to providing land for poor farmers who wish to homestead. Many of these issues involve conflicts between groups and goals that are not easily resolved.

Any effort to resolve these conflicts over the proper management of forests ultimately involves the dynamic ecology of forests. Elements impacting this include the role of fire, the role of pest management, and the methods and techniques of cutting trees when there is harvesting of timber, especially regarding the patch size of the cuts. Considerable experience and literature indicate that a variety of complex dynamics are involved in these latter elements. Multiple equilibria exist with the implied possibility of sudden and discontinuous changes in the nature of a forest, much as described in the opening quotation above from famous ecologist, Aldo Leopold. Deep tradeoffs exist between the local stability of forest ecosystems and their global resilience, tradeoffs that manifest themselves in such contradictions as that efforts to prevent forest fires can make forest fires worse, and efforts to eradicate pests can make their attacks worse and more destructive. This idea that in ecosystems there might exist such a tradeoff has become very widespread and influential, but it was initially due to C.S. Holling (1973) who derived it from the study of forestry ecology.

This paper will review these questions within the general framework of what the complex dynamics of the interaction between forestry ecology and forestry economics imply for forestry management policies. The paper will first review the basic model of optimal forestry rotation, initially from the standpoint of the Faustmann (1849) model of simple timber use of a forest. This will then be modified to account for other uses of the forest as modeled by Hartman (1976), with the possibility of multiple solutions arising in this context due to the different patterns over time of the various uses of the forest. The paper will then consider a series of more specific issues in the dynamics of forestry management, especially the problems of pest and fire management and patch size of cutting (especially the problem of clearcutting), while taking into account the complications introduced by considering hunting, fishing, grazing, biodiversity, carbon sequestration, and social issues.

II. The Basic Model

In the English language tradition, Irving Fisher (1907) proposed that the optimal time to cut a tree for timber use is when its growth rate equals the real rate of interest. However this is true only for the generally unrealistic case where there will be no replanting of the forest after the harvest, and, indeed, there will be no use of the land on which the trees were growing whatsoever after they are cut. The correct solution had been solved within the German language tradition much earlier by Faustmann (1849), who found that if one has an infinite time horizon and plans to replant the forest after harvesting, then the optimal time will be sooner than the Fisher solution implies.[1] This is because the rate of growth of trees decelerates with age, so that cutting sooner means that one can get the more rapidly growing younger trees in place sooner again.

To look at these arguments more closely let us identify the following variables: f(t) will be the growth function of a forest with t being time, and T being the optimal rotation time for a forest; p will be the price of timber, assumed to be constant;[2] r will be the real discount rate, assumed for now to equal a real market rate of interest, and c will be the marginal timber cost (the cost of cutting down the trees). We note that for the forest growth function, f(t), we can expect $f'(t) > 0$ for a considerable period of time, although eventually a forest will cease to grow and f'(t) will eventually become negative. In many forests there will be a shorter period after planting when f''(t) will also be positive, but it will tend to turn negative much sooner than will f'(t). Figure 1 shows a stylized version of a typical tree growth function, with the critical points of this function varying considerably across different tree species.

[insert Figure 1 here]

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[1] Samuelson (1976) provides an account of the historical development of these two approaches. Many other well-known economists advocated the Fisher approach prior to the translation into English of Faustmann's paper in 1968, including Harold Hotelling (1925) and Kenneth Boulding (1935), and in the German language tradition, Faustmann's predecessor, Johann Heinrich von Thünen (1826). However, there were some in English who understood that Fisher was not correct prior to 1968 (Alchian, 1952; Gaffney, 1957).

[2] Constancy of the price is a non-trivial assumption. A considerable literature exists that assumes that the price and other forest values follow stochastic processes, usually some variant of Brownian motion. Option theory using Ito's lemma then is used to provide solutions for optimal control stopping problem for various such models under different assumptions (Reed and Clarke, 1990; Zinkhan, 1991; Reed, 1993; Conrad, 1997; Willassen, 1998; Saphores, 2003). Arrow and Fisher (1974) first suggested the use of option theory to deal with the possibility of irreversible loss of uncertain future forest values.

The Fisher solution for the optimal rotation time, T, that maximizes the present value for the single planting of a forest without reforestation or any valuable alternative use of the land after harvesting in the deterministic case is given by

$$pf'(T) = rpf(T). \tag{1}$$

Removing the price term from both sides gives the simpler and clearer

$$f'(T) = rf(T), \tag{2}$$

which has the famously intuitive interpretation that the forest (or tree) should be cut when its growth rate equals the real market rate of interest, the argument being that after this the forest owner will deposit the proceeds in a safe financial asset and earn the interest.

Allowing for replanting, or an alternative valuable use of the land, shortens the optimal rotation period of time as the owner wishes to replant sooner the more rapidly growing younger trees. Faustmann correctly analyzed this for the infinite time horizon case with a sum of discounted future earnings from the harvesting of the future plantings. This optimal solution for this infinite sum reduces to

$$pf'(T) = rpf(T) + r[(pf(T) - c)/(e^{rT} - 1)]. \tag{3}$$

Given that the additional term on the right hand side can be expected to be positive, this means that the growth rate of the forest will be greater than the real rate of interest at time T, which means the forest should be cut sooner than in the Fisher case given the tendency for forest growth rates to decline after awhile.[3]

The next step in the analysis was due to Hartman (1976) who noted that non-timber amenity values should be taken into account as well, with some of these possibly

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[3] It is possible for there to be multiple solutions to (3) given that forest growth rates initially increase. However any solution when growth rates rise will violate second-order conditions for optimality.

not being marketed, although some might be. If g(t) describes the time pattern of the flow of such amenities, then the Hartman solution for T is given by

$$pf'(T) = rpf(T) + r[(pf(T) - c)/(e^{rT} - 1)] - g(T). \qquad (4)$$

To the extent that g(t) remains positive, this implies an offsetting of the modification provided by Faustmann of the Fisher solution; T will now be greater than for forests whose only value is for timber harvest. Indeed, if g(t) is sufficiently large and remains so as t increases, it may be optimal not to harvest the forest at all and to leave it as a permanently old-growth forest. Clearly what elements are entering into the determination of g(t) is very important, and we turn now to a discussion of this question.

II. Models of Non-Timber Amenity Values

A simple example that avoids the problems of non-marketed amenities can show how taking account of them can introduce nonlinearities that allow for multiple equilibria and various complexities. Swallow, Parks, and Wear (1990) have examined the case of the National Forest in Western Montana. This forest offers cattle grazing opportunities during the early stages of forest growth before the forest canopy covers the grazing areas. This grazing amenity has been estimated to reach a maximum at 12.5 years, with the function taking the following form

$$g(t) = \beta_0 t e^{-\beta_1 t}, \qquad (5)$$

with estimates of the parameter values being $\beta_0 = 1.45$ and $\beta_1 = 0.08$. The peak grazing value is found as $T = 1/\beta_1$. For this case the grazing benefits function can be depicted as Figure 2. When this g(t) is inserted into the Hartman formulation, one gets solutions such as that depicted in Figure 3, with MOC representing the marginal opportunity cost and

MBD representing the marginal benefit of delaying harvest. The global maximum turns out to be at 73 years for this case, which compares with 76 years for the Faustmann solution.[4]

[insert Figure 2]

[insert Figure 3]

Grazing is not the only such non-timber amenity that forests generate, although it is one that has a definite private market value. Other such amenities may involve less clearly marketed phenomena, although in some cases the amenities may be brought to have a market value through appropriate public innovation in institutions managing the forests. Besides grazing, hunting is an activity that may be able to generate income for the private owner of a forest, or possibly a public owner as well, although controlling access to forests by hunters is not always easy, as the long history from the feudal period of aristocrats attempting to control poaching by peasants on their feudal estates demonstrates.[5] In the context of a modern national forest manager such as in the United States, although some of these amenity values may be captured by the state through issuing hunting licenses, most are captured by the hunters themselves. However, the amount of such amenity values have been estimated for many national forests in the U.S. as part of the FORPLAN planning process to determine land use allocation in the U.S. national forests (Johnson, Jones, and Kent, 1980; Bowes and Krutilla, 1985). As well as hunting, fishing is also a non-timber amenity from forests and faces similar questions regarding measuring the scale of its value.

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[4] That such multiple solutions can arise due to non-monotonicity of the time pattern of net benefits for forestry cases was first observed by Porter (1982), who was specifically concerned with the wilderness benefits of forests. Prince and Rosser (1985) have noted the link between such patterns in natural resource use and the reswitching question in capital theory.

[5] It is somewhat ironic that many national forests in Europe were formerly owned by aristocratic families.

Somewhat more difficult to measure but very important in public policy discussions and debates is broader biodiversity that involves species that do not provide a direct use for human beings as do those that are harvested somehow, either as timber or as food as with grazing or as the objects of recreational pursuit and killing as with hunting and fishing.[6] A wide variety of indirect amenities are associated with biodiversity in forests (Perrings, Mäler, Folke, Holling, and Jansson, 1995), some more potentially marketable, such as potential sources of medicines, than others. An especially controversial has involved the preservation of endangered species, with the controversy over preserving the spotted owl in the forests of the northwestern U.S. having been an especially controversial case. In some less developed countries ecotourism has risen as a way of satisfying the conflict between preserving endangered species or habitats and providing for the economic welfare of indigenous communities that use the habitat. More generally other kinds of recreation besides hunting and fishing in forests are sources of amenity values, including purely aesthetic ones such as people viewing beautiful leaves during the fall season. The measurement of some of these amenity values may involve estimation of "existence values" through a variety of methods, many of these highly controversial. These methods of valuation become even more complicated and controversial when they involve traditional populations in rain forests in poorer nations (Gram, 2001).

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[6] Of course in less developed countries hunting and fishing in forests by aboriginal or more generally poor people may well be a major source of their food and not a matter of recreation at all. Kant (2000) provides a discussion of how to integrate the broader socio-economic problems facing traditional communities into an intertemporal optimization framework for forestry management. Especially for aboriginal populations in tropical rain forests, questions of cultural survival enter in as well as more standard ones of economic development or environmental sustainability. For discussions of the coevolution of culture and ecology in tropical rain forests see Norgaard (1981, 1994).

Yet another source of non-timber amenities that has increasingly attracted attention is that of carbon sequestration and oxygen generation, with the former viewed as more critical given the problem of global warming, although again the exact value of this amenity is difficult to measure and very controversial. Although there are some distinct complications in the time patterns involved, it appears that in most forests the amenity value of carbon sequestration tends to rise with the length of the rotation period (Alig, Adams, and McCarl, 1998). A crucial aspect of this is that when a forest is cut for timber there tends to be a substantial release of carbon back into the atmosphere. Indeed, the carbon sequestration amenity value can continue to increase even after the forest has not only stopped growing but actually begun to decline. Thus, in contrast to the grazing example considered above, considering carbon sequestration tends to lengthen the time of an optimal rotation within the Hartman framework.

A somewhat complicated question arises regarding the relationship between carbon sequestration and biodiversity. It turns out that this very much depends on the forest and also the nature of the reforestation (or afforestation) policies after cutting down the forest for timber harvest purposes occurs. Many observers argue that biodiversity tends to increase with age of forests and thus goes hand in hand with carbon sequestration and that in some areas forests of tree species that support more biodiversity are also better at carbon sequestration as with the longleaf pine native to the U.S. Southeast now largely replaced by the more rapidly growing loblolly pine (Alavalapati, Stainback, and Carter, 2002). Furthermore longer lasting forests with more carbon sequestration may also provide external benefits in terms of reduced soil erosion, flooding, and other environmental benefits (Plantinga and Wu, 2003). However, if reforestation policies involve replacing multi-species and biodiverse forests with mono-species and less biodiverse ones, then a policy oriented towards carbon sequestration may conflict with one oriented toward preserving biodiversity (Caparrós and Jacquemont, 2003).

However, more detailed analysis of some forests as carried out by the FORPLAN process in the United States has revealed that in some areas some of the generalizations listed above do not hold. What is clear is that for a given forest type, the patterns of these non-timber amenities may vary in a much more complicated manner over time as the succession process within a given forest type proceeds. Thus in the deciduous forests found in the George Washington National Forest in Virginia and West Virginia, the patterns of biodiversity and the patterns of hunting and other amenities follow quite a complicated pattern.[7]

The initial pattern after a clearcut in terms of huntable wild animal populations is for deer to reach a maximum population in the neighborhood of five to ten years afterwards. This essentially resembles the pattern for grazing in Western Montana, and what is involved is the deer doing especially well on the edges of areas that have been clearcut at this time afterwards. There is plenty of food for the deer but not so much cover that they cannot do well. In the George Washington National Forest, deer hunting is by far the most popular and thus weighs quite heavily in this amenity value.

A second peak of non-timber amenity value arrives at around 25 years after a clearcut. At this point aggregate biodiversity in terms of the sheer number of different species living in the forest is maximized. This contradicts the widespread view that

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[7] The source of this information is from the author's own unpublished work on the FORPLAN model developed for use in managing the George Washington National Forest in 1980.

biodiversity is maximized in old growth forests.[8] At this point the initial growth of oak trees has reached a substantial level, but new species of trees, such as maples, have begun to grow also, and there is a great deal of general undergrowth, the latter especially important. In the George Washington National Forest the hunted species that are especially prevalent at this time tend to wild turkeys and grouse.

Finally there is a later maximum that emerges after about 60 years for bear populations. Bears do well in forests with large fallen-down trees. These older forests have generally lost much of their undergrowth and also have less variety of tree species. Hence they are lower in overall biodiversity, but of course there are people who desire to hunt bears and others who view bears as especially valuable for more general reasons, some of these possibly irrational.[9] Figure 4 depicts the pattern just described for the George Washington National Forest for its non-timber amenities, which suggests a more complicated set of possible multiple equilibria than observed for the Western Montana case. Needless to say, it is unsurprising that the Supervisor of the George Washington National Forest would complain that his most difficult problem was adjudicating between those who wished to hunt deer (and thus supported road building and clearcutting) and those who wished to hunt bears (and thus supported preserving old growth forests), with both groups being heavily armed and vigorous in their presentations.

[insert Figure 4]

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[8] What is true is that in many forests certain endangered species fare well in old growth forests, the spotted owl example in the northwestern forests of the U.S. being a famous example, even if in some of these cases there is less aggregate biodiversity. To the extent that old growth forests become rare, this fact alone can increase the amenity value of the few species that thrive only in such forests.

[9] It is well known that surveys of the public show much greater willingness to pay to preserve large mammals than other species, especially ones perceived to be “cuddly,” which is certainly the case for bears (Weitzman, 1992). It is no accident that the World Wild Fund uses Panda bears as a symbol in their fundraising activities, although Pandas are not technically bears but more closely related to raccoons.

IV. Management Problems Beyond Rotation of Timber Harvest

A. Fire Rotation

Whereas previously we have considered the problem of optimal rotation of timber harvesting while accounting for non-timber amenity values, now we consider an alternative rotation scheme. This involves the use of fire to maximize the persistence of endangered species whose populations peak in a midsuccessional stage of a forest ecosystem, much as the wild turkey and grouse in the previous discussion. One such example is eastern bristlebirds in the U.S. (Pyke, Saillard, and Smith, 1995) with Johnson (1992) and Whelan (1995) providing more extended discussions and cases. Stochastic dynamic programming has been used to study optimal fire rotation systems by Possingham and Tuck (1997) and by Clark and Mangel (2000).

Clark and Mangel (pp. 176-181) consider an endangered population wherein habitat quality is given by q(t) since the time of the last fire, r is the litter size, s_a is the probability that an adult survives in the absence of fire, s_j is the probability that a juvenile survives in the absence of fire, and N(t) is the adult population in time t since a fire. The population equation after a fire then becomes

$$N(t+1) = [s_a + s_j r q(t)]N(t). \qquad (6)$$

Letting r = 2, $s_a = 0.7$, $s_j = 2$, with q(t) reaching a maximum between five and ten years, Clark and Mangel (p. 178) find the trajectory of average population to follow that shown in Figure 5.

[insert Figure 5]

Letting a fitness parameter, f, be the percent of the population (both adult and juvenile) that survives a burn, and letting f = 0.8, Clark and Mangel study a case over time of 20 years and population that reaches a maximum of 50 but which must kept at least as great as 3. They compute at each time t the maximum probability of the survival of the species based on starting a fire or not starting a fire for a given population size. Their solution is depicted in Figure 6, which divides the time-population space into zones of starting a fire or not starting a fire Clark and Mangel, p. 181).

[insert Figure 6]

The problem of fire rotation and management has become highly controversial among forest managers within the United States, with the issues going well beyond those of preserving endangered species and involving the costs of the fires themselves and their degree of general destructiveness. Muradian (2001) argues that the relationship between fire frequency and vegetative density is one of multiple states, allowing for the possible of catastrophic dynamics. This follows rather closely the argument of Holling (1973) regarding the tradeoff between resilience and stability. Thus, traditionally policy in the U.S. was to attempt to fight all fires that appeared on national forest or park lands. However, the truly catastrophic fires that have broken out in several national parks, most famously in Yellowstone Park during the 1990s, have made policymakers aware that not allowing any fires at all leads to a dangerous accumulation of underbrush and dead branches and trunks that can lead to a much greater fire when one finally breaks out. The short term stability of fighting all fires leads to the longer term decline in resilience of the forest to catastrophic fire. So a new policy of actively starting fires to maintain resiliency in the forests has been adopted, although this has also become controversial since one of these got out of control in Arizona and ended up destroying property.

B. Pest Management

Holling (1965, 1973) initially posed his hypothesis mentioned above after contemplating the dynamics of spruce-budworm outbreaks in western Canadian coniferous forests. Such outbreaks occur in a fairly regular pattern approximately every 40 years or so. Among those looking at this have included May (1977), Ludwig, Jones, and Holling (1978), with Casti (1989) and Rosser (1991) putting the argument into an explicit context of catastrophe theory. What is involved is essentially a three-level predator-prey model, with the budworms feeding on the tree leaves, which grow larger as the trees grow larger, and migratory birds limiting the budworm population by preying on them. The trigger mechanism for the periodic outbreak is that there is an upper limit to the ability of the birds to concentrate in the trees, while the budworms can keep increasing with the leaf size. So, as the bird population becomes limited at a crucial level, they cease being able to limit the budworms which then break out into a rapid increase that in turn triggers a crash in the tree population and thus also a subsequent crash in the budworm population.

Following Ludwig, Jones, and Holling, let B equal the budworm population, r_B equal to the natural growth rate of the budworms, K_B equal the budworm carrying capacity (determined by the amount of leaves), α = the predator saturation parameter (a proportion of the budworm carrying capacity), and β equal the maximum rate of predation by the birds upon the budworms. The budworm dynamics in their early stages are given by

$$dB/dt = r_B B(1 - B/K_B) - \beta B^2/(\alpha^2 + B^2). \qquad (7)$$

Nonzero equilibria are solutions of

$$(r_B K_B/\beta) = u^*/[(\alpha/K_B)^2 + u^{*2})(1 - u^*). \qquad (8)$$

This set of solutions is depicted in Figure 7, with the zone of multiple equilibria and associated catastrophic hysteresis loops being that of an infected forest.

[insert Figure 7]

Holling in particular (1986) has discussed at length policy responses to this problem. Whereas many policymakers are inclined to spray the budworms, this tends to happen to late in the cycle. When they have become visible they are already in the epidemic phase and spraying simply holds the system in that very unstable state. Holling focuses on the larger system, especially the idea of encouraging greater bird population to constrain the budworms. This led him to consideration of how events at great distances might affect the system, e.g. how a failure to maintain wetlands in the U.S. that are used by the birds when they migrate might trigger an outbreak of the budworms by reducing the bird population below a critical size. For Holling this was example represents the very essence of "local surprise and global change" in ecosystems.[10]

C. Patch Size Management

Finally let us consider the problem of the size of cut areas in forests, which is closely related to the sizes of patches within forests. It has long been known that timber harvesters prefer to harvest by cutting large clearcuts that are then replanted with a single species of all the same age. This is the least expensive method of harvesting for a pure lumber producer unconcerned about any other amenity values of the forest. However, it is known that large clearcuts can reduce the maximum size of patches in a forest and that the survival of species may depend on there being sufficiently large patches to sustain the

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[10] A further aspect of this involves studying the relations between the different hierarchical levels of the system, with the question of effects passing from lower levels to higher levels becoming very important in this phenomenon of destabilization due to seemingly minor causes. Among those studying this aspect of such dynamics include Allen and Starr (1982), Holling (1992), and Rosser, Folke, Günther, Isomäki, Perrings, and Puu (1994).

more fragile species. More particularly it has been argued that there is a nonlinear relationship between habitat destruction (or fragmentation) and largest patch size (Tilman, May, Lehman, and Nowak, 1994; Bascompte and Solé, 1996; Metzger and Décamps, 1997; Muradian, 2001). Below a certain threshold of patch size there is a relative sudden collapse of population for the fragile species.

Figure 8 depicts the basic situation involved here. It combines figures found in the above references with the private cost of timber harvesting. The horizontal axis represents the size of the cuts and the vertical axis represents quantities of value units, assuming these can be estimated for the species affected by the timber harvesting. Line A represents the benefits per forest of not cutting at a certain scale, the value of the species preserved at that scale of patch-size of cutting, which shows the nonlinear and catastrophically dropping off aspect. Line B shows the private costs of timber harvesting per forest, which steadily decline with the size of the cut. Clearly there is a zone of sizes where it would be socially superior to cut although these would entail higher costs for the private timber harvester. Unsurprisingly this issue is one that continues to be very controversial in forestry management.

[insert Figure 8]

V. Conclusions

We have reviewed a variety of complex ecological-economic problems associated with dynamically managing forests around the world. Needless to say we have barely scratched the surface of these issues and it must be noted that we have mostly dealt with fairly stylized cases or specific examples. Many cases and situations may appear to be very different from the ideas presented here. We must also note that we have largely avoided any detailed discussion of issues arising from conflicts or ambiguities about property rights or access, although these are very serious matters in the traditional communities living in the tropical rain forests, and are also issues even in the temperate forests in the more high income countries such as Canada and the United States.

We began by reviewing the basic literature on optimal rotation of a forest being harvested solely for timber use, resulting in the Faustmann solution. This was modified by considering the analysis of Hartman, which allows for accounting for non-timber amenities of the forest, including grazing, hunting, fishing, recreation, social values of traditional communities, carbon sequestration, and more general existence and aesthetic values. A central issue involves the fact that these other amenities may exhibit complicated time patterns over the life of a forest that do not correspond directly with the growth rate of the trees, the factor that underlies decisionmaking about timber harvesting by a private owner for whom timber is the sole value of interest. This leads to the possibility of multiple equilibria and considerable dynamic complications. These complications have led to great difficulties for actual decisionmakers responsible for the management of forests that have multiple uses, as in the national forests of the United States.

We also considered a further set of management issues, notably fire rotation, pest management, and patch size management in regard to the size of cuts made during timber harvesting. In all of these cases the presence of nonlinearities and discontinuities impose heightened difficulties for forest managers. The presence of critical thresholds in all of these cases is a pervasive phenomenon, and one that poses deep problems not easily solved. Although we did not examine cases involving chaotic or other more erratic dynamics in forestry management, such phenomena are possible. However they would tend to operate over relatively long time scales, unlike for much shorter-lived biological populations. For forests the more dramatic and compelling problems arise from the discontinuities that are studied more by catastrophe theory, collapses of species populations, sudden collapses of entire ecosystems due to fire or outbreaks of pests, or the more insidious damage of badly managed or timed timber harvesting (Rosser, 2001). Dealing with these problems will challenge forest managers for the foreseeable future.

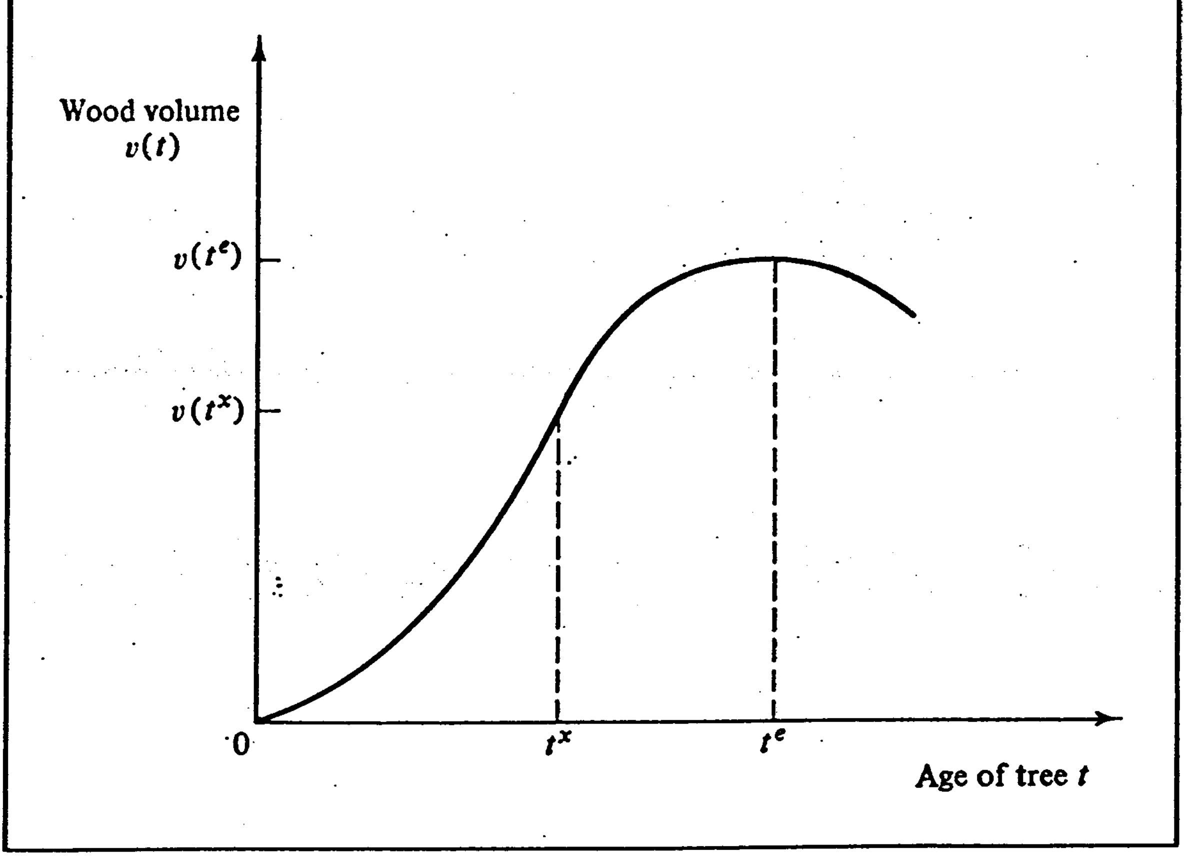
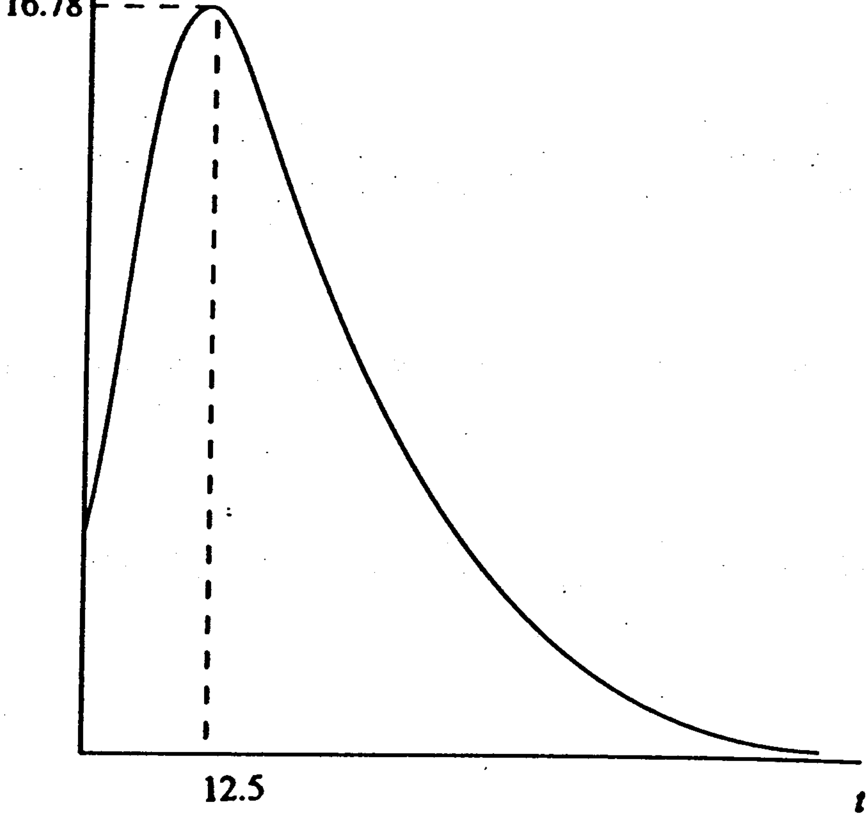
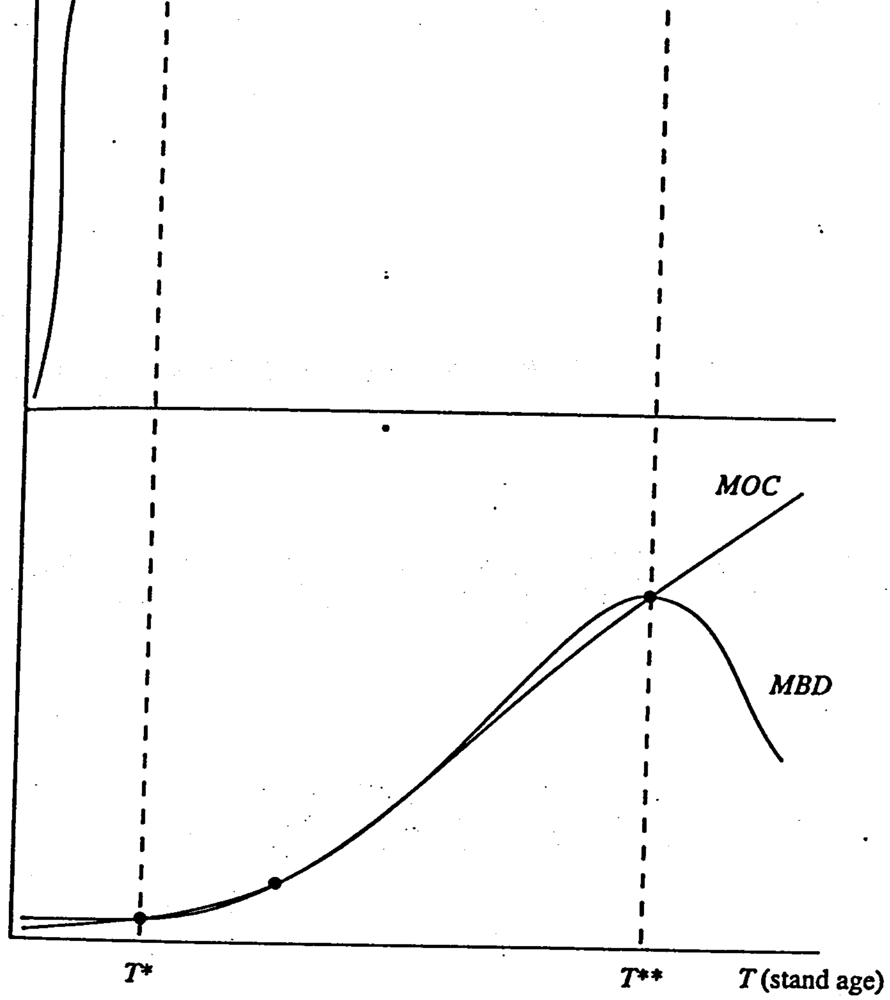


Figure 1: Typical Tree Growth Pattern



Grazing benefit function : Figure 2



Optimal Hartman rotation; Figure 3

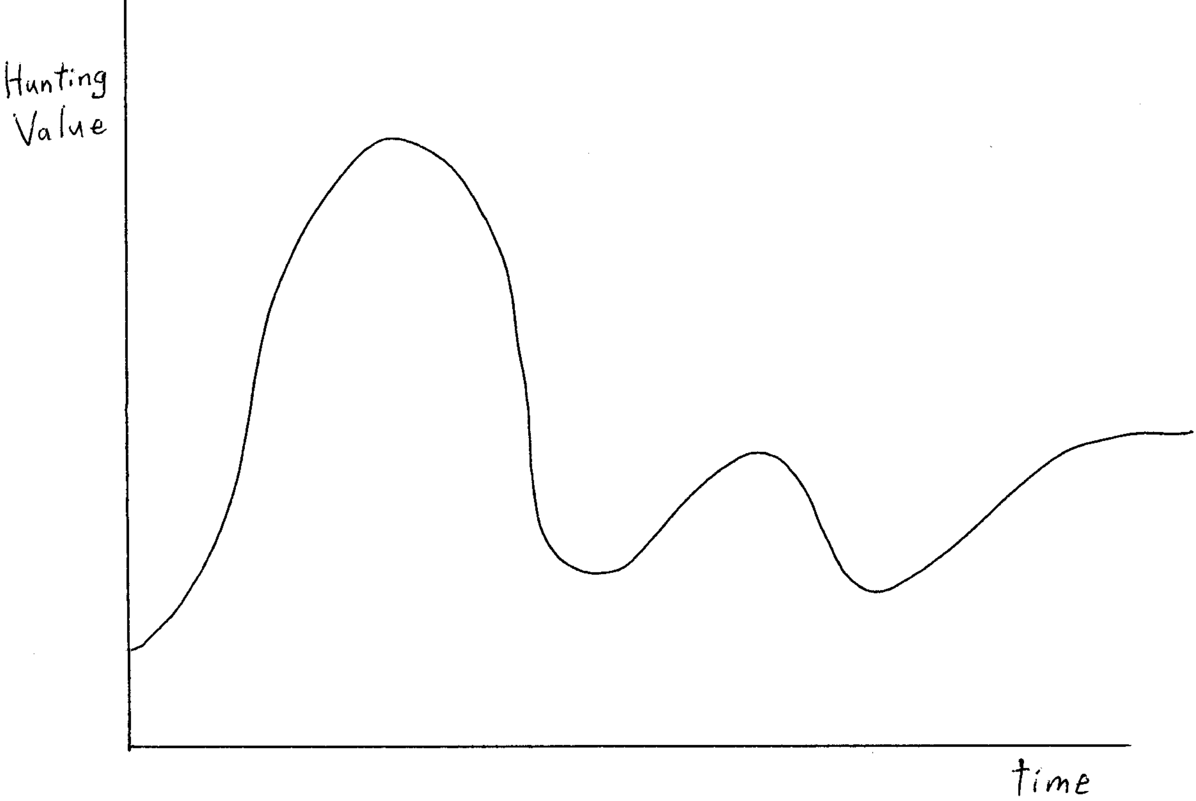


Figure 4: Virginia Deiducus Forest Hunting Amenity

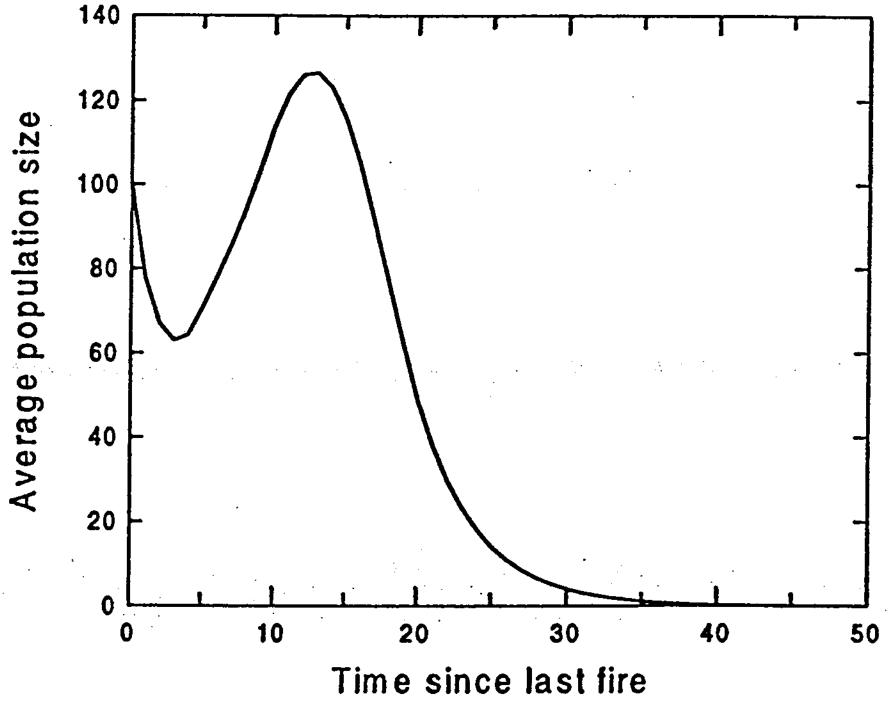


Figure 5: Average Population Path

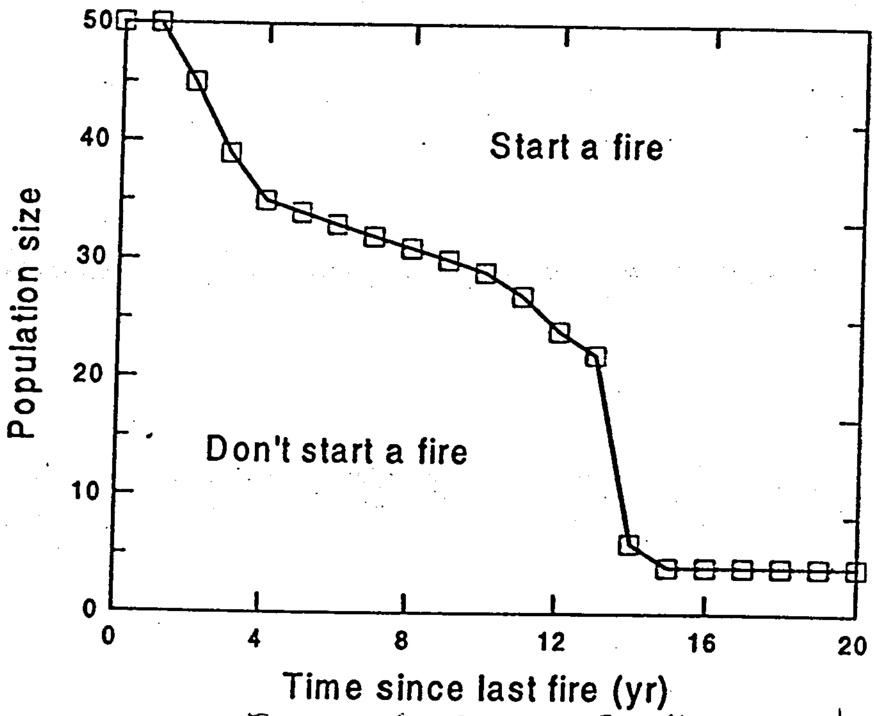


Figure 6: Optimal Fire Management

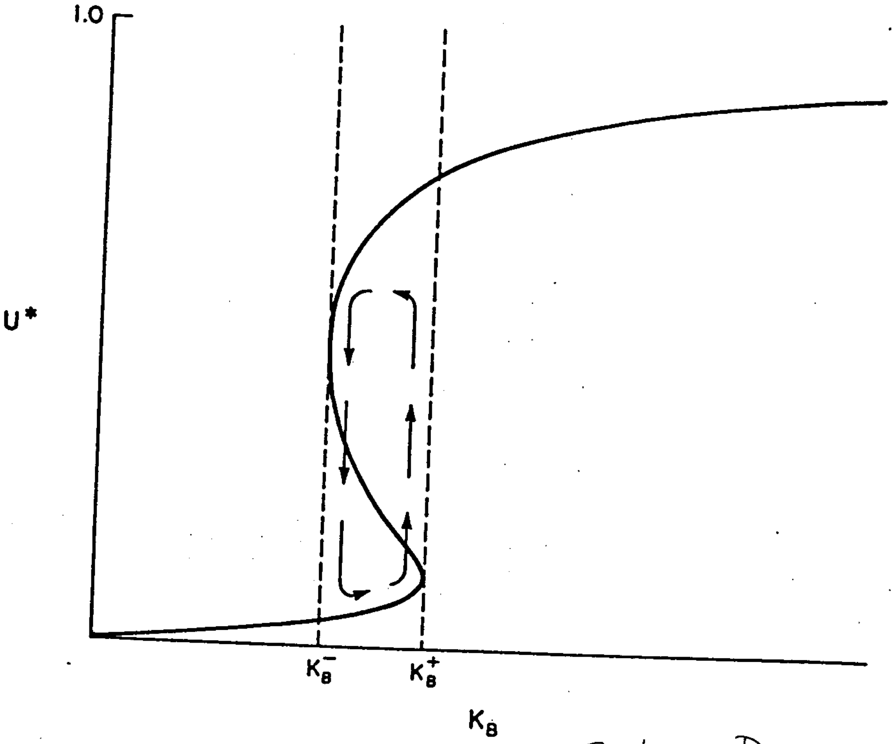


Figure 7: Spruce-Budworm Dynamics

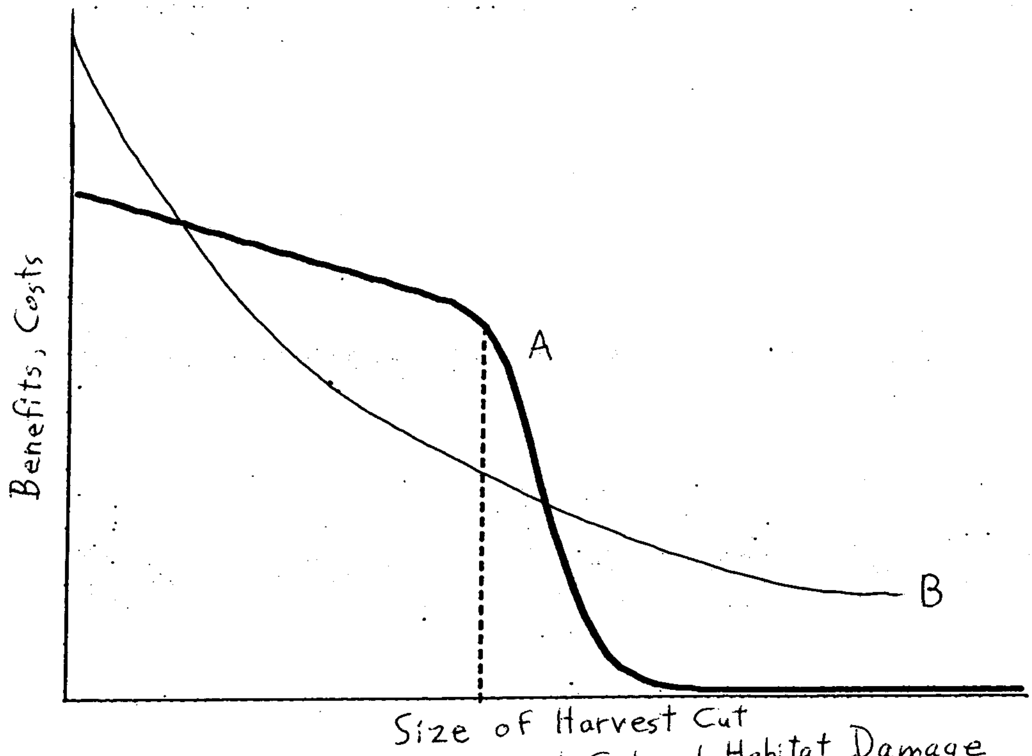


Figure 8: Harvest Cut and Habitat Damage