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Exchange rate and tax policies in an economic development model with public capital

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Abstract

The purpose of this study is to examine the effects of exchange rate and tax policies on economic growth in a model of economic growth with public infrastructures. Public infrastructure, such as roads, airports, bridges, ports and electricity, gas and water services, is, in general, considered to have a positive influence on economic activities and there have been several empirical studies to evaluate this positive impact. On the basis of these preceding works, our present study provides a model of economic development in which public infrastructure enhances productivity. Our analysis shows that wage flexibility affects economic stability. Specifically, we find that there exists a threshold value in the degree of wage flexibility; that the stationary point is unstable or stable, respectively, when the degree is less than or more than the threshold value; that cyclical fluctuations arise when the degree is sufficiently close to the threshold value.

Keywords: Public capital; Economic development; Exchange rate policy; Tax policy.

JEL classification: O11; O41; H54.

1 Introduction

There are, probably, no economists who deny the importance of public infrastructure in economic development. The existence of proper public infrastructure is a necessity for sound economic development or economic growth as advanced nations have experienced. It is thus natural that a lot of economists have been fascinated by inquiry into investment on public infrastructure. After the birth of the so-called endogenous growth theory, in particular, a lot of studies have been conducted in relation to public infrastructure. Empirical studies (e.g., Aschauer, 1989; Gramlich, 1994; Albara-Bertrand and Mamatzakis, 2004) have evaluated the impact on economic growth of public infrastructure, such as roads, bridges, ports, airports and electricity, gas and water services and concluded that public infrastructure surely improves productivity. Theoretical studies (e.g., Barro, 1990; Barro and Sala-i-Martin, 1992;

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Baxter and King, 1993; Futagami et. al. 1993; Turnovsky and Fisher, 1995; Fisher and Turnovsky, 1998; Gómez, 2004; Chatterjee 2007; Agénor, 2008; Maebayashi et al. 2016) have been conducted to explain the mechanism of public infrastructure providing huge productivity. Employing the framework of endogenous growth theory, they took public infrastructure as a public good and emphasized the externality or spillover effect of it.

In the preceding works on public capital, there are two types of treatments of it. One is to take it as a flow variable, while the other is to regard it as a stock variable. The former approach puts much stress on the level of investment on public capital (infrastructure), while the latter emphasizes the amount of public capital itself. Given that public capital, represented by public infrastructure such as roads, bridges and ports, is by definition stock, it is natural to treat it as a stock variable, but it is the fact that what matters in private and public production is services drawn from public capital rather than public capital itself that can justify the flow-variable approach.[1] The flow-variable approach was adopted by, for example, Barro (1990), Barro and Sala-i-Martin (1992), Turnovsky and Fisher (1995) or Agénor (2008), while the stock-variable was introduced in, for instance, Baxter and King (1993), Futagami, et al. (1993), Fisher and Turnovsky (1998), Turnovsky (2004), Gómez (2004), Chatterjee (2007) or Maebayashi et al. (2016). Thus, there are two ways to treat public capital in theory, but, logically speaking, public capital should be viewed as stock and public investment should be regarded as process of accumulation or decumulation of public capital.

The purpose of the present study is to examine, along the line of the aforementioned stock-variable approach, the effects of exchange rate and tax policies on economic development. More specifically, we intend to study the dynamics of public capital, private capital and nominal wages, through public investment financed by tax revenues, private fixed investment and the traditional Phillips relationship, respectively, and the influence of changes in the exchange rate and the tax rate on this dynamics. The model we use is similar to those of Ros and Skott (1998) and of Neto and Lima (2017), but it possesses a notable difference from both of them. First, our model includes the dynamics of public capital unlike Ros and Skott's (1998). Second, public capital is treated as a stock variable in our model, though it was a flow variable in Neto and Lima's (2017) model. The first point is due to the difference in objective of research because Ros and Skott (1997) did not focus on the influence of public capital. More importantly, the second one reflects the aforementioned difference in treatment of public investment or public capital. Indeed, Neto and Lima (2017) identified public infrastructure with government expenditure on public investment, which is a flow variable, though they dealt with the effect of infrastructure as public capital on economic development. As we have already argued, public investment, rather than, public capital should be taken as a flow variable, and the fatal flaw made by Neto and Lima (2017) should be removed.

This paper is organized as follows. In Section 2, we shall build a model of a small open developing economy with public capital. In so doing, we shall describe public investment, the level of which is controlled by adjustments

[1] Precisely speaking, the flow-variable approach puts government expenditure on public investment in the production function, while the stock-variable one takes public capital stock accumulated as a result of public investment as a production input. Theoretically, the difference appears in the number of dimensions of dynamical systems and can make a significant difference in analytical results.

in the tax rate, as an accumulation process of public capital and also distinguish tradable and non-tradable goods to analyze the influence of changes in the exchange rate through international trade. In Section 3, we shall study the characteristics of the model formulated in Section 2. In so doing, we shall focus on the (comparative) static properties and dynamic properties of our model. In particular, we shall conduct the comparative static studies of the steady state with respect to changes in the exchange rate and in the tax rate and examine, in relation to wage flexibility, the stability and instability of the steady state and the possibility of periodical motions being generated. In Section 4, we shall perform some numerical simulations to see if the analysis in Section 3 is valid. In Section 5, we shall summarize our analysis and conclude this paper.

2 The model

In this section, we shall set up a simple model of a small open developing economy with public capital to discuss the effects of exchange rate and tax policies on economic development. In the model presented below, we shall distinguish between tradable and non-tradable goods to analyze properly the impact of an exchange rate policy on an economy.

First, we specify the production techniques of the tradable good, which can be exported or imported between countries, and of the non-tradable good, which can only domestically be purchased or sold. As regards the tradable good, we assume that the production technique of it can be represented in the following way:

$$Y_T = A_T K_P^{\chi_T} K_T^{\alpha_T} L_T^{1-\alpha_T}, \tag{1}$$

where α_T and χ_T are positive constants with $\alpha_T < 1$. In (1), $Y_T,\ K_T,\ K_P,\ L_T$ and A_T stand for the level of output of the tradable good, the volume of private capital stock of the tradable good sector, the volume of public capital stock, the amount of employment engaged in the tradable good sector and the total factor productivity for the tradable good sector, respectively; α_T and χ_T are the share of profits in the tradable good sector ($1 - \alpha_T$ is equal to the share of wages) and the elasticity of public capital on production of the tradable good sector, respectively. It is supposed in (1) that the production function of the tradable good is of the Cobb-Douglas type and of the Hicks-neutral type of technical progress.[2] Also, the private capital stock is assumed to be utilized in production of the tradable good. For our analysis, we also assume that the two elasticities satisfy the following condition:

$$\alpha_T + \chi_T < 1. \tag{2}$$

Condition (2) is empirically not so restrictive and it can be justified as a realistic assumption. As for the non-tradable

[2] It does not matter at all whether technical progress is of the Hicks-neutral type or the Harrod-neutral one. Note, however, that production functions with homogeneity of degree one are both of the Hicks-neutral type and of the Harrod-neutral type if and only if they are of the Cobb-Douglas type. For details, see Uzawa (1961). This argument also applies to the production function of the non-tradable good.

good, on the other hand, the production function of it is assumed to be given by the following form:

$$Y_N = A_N K_P^{\chi_N} L_N^{1-\alpha_N}, \tag{3}$$

where α_N and χ_N are positive constants with $\alpha_N < 1$. In (3), Y_N, L_N and A_N stand for the level of output of the non-tradable good, the amount of employment engaged in the non-tradable good sector and the total factor productivity for the tradable good sector, respectively; α_N and χ_N are the share of profits in the non-tradable good sector ($1-\alpha_N$ is equal to the share of wages) and the elasticity of public capital on production of the non-tradable good sector, respectively. Unlike in the production function of the tradable good, private capital is supposed to play no role in production of the non-tradable good. This assumption reflects the fact that the tertiary industry, most of which is composed of those of non-tradable goods (services), is a comparatively labor-intensive industry.[3]

Second, we have a look at the price of the tradable good and the interest rate. With the price of the tradable good produced in the foreign countries given, we can normalize it as unity.[4] Letting E be the exchange rate measured in the home currency, the price of the tradable good produced in the home country, denoted by P_T, is assumed to be determined internationally in the following way:

$$P_T = E. \tag{4}$$

Regarding the interest rate, we can assume from the interest rate parity that the interest rate of the home country is equal to that of the foreign countries because the home country is, by assumption, a small open economy with perfect capital mobility. Letting i_H and i be, respectively, the interest rates of the home country and of the foreign countries, it follows that the following relationship holds:

$$i_H = i. \tag{5}$$

Third, we take a look at expenditure behavior of consumers. For simplicity, we suppose that the share of expenditure on the tradable good (in total expenditure) is constant. Denoting the share of the tradable good by ϕ, we have the following relationship:

$$P_N C_N = \frac{1-\phi}{\phi} P_T C_T, \tag{6}$$

where ϕ is a constant that lies between 0 and 1. In (6), P_N, C_N and C_T stand for, respectively, the price of the non-tradable good, the consumption of the non-tradable good and the consumption of the tradable good in the home country.

[3] This assumption was adopted in Ros and Skott (1998) and Neto and Lima (2016).

[4] In our model, "money" does not explicitly appear and so we do not need to care about absolute prices.

Fourth, we assume, following the classical orthodox, that there are two classes in our model economy; one is workers, who spend all of their income (wage income) on consumption, and the other is capitalists, who save a portion of their income (profit income). According to the budget constraint of the home country, we can see that the following holds:

$$P_T C_T + P_N C_N = (1-\tau)[W(L_T + L_N) + (1-s)(P_T r K_T + \Pi_N)], \tag{7}$$

where s and τ are constants that lie between 0 and 1. In (7), $W,\ \Pi_N,\ r,\ s$ and τ represent the nominal wage, the profit in the non-tradable good sector, the profit rate on capital in the tradable good sector, the saving rate and the tax rate, respectively. This equation functions as the demand function of the tradable good.

Fifth, we consider the market clearing condition of the non-tradable good. This condition can be written as follows:

$$Y_N = C_N. \tag{8}$$

Note that the non-tradable good cannot be purchased by the foreign countries.

Sixth, we turn to the dynamics in our model. We assume that investment on public capital is financed by the tax revenue in the following way:

$$\dot{K}_P = \tau\Big(Y_T + \frac{P_N Y_N}{P_T}\Big) - \delta K_P, \tag{9}$$

where δ is a positive constant that represents the depreciation rate of public capital. Unlike in Neto and Lima (2016), public capital, which is a proxy of infrastructure, is treated as a stock variable, not as a flow one. Since almost of all infrastructure is durable, our treatment can safely be justified. In (9), it is implicitly assumed that the price of public capital is equal to that of the tradable good. This is because, in our model, where there are only two types of goods, the non-tradable good cannot be used in investment due to the existence of (8), which implies that the non-tradable good is utilized for consumption purposes alone.

Seventh, we formalize the process of investment on private capital. In general, the gross rate of capital accumulation can be considered to be determined by comparing the ratio of the (after-tax) profit rate on private capital $r(1-\tau)$ and the (after-tax) home interest rate $i_H(1-\tau) = i(1-\tau)$ (by (5)) with each other. Then, we can postulate that the dynamics of private capital K_T is expressed in the following form:

$$\frac{\dot{K}_T}{K_T} = g\Big(\frac{r}{i}\Big) - \delta, \tag{10}$$

where g is twice continuously differentiable with

$$g(0) = 0, \ \lim_{q \to \infty} g(q) = \infty, \tag{11}$$

$$g'(q) > 0 \ \text{ for } q \geq 0. \tag{12}$$

In (10), δ also stands for the depreciation rate of private capital stock. Note that, since the price of public capital is equal to that of the tradable good and to that of private capital, the depreciation rate of private capital should be equal to that of public capital.[5] In our model, public and private capitals are categorized as the tradable good, but they differ from each other in that the former has the so-called externality while the latter does not.

Finally, we allow for changes in the nominal wage W. Following the usual Phillips curve, we formalize the dynamics of W in the following way:

$$\frac{\dot{W}}{W} = \beta(e - e^*), \tag{13}$$

where e^* is a positive constant; β is a positive parameter. In (13), e and e^* represent, respectively, the employment rate and the natural rate of employment, for which the rate of changes in W remains at 0; β stands for the so-called wage flexibility.

3 Analysis

In this section, we shall analyze the model formalized in the previous section.

To begin, we derive the optimal levels of employment in the tradable good sector and in the non-tradable good one. It follows from (1), (3) and (4) that, for given K_P and K_T, the first order conditions for optimality give

$$L_T = (1 - \alpha_T)^{\frac{1}{\alpha_T}} A_T^{\frac{1}{\alpha_T}} E^{\frac{1}{\alpha_T}} K_P^{\frac{\chi_T}{\alpha_T}} K_T W^{-\frac{1}{\alpha_T}}, \tag{14}$$

$$L_N = (1 - \alpha_N)^{\frac{1}{\alpha_N}} A_N^{\frac{1}{\alpha_N}} P_N^{\frac{1}{\alpha_N}} K_P^{\frac{\chi_N}{\alpha_N}} W^{-\frac{1}{\alpha_N}}. \tag{15}$$

Substituting (14) and (15) in (1) and (3) respectively, we obtain

$$Y_T = (1 - \alpha_T)^{\frac{1-\alpha_T}{\alpha_T}} A_T^{\frac{1}{\alpha_T}} E^{\frac{1-\alpha_T}{\alpha_T}} K_P^{\frac{\chi_T}{\alpha_T}} K_T W^{-\frac{1-\alpha_T}{\alpha_T}}, \tag{16}$$

$$Y_N = (1 - \alpha_N)^{\frac{1-\alpha_N}{\alpha_N}} A_N^{\frac{1}{\alpha_N}} P_N^{\frac{1-\alpha_N}{\alpha_N}} K_P^{\frac{\chi_N}{\alpha_N}} W^{-\frac{1-\alpha_N}{\alpha_N}}, \tag{17}$$

Also, we can calculate the profit rate on private capital r and the level of profits in the non-tradable good sector

[5] The assumption that private capital and public capital have a common depreciation rate is not essential in our analysis. The consequences will not dramatically be altered even if this assumption is relaxed.

Π_N as follows:

$$r = \alpha_T(1-\alpha_T)^{\frac{1-\alpha_T}{\alpha_T}} A_T^{\frac{1}{\alpha_T}} E^{\frac{1-\alpha_T}{\alpha_T}} K_P^{\frac{\chi_T}{\alpha_T}} W^{-\frac{1-\alpha_T}{\alpha_T}}, \tag{18}$$

$$\Pi_N = \alpha_N(1-\alpha_N)^{\frac{1-\alpha_N}{\alpha_N}} A_N^{\frac{1}{\alpha_N}} P_N^{\frac{1}{\alpha_N}} K_P^{\frac{\chi_N}{\alpha_N}} W^{-\frac{1-\alpha_N}{\alpha_N}}. \tag{19}$$

Next, we make use of (6)-(8) to derive the level of consumption of the tradable good C_T and the price of the non-tradable good P_N as follows:

$$C_T = (1-\alpha_T)^{\frac{1-\alpha_T}{\alpha_T}}(1-\alpha_T s)\phi(1-\tau)[1-(1-\alpha_N s)(1-\phi)(1-\tau)]^{-1} A_T^{\frac{1}{\alpha_T}} E^{\frac{1-\alpha_T}{\alpha_T}} K_P^{\frac{\chi_T}{\alpha_T}} K_T W^{-\frac{1-\alpha_T}{\alpha_T}}, \tag{20}$$

$$\begin{aligned} P_N = {} & (1-\alpha_T)^{\frac{(1-\alpha_T)\alpha_N}{\alpha_T}}(1-\alpha_N)^{-(1-\alpha_N)}(1-\alpha_N s)^{\alpha_N}(1-\phi)^{\alpha_N}(1-\tau)^{\alpha_N} \\ & \times [1-(1-\alpha_N s)(1-\phi)(1-\tau)]^{-\alpha_N} A_T^{\frac{\alpha_N}{\alpha_T}} A_N^{-1} E^{\frac{\alpha_N}{\alpha_T}} K_P^{\frac{\alpha_N \chi_T}{\alpha_T}-\chi_N} K_T^{\alpha_N} W^{1-\alpha_N-\frac{(1-\alpha_T)\alpha_N}{\alpha_T}} \end{aligned} \tag{21}$$

Thirdly, based upon (14) and (15), we define the employment rate e in the following way:

$$e = \frac{L_T + L_N}{N} = \frac{1-\alpha_T+(\alpha_T-\alpha_N)(1-s)(1-\phi)(1-\tau)}{[1-(1-\alpha_N s)(1-\phi)(1-\tau)]N} A_T^{\frac{1}{\alpha_T}} E^{\frac{1}{\alpha_T}} K_P^{\frac{\chi_T}{\alpha_T}} K_T W^{-\frac{1}{\alpha_T}}, \tag{22}$$

where N is a positive constant. In (22), N stands for the level of labor force, which is assumed to be constant in our model.

Thus, putting (14)-(22) in (9)-(13), we can reach the following system of equations:

$$\dot{K}_P = \frac{[1-(\alpha_T-\alpha_N)s(1-\phi)(1-\tau)]\tau}{1-(1-\alpha_N s)(1-\phi)(1-\tau)}(1-\alpha_T)^{\frac{1-\alpha_T}{\alpha_T}} A_T^{\frac{1}{\alpha_T}} E^{\frac{1}{\alpha_T}} K_P^{\frac{\chi_T}{\alpha_T}} K_T W^{-\frac{1-\alpha_T}{\alpha_T}} - \delta K_P, \tag{23}$$

$$\dot{K}_T = g\Big(\alpha_T(1-\alpha_T)^{\frac{1-\alpha_T}{\alpha_T}} i^{-1} A_T^{\frac{1}{\alpha_T}} E^{\frac{1-\alpha_T}{\alpha_T}} K_P^{\frac{\chi_T}{\alpha_T}} W^{-\frac{1-\alpha_T}{\alpha_T}}\Big) K_T - \delta K_T, \tag{24}$$

$$\dot{W} = \beta\Big\{\frac{1-\alpha_T+(\alpha_T-\alpha_N)(1-s)(1-\phi)(1-\tau)}{[1-(1-\alpha_N s)(1-\phi)(1-\tau)]N} A_T^{\frac{1}{\alpha_T}} E^{\frac{1}{\alpha_T}} K_P^{\frac{\chi_T}{\alpha_T}} K_T W^{-\frac{1}{\alpha_T}} - e^*\Big\} W. \tag{25}$$

In what follows, the system of equations (23)-(25) is simply denoted by "System."

3.1 Characteristics of the steady state

A steady state of System, (K_P^*, K_T^*, W^*), is defined as a point $(K_P, K_T, W) \in \mathbb{R}^3_{++}$ that satisfies $\dot{K}_P = \dot{K}_T = \dot{W} = 0$. Then, a steady state of System, (K_P^*, K_T^*, W^*), can be redefined as a solution of the following simultaneous equations:

$$\delta = \frac{[1-(\alpha_T-\alpha_N)s(1-\phi)(1-\tau)]\tau}{1-(1-\alpha_N s)(1-\phi)(1-\tau)}(1-\alpha_T)^{\frac{1-\alpha_T}{\alpha_T}} A_T^{\frac{1}{\alpha_T}} E^{\frac{1}{\alpha_T}} K_P^{\frac{\chi_T}{\alpha_T}-1} K_T W^{-\frac{1-\alpha_T}{\alpha_T}}, \tag{26}$$

$$\delta = g\Big(\alpha_T(1-\alpha_T)^{\frac{1-\alpha_T}{\alpha_T}} i^{-1} A_T^{\frac{1}{\alpha_T}} E^{\frac{1-\alpha_T}{\alpha_T}} K_P^{\frac{\chi_T}{\alpha_T}} W^{-\frac{1-\alpha_T}{\alpha_T}}\Big), \tag{27}$$

$$e^* = \frac{1-\alpha_T+(\alpha_T-\alpha_N)(1-s)(1-\phi)(1-\tau)}{[1-(1-\alpha_N s)(1-\phi)(1-\tau)]N} A_T^{\frac{1}{\alpha_T}} E^{\frac{1}{\alpha_T}} K_P^{\frac{\chi_T}{\alpha_T}} K_T W^{-\frac{1}{\alpha_T}}. \tag{28}$$

To find a solution of (26)-(28), it is convenient to rewrite them in the logarithm forms:[6]

$$
\begin{aligned}
&(\chi_T - \alpha_T)\log K_P + \alpha_T \log K_T - (1-\alpha_T)\log W \\
=& -\log E - \alpha_T \log\tau - \alpha_T \log[1-(\alpha_T-\alpha_N)s(1-\phi)(1-\tau)] + \alpha_T\log[1-(1-\alpha_N s)(1-\phi)(1-\tau)] + \text{const.}
\end{aligned} \tag{29}
$$

$$
\chi_T \log K_P - (1-\alpha_T)\log W = -(1-\alpha_T)\log E + \text{const.} \tag{30}
$$

$$
\begin{aligned}
&\chi_T \log K_P + \alpha_T \log K_T - \log W \\
=& -\log E - \alpha_T \log[1-\alpha_T+(\alpha_T-\alpha_N)(1-s)(1-\phi)(1-\tau)] + \alpha_T\log[1-(1-\alpha_N s)(1-\phi)(1-\tau)] + \text{const.}
\end{aligned} \tag{31}
$$

The following proposition states that it is possible to find a unique steady state of System.

Proposition 1. *There exists a unique steady state of System,* $(K_P^*, K_T^*, W^*) \in \mathbb{R}^3_{++}$.

Proof. The system of simultaneous equations (29)-(31) can be written as

$$
\begin{pmatrix} \chi_T - \alpha_T & \alpha_T & -(1-\alpha_T) \\ \chi_T & 0 & -(1-\alpha_T) \\ \chi_T & \alpha_T & -1 \end{pmatrix}
\begin{pmatrix} \log K_P \\ \log K_T \\ \log W \end{pmatrix}
=
\begin{pmatrix} \text{const.} \\ \text{const.} \\ \text{const.} \end{pmatrix}.
$$

The determinant of the coefficient matrix is given by

$$
-\alpha_T^2(1-\alpha_T-\chi_T) < 0,
$$

where the inequality follows from (2). Then, the system of simultaneous equations (29)-(31) has a unique solution $(\log K_P^*, \log K_T^*, \log W^*)$. □

Proposition 1 guarantees the existence and uniqueness of a steady state in System. In the related models presented by Skott and Lima (1997) and Neto and Lima (2017), there exist two (or more) steady states, one of which is stable and the other of which is unstable, while in our System, the uniqueness of a steady state can be established. This difference is due to the existence of a new state variable, K_P, public capital.

We can evaluate the effects of the exchange rate E and of the tax rate τ on the unique steady state (K_P^*, K_T^*, W^*)

[6] Note that, because of (11) and (12), $g^{-1}(\delta)$ exists and is positive.

as follows:

$$
\begin{aligned}
\frac{\partial \log K_P^*}{\partial \log E} &= \frac{1-\alpha_T}{1-\alpha_T-\chi_T} > 0, && (32)\\
\frac{\partial \log K_T^*}{\partial \log E} &= \frac{\chi_T}{1-\alpha_T-\chi_T} > 0, && (33)\\
\frac{\partial \log W^*}{\partial \log E} &= \frac{1-\alpha_T}{1-\alpha_T-\chi_T} > 0, && (34)\\
\frac{\partial \log K_P^*}{\partial \log \tau} &= \frac{1-\alpha_T}{1-\alpha_T-\chi_T} \\
&\quad + \frac{(1-\alpha_T)[2-(1+\alpha_T)s](\alpha_T-\alpha_N)(1-\phi)\tau}{(1-\alpha_T-\chi_T)[1-(\alpha_T-\alpha_N)s(1-\phi)(1-\tau)][1-\alpha_T+(\alpha_T-\alpha_N)(1-s)(1-\phi)(1-\tau)]}, && (35)\\
\frac{\partial \log K_T^*}{\partial \log \tau} &= \frac{\chi_T}{1-\alpha_T-\chi_T} + \frac{(1-\alpha_N s)(1-\phi)\tau}{1-(1-\alpha_N s)(1-\phi)(1-\tau)} \\
&\quad + \frac{(1-\alpha_T)(\alpha_T-\alpha_N)(1-s)(1-\phi)\tau}{(1-\alpha_T-\chi_T)[1-\alpha_T+(\alpha_T-\alpha_N)(1-s)(1-\phi)(1-\tau)]} && (36)\\
&\quad + \frac{(\alpha_T-\alpha_N)\chi_T s(1-\phi)\tau}{(1-\alpha_T-\chi_T)[1-(\alpha_T-\alpha_N)(1-s)(1-\phi)(1-\tau)]}, \\
\frac{\partial \log W^*}{\partial \log \tau} &= \frac{\chi_T}{1-\alpha_T-\chi_T} \\
&\quad + \frac{[2-(1+\alpha_T)s](\alpha_T-\alpha_N)\chi_T(1-\phi)\tau}{(1-\alpha_T-\chi_T)[1-(\alpha_T-\alpha_N)s(1-\phi)(1-\tau)][1-\alpha_T+(\alpha_T-\alpha_N)(1-s)(1-\phi)(1-\tau)]}. && (37)
\end{aligned}
$$

It is seen from (32)-(34) that a rise in the exchange rate E (a devaluation of the home currency) has a positive influence on the steady state values of K_P^*, K_T^* and W^*, while it is ambiguous whether an increase in the tax rate τ has a favorable effect on the steady state value of K_P^*, K_T^* or W^*. If the capital share of the tradable good sector α_T is sufficiently close to that of the non-tradable good sector α_N, or if the former is greater than the latter (this case is reasonable because only the tradable-good industry uses public capital), however, we can safely argue from (35)-(37) that an increase in τ affects positively K_P^*, K_T^* and W^*.

Now, we turn to the effects of changes in the exchange rate E and the tax rate τ on the steady state level of real aggregate output (GDP). With the price of the tradable good P_T being the deflater, the real level of aggregate output, denoted by Y, can be defined as

$$Y = Y_T + \frac{P_N}{P_T} Y_N. \tag{38}$$

Let Y^* be the level of real aggregate output at the steady state. Then, since we have $\dot{K}_P = 0$ at the steady state, we find from (4), (9) and (38) that

$$Y^* = \frac{\delta}{\tau} K_P^*, \tag{39}$$

Taking the logarithm of (39) and differentiating it with respect to $\log E$ and $\log \tau$, we obtain the following relationships (by utilizing (32) and (35)):

$$\frac{\partial \log Y^*}{\partial \log E} = \frac{\partial \log K_P^*}{\partial \log E} = \frac{1-\alpha_T}{1-\alpha_T-\chi_T} > 0, \tag{40}$$

$$\begin{aligned}\frac{\partial \log Y^*}{\partial \log \tau} &= \frac{\partial \log K_P^*}{\partial \log \tau} - 1 \\ &= \frac{\chi_T}{1-\alpha_T-\chi_T} \\ &+ \frac{(1-\alpha_T)[2-(1+\alpha_T)s](\alpha_T-\alpha_N)(1-\phi)\tau}{(1-\alpha_T-\chi_T)[1-(\alpha_T-\alpha_N)s(1-\phi)(1-\tau)][1-\alpha_T+(\alpha_T-\alpha_N)(1-s)(1-\phi)(1-\tau)]}.\end{aligned} \tag{41}$$

We can easily find from (40) that the steady state value of real aggregate output, Y^*, is influenced positively by a rise in the exchange rate E, but we cannot determine definitely whether an increase in the tax rate τ has a favorable influence on Y^*. But if the capital share of the tradable good sector α_T is sufficiently close to that of the non-tradable good sector α_N, however, we can ensure from (41) that an increase in τ affects positively Y^*. Provided that α_T is sufficiently near α_N, which does not seem unrealistic, we can maintain that both a devaluation of the home currency and a tax increase improve the economic condition represented by the steady state value of real aggregate output.

To conclude this subsection, we explore the exchange rate E or the tax rate τ that is able to maximize the level of (per capita) real consumption evaluated at the steady state. To do so, we evaluate the effects of changes in E and τ on the steady-state consumption. With the price of the tradable good P_T being the deflater, the real level of consumption C can be defined as in the case of Y:

$$C = C_T + \frac{P_N}{P_T} C_N. \tag{42}$$

Letting C^* be the steady state value of C, it follows from (6), (20) and (42) that

$$C^* = (1-\alpha_T)^{\frac{1-\alpha_T}{\alpha_T}} (1-\alpha_T s)(1-\tau)[1-(1-\alpha_N s)(1-\phi)(1-\tau)]^{-1} A_T^{\frac{1}{\alpha_T}} E^{\frac{1-\alpha_T}{\alpha_T}} (K_P^*)^{\frac{\chi_T}{\alpha_T}} K_T^* (W^*)^{-\frac{1-\alpha_T}{\alpha_T}}. \tag{43}$$

As for the impact of changes in E on C^*, we can evaluate it by taking the logarithm of (43), differentiating it with respect to $\log E$ and substituting (32)-(37):

$$\frac{\partial \log C^*}{\partial \log E} = \frac{\chi_T}{1-\alpha_T-\chi_T} > 0. \tag{44}$$

We know from (44) that an increase in E (a devaluation of the home currency) always raises the steady state real consumption C^*. As regards the influence of changes in τ on C^*, on the other hand, we reach a different conclusion. To see this in a simple way, assume that the capital shares of the tradable good and non-tradable good sectors are equal to each other, $\alpha_T = \alpha_N$. Under this assumption, we can obtain the following in the same way as above:

$$\frac{\partial \log C^*}{\partial \log \tau} = \frac{\chi_T}{1-\alpha_T} - \frac{\tau}{1-\tau}. \tag{45}$$

It is seen from (45) that a rise in τ does not necessarily increase (or decrease) C^*. More interestingly, it is possible to derive the tax rate, denoted by τ^*, that maximizes C^*. This tax rate τ^* can be given by

$$\tau^* = \frac{\chi_T}{1-\alpha_T} < 1, \tag{46}$$

where the inequality follows from (2). We find from (46) that the "optimal" tax rate, which maximizes the steady state real consumption, exists (under the assumption of $\alpha_T = \alpha_N$).

3.2 Stability and instability of the steady state and existence of a periodic orbit

We are now ready to examine the local asymptotic stability of the unique steady state of System (K_P^*, K_T^*, W^*). To do so, we calculate the Jacobian matrix of System evaluated at (K_P^*, K_T^*, W^*) as follows:

$$J^* = \begin{pmatrix} \alpha_T^{-1}(\chi_T - \alpha_T)\delta & \delta K_P^*/K_T^* & -\alpha_T^{-1}(1-\alpha_T)\delta K_P^*/W^* \\ \alpha_T^{-1}\chi_T g'(g^{-1}(\delta))g^{-1}(\delta)K_T^*/K_P^* & 0 & -\alpha_T^{-1}(1-\alpha_T)g'(g^{-1}(\delta))g^{-1}(\delta)K_T^*/W^* \\ \alpha_T^{-1}\chi_T\beta e^* W^*/K_P^* & \beta e^* W^*/K_T^* & -\alpha_T^{-1}\beta e^* \end{pmatrix}.$$

The characteristic equation associated with J^* can be calculated as

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \tag{47}$$

where

$$a_1 = \frac{(\alpha_T - \chi_T)\delta}{\alpha_T} + \frac{e^*}{\alpha_T}\beta, \tag{48}$$

$$a_2 = -\frac{\chi_T\delta g'(g^{-1}(\delta))g^{-1}(\delta)}{\alpha_T} + \frac{[(1-\chi_T)\delta + (1-\alpha_T)g'(g^{-1}(\delta))g^{-1}(\delta)]e^*}{\alpha_T}\beta, \tag{49}$$

$$a_3 = \frac{(1-\alpha_T-\chi_T)\delta e^* g'(g^{-1}(\delta))g^{-1}(\delta)}{\alpha_T}\beta > 0. \tag{50}$$

According to the Routh-Hurwitz criterion, the unique steady state is locally asymptotically stable if and only if the following conditions hold:

$$a_1 > 0, \ a_3 > 0,$$

$$a_1 a_2 > a_3,$$

which can, under our assumptions and by (48)-(50), be reduced to[7]

$$\frac{(\alpha_T - \chi_T)\delta}{\alpha_T} + \frac{e^*}{\alpha_T}\beta > 0, \tag{51}$$

$$\begin{aligned} & h(\beta)(\equiv a_1 a_2 - a_3) \\ =& \frac{[(1-\chi_T)\delta + (1-\alpha_T)g'(g^{-1}(\delta))g^{-1}(\delta)](e^*)^2}{\alpha_T^2}\beta^2 + a_*\beta - \frac{(\alpha_T - \chi_T)\chi_T\delta g'(g^{-1}(\delta))g^{-1}(\delta)}{\alpha_T^2} > 0. \end{aligned} \tag{52}$$

Furthermore, we can explore the possibility that a non-constant periodic orbit is generated in System by way of a Hopf bifurcation. Let the wage flexibility β be a varying parameter. According to Liu (1994, p. 252, Theorem), a Hopf bifurcation arises and then a non-constant periodic orbit is brought out if the following conditions, as well as (51), hold:

$$h(\beta) = 0, \tag{53}$$

$$h'(\beta) \neq 0. \tag{54}$$

As regards the stability of the unique equilibrium and the possibility of existence of a periodic orbit, we can establish the following proposition.

Proposition 2. *(i) The unique steady state of System (K_P^*, K_T^*, W^*) is locally asymptotically stable if β is sufficiently large.*

(ii) Assume further that

$$\chi_T < \alpha_T. \tag{55}$$

Then, there exists a positive β^* such that

(a) the unique steady state of System (K_P^*, K_T^*, W^*) is locally asymptotically stable (resp. unstable) for $\beta > \beta^*$ (resp. $\beta < \beta^*$) and that

(b) a non-constant periodic orbit is generated by way of a Hopf bifurcation for β sufficiently close to β^*.

Proof. (i) For β sufficiently large, conditions (51) and (52) are satisfied because the coefficient of β^2 of h is positive. Therefore, Assertion (i) holds true.

(ii) To begin, we have (48) for every $\beta > 0$ under (51). Then, for the local asymptotic stability, condition (52) is necessary and sufficient.

Next, we know from (52) that the coefficient of β^2 and the constant term of $h(\beta)$ are positive and negative, respectively. Then, the quadratic $h(\beta) = 0$ has positive and negative real roots. Letting β^* be the positive root, we find, condition (52) is fulfilled for $\beta > \beta^*$ but violated for $\beta < \beta^*$. Thus, Assertion (ii-a) is proved.

[7] Since the exact value of a_* is not necessary for our analysis, it is omitted.

Finally, we see that $h'(\beta^*) \neq 0$ because the quadratic $h(\beta) = 0$ has a negative root other than $\beta = \beta^*$. Then, conditions (53) and (54) are fulfilled for $\beta = \beta^* > 0$. Hence, Assertion (ii-b) is verified. □

Proposition 2 says that the unique steady state is locally asymptotically stable if the wage flexibility β is sufficiently large and that, if the elasticity on production of public capital χ_T is less than that of private capital α_T, there is a (Hopf) bifurcation value of β or β^*, for which a non-constant periodic orbit is generated. This proposition implies that high flexibility of wages contributes to stability in our model. It is a natural consequence because our model is built completely upon the neoclassical theory, which maintains that flexibility of prices is conducive to economic stability.[8]

More interestingly, Proposition 2 mentions the possibility of emergence of persistent cyclical fluctuations (a non-constant periodic orbit). These cyclical motions can be interpreted as business cycles because the level of real aggregate output also undergoes ups and downs along these cycles. The mechanism of these persistent cycles emerging can be explained as follows. If the parameter of wage flexibility β is set at 0, the steady state of the dynamical system of K_P and K_T, (23) and (24), is a saddle point and unstable, provided that condition (55) is fulfilled;[9] as β gets larger, the stabilizing force of wage dynamics (25) comes to dominate the above destabilizing one; at the threshold $\beta = \beta^*$, the steady state turns from being unstable to being stable and a non-constant periodic orbit is generated by this (Hopf) bifurcation. In the related preceding works mentioned in Section 1, this possibility was not touched on and only the characteristics of a steady state were discussed. In this respect, this fact may be said to constitute one of the major findings in this research.

4 Numerical analysis

In this section, we perform numerical simulations to check the validity of results in Section 4. For this purpose, we specify the parameter values and the capital accumulation function g.

First, we normalize the values of A_T and N as unity:

$$A_T = 1, \tag{56}$$

$$N = 1. \tag{57}$$

Since we assume that the total factor productivity for the tradable good sector A_T and the number of population N are both constant over time, these normalizations can be justified.

Second, we fix the capital shares of the tradable good and non-tradable good sectors, α_T and α_N. According to Golin's (2002) three ways of calculations, the labor share in GDP was about 0.65 among 31 countries.[10] Therefore,

[8] In the Keynesian theory, flexibility of prices can work as a destabilizer. For theoretical analysis of the destabilizing force of flexibility of prices in Keynesian systems, see Murakami (2014).

[9] This implies that our System possesses instability in the absence of wage flexibility or in the presence of wage rigidity.

[10] Golin (2002) argued that naive calculations of labor shares can yield lower values than actual ones because employee compensation

we set α_T and α_N as follows:

$$\alpha_T = 0.35, \tag{58}$$

$$\alpha_N = 0.35. \tag{59}$$

For simplicity, it is assumed that the capital share is the same among the tradable good and non-tradable good sectors.

Third, following Turnovsky (2004), we set the value of the elasticity of public capital on production of the tradable good χ_T as follows:

$$\chi_T = 0.2. \tag{60}$$

Fourth, we adopt the following as the value of the (annual) depreciation rate of capital δ:

$$\delta = 0.1. \tag{61}$$

This value fits with empirical data.[11]

Fifth, we make use of Naastepad and Storm (2007) to determine the value of the saving rate of capitalists s. According to them, the saving rate out of capital income is, on average, around 0.48 among the seven countries in OECD.[12] Then, we set s as follows:

$$s = 0.5. \tag{62}$$

Sixth, we set the value of the interest rate i as

$$i = 0.05. \tag{63}$$

Seventh, we tentatively set the share of expenditure on the tradable good ϕ and the natural rate of employment e^* as follows:

$$\phi = 0.5, \tag{64}$$

$$e^* = 0.9. \tag{65}$$

shares, which are usually identified with labor shares, do not include pays for self-employed workers and he did make some adjustments to correct it.

[11] In calibrations of dynamic stochastic general equilibrium (DSGE) models, the number of 0.025 is usually adopted as the quarterly rate of capital depreciation, which implies that the annual rate of capital depreciation is about 0.1. For values in parameter calibrations, see, for instance, Juillard and Villemot (2010).

[12] These seven countries are France, Germany, Italy, Japan, Netherlands, Spain, United Kingdom and United States.

Finally, we specify the capital accumulation function g as follows:

$$g(q) = 0.25q. \tag{66}$$

Note that the capital accumulation function g given in (66) satisfies (11) and (12).

Under the specifications of (56)-(66), the unique steady state of System (K_P^*, K_T^*, W^*) can be characterized by the following:

$$\frac{\partial \log K_P^*}{\partial \log E} = 1.444, \ \frac{\partial \log K_P^*}{\partial \log \tau} = 1.444, \tag{67}$$

$$\frac{\partial \log K_T^*}{\partial \log E} = 0.4444, \ \frac{\partial \log K_T^*}{\partial \log \tau} = 0.5676, \tag{68}$$

$$\frac{\partial \log W^*}{\partial \log E} = 1.444, \ \frac{\partial \log W^*}{\partial \log \tau} = 0.4444. \tag{69}$$

Thus, we can easily see that both a rise in the exchange rate E and that in the tax rate τ have a positive impact on the steady state values K_P^*, K_T^* and W^*. These results (67)-(69) are, of course, consistent with (32)-(37).

Now that the effects of changes in the exchange rate E and the tax rate τ are numerically evaluated, we fix these values as follows:

$$E = 1, \tag{70}$$

$$\tau = 0.2. \tag{71}$$

The specification in (70) simply normalizes the exchange rate E and the value in (71) is chosen based upon Baxter and King (1993).

Under the specifications of (56)-(71), the steady state of System can be calculated as follows:

$$(K_P^*, K_T^*, W^*) = (6.818, 39.97, 5.480). \tag{72}$$

We proceed to confirm the validity of Proposition 2. Under (56)-(71), the hypothesis of Proposition 2 (ii), condition (55), holds. We are easily able to find the threshold value of β, β^*, as mentioned in Proposition 2:

$$\beta^* = 0.02222. \tag{73}$$

Note that β^* is the positive root of $h(\beta)$ in (52).

We are now ready to perform numerical simulations of System with (56)-(71) to check the validity of Proposition 2. We consider the cases of $\beta = 0.01$ $(< \beta^*)$, of $\beta = 0.02223$ $(\approx \beta^*)$ and of $\beta = 0.1$ $(> \beta^*)$, where β^* is given in (73). In the following figures illustrated are the solution paths of $K_P(t)$, $K_T(t)$ and $W(t)$ of System with (56)-(71)

in these three cases.[13]

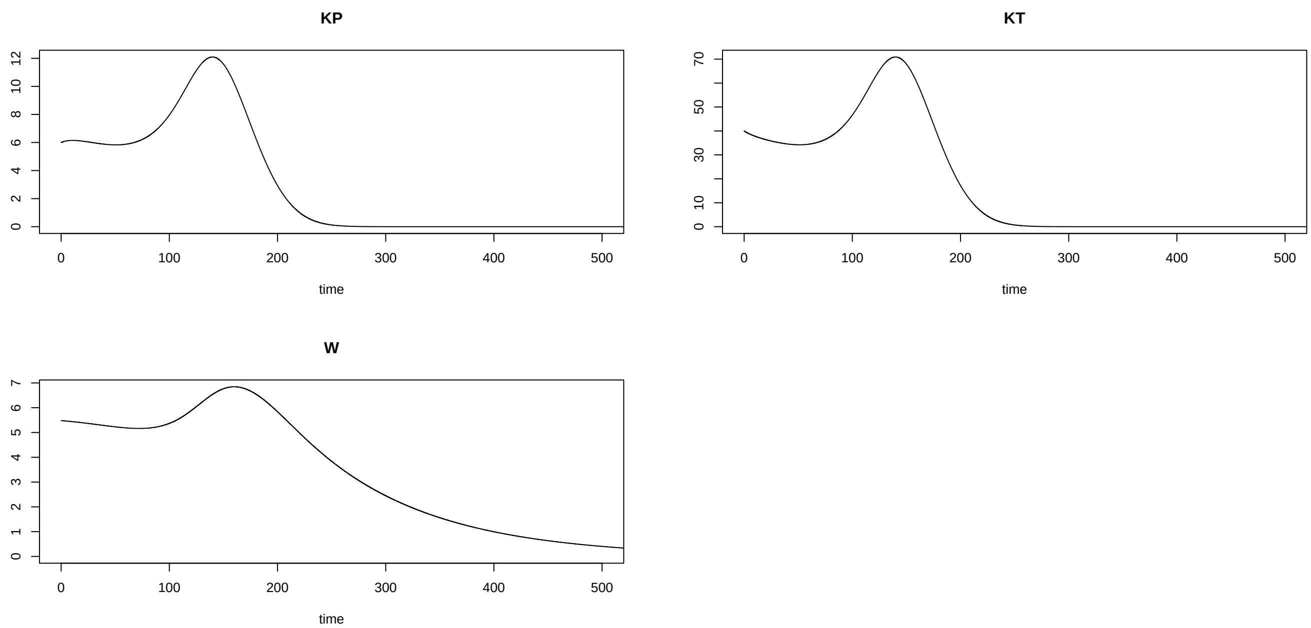

Figure 1: Solution paths with $\beta = 0.01$

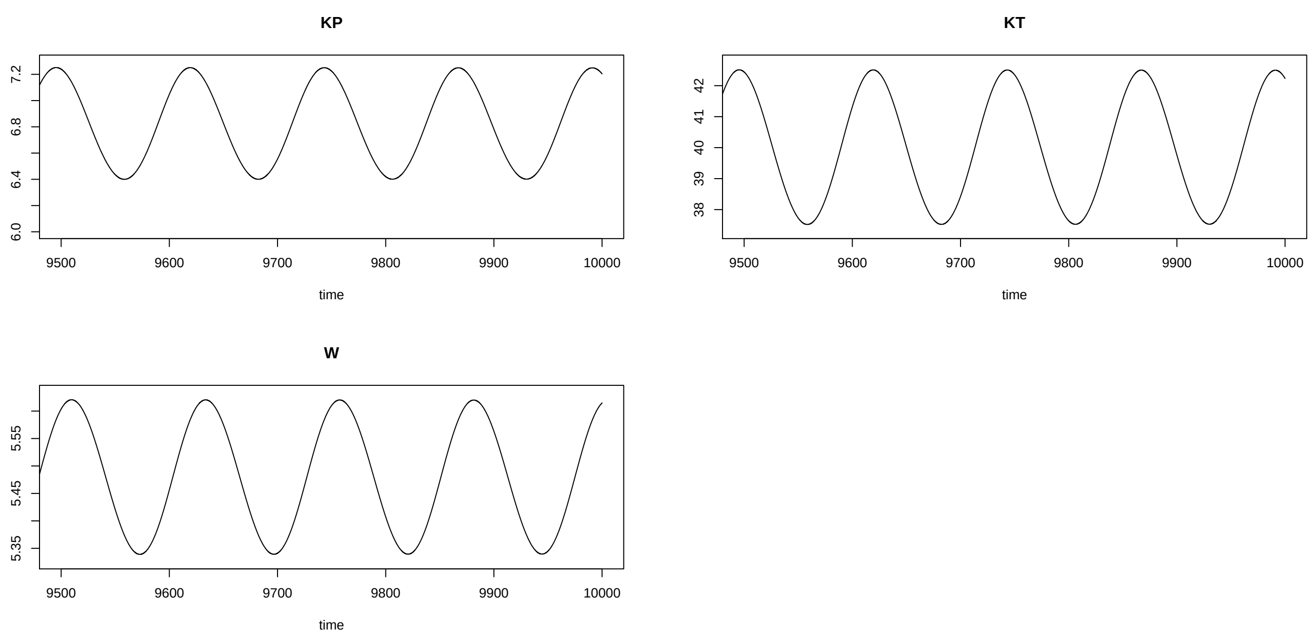

Figure 2: Solution paths with $\beta = 0.02223$

[13]The initial condition is set at $(K_P(0), K_T(0), W(0)) = (6, 39.97, 5.480)$ for all cases.

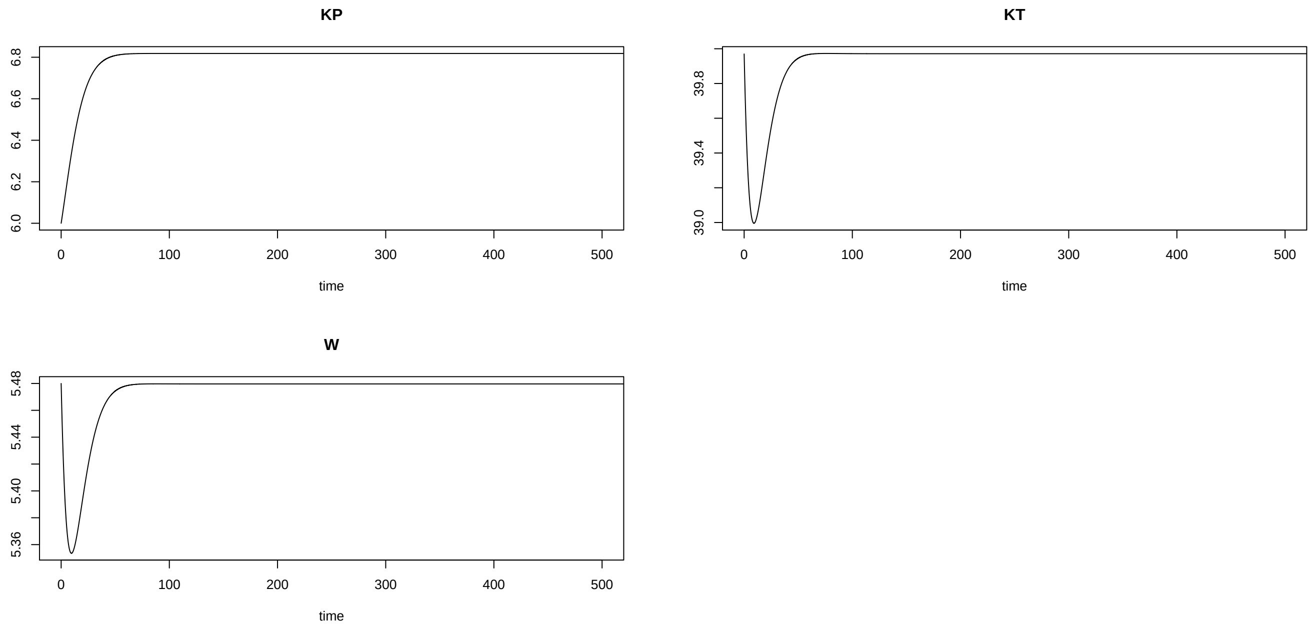

Figure 3: Solution paths with $\beta = 0.1$

We can see from figures 1-3 that the solution paths of $K_P(t)$, $K_T(t)$ and $W(t)$ diverge if β is sufficiently small, converge to periodic orbits if β is sufficiently close to the bifurcation value β^* and converge to the steady state values if β is sufficiently large. We can thus conclude that Proposition 2 is valid.

5 Conclusion

We are now ready to summarize our analysis.

Throughout this paper, we have, making use of a dynamic model of a small open economy with two kinds of goods, investigated the effects of exchange rate and tax policies on economic development, especially on the dynamics of public and private capital stocks and nominal wages. In our analysis, we have treated public capital as a stock variable and supposed that it, rather than its services, is a production input. By taking this stock-variable approach, we have been able to remove the theoretical flaws about the treatment of public capital and obtain richer analytical results than the related preceding works.

We have found that a devaluation of the home currency increases the steady state values of public and private capital stocks and the nominal wage and a rise in the tax rate is highly likely to raise these values. Also, we have seen that the same consequence can be applied to the steady state value of real aggregate output, but that a devaluation of the home currency increases the steady state value of real consumption while there can exist the optimal tax rate that maximizes this value. Furthermore, we have shown that the stability of the steady state is more likely to be attained the more flexible the nominal wage is and that a non-constant periodic orbit, representing persistent cyclical fluctuations, can arise if the degree of wage flexibility is around some threshold value. These conclusions have never been obtained in the related preceding works and may imply that exchange rate and tax policies can cause complex dynamics depending upon the degree of wage flexibility.

To conclude this paper, we mention some possible future extension of our present study. In our present analysis, we have not dwelt on rigorous microeconomic foundations, for simplicity. Theoretically, microeconomic foundations may be desirable because they enable us to discuss the effects of policies on social welfare. In this respect, it may be worthwhile to provide microeconomic foundations and welfare analysis for our framework.

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