Stability, Instability and Cycles in a Wage-led Economy and a Profit-led Economy

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Abstract

In this paper, we study the dynamic properties of a wage-led economy and a profit-led economy by using an analytical framework of the high-dimensional dynamic Keynesian model. An economy in which the aggregate effective demand is an increasing function of the real wage rate is called the 'wage-led' economy, while an economy in which the aggregate effective demand is a decreasing function of the real wage rate is called the 'profit-led' economy. We adopt an approach with two Phillips curves, which implies that the wage Phillips curve and the price Phillips curve are distinguished, and the dynamic movement of the real wage rate is governed by these two Phillips curves. We investigate the dynamic stability, instability, and the occurrence of cyclical fluctuations analytically and show that the dynamic properties of these two economies are very different. We also present some numerical simulations which support our analytical results.

Key words : wage-led economy, profit-led economy, high-dimensional dynamic Keynesian model, two Phillips curves, stability, instability, cycles

1. Introduction

In this paper, we study how the changes of the real wage rate affect the macroeconomic stability or instability theoretically by using a high dimensional Keynesian macrodynamic model with two Phillips curves. Following the procedure by Asada, Chen, Chiarella and Flaschel(2006), we formulate the wage Phillips curve and the price Phillips curve separately, and study the stabilizing and the destabilizing effects of the changes of the real wage rate, the results of which depend on the difference of the adjustment speeds of wages and prices. The important conclusion in our analysis is that the dynamic properties of the macroeconomic system crucially depend on whether the economy is the 'wage-led' or the 'profit-led'.

An economy in which the aggregate effective demand is an *increasing* function of the real wage rate is called the 'wage-led' economy, and an economy in which the aggregate effective demand is a *decreasing* function of the real wage rate is called the 'profit-led' economy.[1] In our model, the *increase* of the real wage rate induces the *increase* of the real consumption expenditure, but it induces the *decrease* of the real investment expenditure. The economy becomes the 'wage-led' if the effect on the real consumption expenditure is relatively strong, and it becomes the 'profit-led' if the effect on the real investment expenditure is relatively strong.

We show under certain conditions that the relatively *high* speed of wage adjustment compared with the speed of price adjustment has a *destabilizing* effect in a wage-led economy, while the opposite conclusion is obtained in the profit-led economy. This means that the *flexible* wage combined with the *sticky* price has a *destabilizing* effect in a wage-led economy, while it has a *stabilizing* effect in a profit-led economy.

This paper is organized as follows. In section 2, two Phillips curves approach due to Asada, Chen, Chiarella and Flaschel(2006) is summarized. From section 3 to section 5, other building blocks of our model are introduced step by step. In section6, it is shown that our model can be reduced to five dimensional nonlinear differential equations, and the properties of the equilibrium solution of this system are studied. In section 7, we study the out of equilibrium dynamics of the system. We investigate the dynamic stability, instability, and the existence of cyclical fluctuations under the 'wage-led' and the 'profit-led' assumptions. In section 8, we present some numerical simulations which

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[1] The distinction between the 'wage-led' economy and the 'profit-led' economy was introduced by the economists of French 'regulation school' such as Aglietta and Boyer for the first time. See, for example, Aglietta(1979) and Boyer(1990). The related papers in the context of static and dynamic macroeconomic analyses are Bhaduri and Marglin(1990), Marglin and Bhaduri(1990), Chen, Chiarella, Flaschel and Hung(2005), Asada, Flaschel, Jaeger and Proaňo(2007), and Bhaduri(2008).

support the analytical results in section 7. Section 9 is devoted to the economic interpretation of the results which were obtained both analytically and numerically in sections 7 and 8. Complicated and lengthy proofs of two main propositions are relegated to the mathematical appendices.

2. Two Phillips curves approach : Wage and price Phillips curves

In this section, we summarize the 'two Phillips curves' approach which was introduced by Asada, Chiarella, Flaschel and Franke(2003) and Asada, Chen, Chiarella and Flaschel(2006). In this approach, the wage Phillips curve and the price Phillips curve are formulated separately as

$$\dot{w}/w = \beta_w(e - \bar{e}) + \kappa_w(\dot{p}/p) + (1 - \kappa_w)\pi^c \tag{1}$$

$$\dot{p}/p = \beta_p(u - \bar{u}) + \kappa_p(\dot{w}/w) + (1 - \kappa_p)\pi^c \tag{2}$$

where the meanings of the symbols are as follows.

$w =$ nominal wage rate. $p =$ price level. $N^s =$ labor supply. $N =$ labor employment. $e = N/N^s =$ rate of employment = 1 − rate of unemployment. $\bar{e} =$ 'natural' rate of employment $(0 < \bar{e} < 1)$. $Y =$ actual real output level (actual real national income). $Y^p = y^p K =$ real output level in case of full capacity utilization of capital stock. $K =$ real capital stock. $y^p =$ output-capital ratio in case of full capacity utilization of capital stock, which is assumed to be a positive constant. $y = Y/K =$ actual output-capital ratio. $u = Y/Y^p = Y/(y^p K) = y/y^p =$ rate of capacity utilization of capital stock. $\bar{u} =$ 'normal' rate of capacity utilization of capital stock $(0 < \bar{u} < 1)$.

The variable π^c is called 'inflation climate' by Asada, Chen, Chiarella and Flaschel(2006). We assume that β_w, β_p, κ_w, and κ_p are parameters such that

$\beta_w > 0$, $\beta_p > 0$, $0 < \kappa_w < 1$, and $0 < \kappa_p < 1$. Later we shall treat β_w and β_p as

bifurcation parameters. A dot over the symbol (·) denotes the derivative with respect to time.

Eq. (1) is the equation of 'wage Phillips curve', which means that the rate of increase of nominal wage rate $(\dot{w}/w)$ depends on the rate of employment (e) and the weighted average of the actual price rate of inflation $(\dot{p}/p)$ and the 'inflation climate' (π^c). Eq. (2) is the equation of 'price Phillips curve', which means that $\dot{p}/p$ depends on the rate of capacity utilization of capital stock (u) and the weighted average of

$\dot{w}/w$ and π^c. These two equations reflect the adjustment of disequilibrium in the labor market and the goods market respectively.

The 'inflation climate' is intimately related to the concept of 'expected rate of inflation', but these two concepts are somewhat different. In this model, it is assumed that the economic agents know the correct values of the current rates of price and wage inflation ($\dot{p}/p$ and $\dot{w}/w$). In this sense, the 'perfect myopic foresight' is assumed in this model.[2] It is assumed, however, that the economic agents do not base their behavior only on the current rate of inflation but they base it also on the 'inflation climate' that is influenced by the intermediate run or long run inflation forecast. We shall specify the way of the formation of 'inflation climate' later.

We can rewrite equations (1) and (2) as follows by using the matrix form.

$$\begin{bmatrix} 1 & -\kappa_w \\ -\kappa_p & 1 \end{bmatrix}\begin{bmatrix} \dot{w}/w \\ \dot{p}/p \end{bmatrix} = \begin{bmatrix} \beta_w(e-\bar{e})+(1-\kappa_w)\pi^c \\ \beta_p(u-\bar{u})+(1-\kappa_p)\pi^c \end{bmatrix} \qquad (3)$$

Solving this system of equations, we obtain the following reduced forms of the wage and price Phillips curves.

$$\dot{w}/w = \begin{vmatrix} \beta_w(e-\bar{e})+(1-\kappa_w)\pi^c & -\kappa_w \\ \beta_p(u-\bar{u})+(1-\kappa_p)\pi^c & 1 \end{vmatrix} / \begin{vmatrix} 1 & -\kappa_w \\ -\kappa_p & 1 \end{vmatrix}$$

$$= \kappa\{\beta_w(e-\bar{e})+\kappa_w\beta_p(u-\bar{u})\}+\pi^c \qquad (4)$$

$$\dot{p}/p = \begin{vmatrix} 1 & \beta_w(e-\bar{e})+(1-\kappa_w)\pi^c \\ -\kappa_p & \beta_p(u-\bar{u})+(1-\kappa_p)\pi^c \end{vmatrix} / \begin{vmatrix} 1 & -\kappa_w \\ -\kappa_p & 1 \end{vmatrix}$$

$$= \kappa\{\kappa_p\beta_w(e-\bar{e})+\beta_p(u-\bar{u})\}+\pi^c \qquad (5)$$

where $\kappa = 1/(1-\kappa_w\kappa_p) > 1$.

From equations (4) and (5), we have the following equation that describes the dynamic of the real wage rate $\omega = w/p$, which is independent of the inflation climate π^c.

$$\dot{\omega}/\omega = \dot{w}/w - \dot{p}/p = \kappa\{(1-\kappa_p)\beta_w(e-\bar{e})-(1-\kappa_w)\beta_p(u-\bar{u})\} \qquad (6)$$

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[2] The term 'perfect myopic foresight' is due to Burmeister(1980).

3. Dynamics of rate of capacity utilization and rate of employment

In this section, we formulate the dynamics of the rate of capacity utilization and the rate of employment following the tradition of the 'dynamic Keynesian model' that is due to Asada, Chiarella, Flaschel and Franke(2003), Asada, Chen, Chiarella and Flaschel(2006), and Asada(2006), which is based on the principle of effective demand.

First, we suppose that the dynamic of production is governed by the following Keynesian 'quantity adjustment' process that is due to Asada(2006).

$$\dot{y} = \alpha[\frac{C+I+G-Y}{K}] = \alpha(c+g+h-y) \ ; \ \alpha > 0 \tag{7}$$

where C = real private consumption expenditure, $I = \dot{K}$ = real private investment expenditure, G = real government expenditure, $c = C/K$ = real private consumption expenditure per capital stock, $g = I/K = \dot{K}/K$ = rate of investment (rate of capital accumulation), $h = G/K$ = real government expenditure per capital stock.[3] We can rewrite Eq. (7) as follows since the rate of capacity utilization of capital stock can be written as $u = y/y^p$.

$$\dot{u} = (\alpha/y^p)(c+g+h-y^p u) \ ; \ \alpha > 0, \ y^p > 0 \tag{8}$$

Next, by definition, we can express the labor employment as follows.

$$N = \frac{(Y/K)K}{Y/N} = \frac{yK}{a} = \frac{uy^p K}{a} \tag{9}$$

where $a = Y/N$ is the average labor productivity, which is assumed as a constant.[4] In this case, we can express the rate of employment as

$$e = N/N^s = (uy^p K)/(aN^s). \tag{10}$$

Differentiating this equation with respect to time, we obtain the following law of the motion of the rate of employment.

$$\dot{e}/e = (\dot{u}/u) + (\dot{K}/K) - (\dot{N^s}/N^s) = (\dot{u}/u) + g - n \tag{11}$$

where $n = \dot{N^s}/N^s$ is the growth rate of labor supply, which is assumed to be a positive constant for simplicity.

4. Specifications of consumption function, investment function and government expenditure

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[3] In this formulation, we neglect capital depreciation and international trade for simplicity.

[4] We abstract from technical progress in this paper for simplicity. See, however, Asada(2006) as for a model of exogenous technical progress and Asada and Ouchi(2009) as for a model of endogenous technical progress in the similar analytical framework as that of the present model.

We must specify how three components of the effective demand c, g, h are determined. First, let us consider the determinant of the consumption expenditure per capital stock (c). In this paper, we adopt the following hypothesis of Kalecki(1971)'s two class model. "Workers spend all of their wage income, and capitalists save all of their profit income". In this case, we obtain the following type of consumption function.

$$C = (1 - \tau_w)\omega N = (1 - \tau_w)\omega Y / a = (1 - \tau_w)\omega y K / a = (1 - \tau_w)\omega u y^p K / a \qquad (12)$$

where τ_w is the average tax rate on wage income, which is assumed as a constant such that $0 < \tau_w < 1$. We can rewrite this equation as

$$c = C / K = (1 - \tau_w)\omega u y^p / a. \qquad (13)$$

Obviously, this 'Kaleckian' consumption function has been derived under the very restrictive assumptions. We can easily generalize, however, this type of consumption function by introducing workers' saving and capitalists' consumption following the procedures by Kaldor(1956) and Pasinetti(1974). The essential point in our model is that the real aggregate private consumption expenditure becomes an increasing function of the real wage rate. For that purpose, we need not adopt Kalecki(1971)'s extreme assumption, but it is sufficient to assume that the average propensity to consume from the wage income is greater than the average propensity to consume from the profit income, although we adopt the simple Kaleckian consumption function (13) for simplicity.

Next, we specify the investment function as follows.

$$g = g(u, \omega, i - \pi^c) \; ; \; g_u = \partial g / \partial u > 0, \; g_\omega = \partial g / \partial \omega < 0,$$

$$g_{i - \pi^c} = \partial g / \partial (i - \pi^c) < 0 \qquad (14)$$

We can rationalize this type of investment function by using the following logic. Let us start from the following more conventional 'Keynesian' investment function, which is also consistent with Tobin's q theory (cf. Yoshikawa 1980).

$$g = g(r^e, i - \pi^e) \; ; \; g_{r^e} = \partial g / \partial r^e > 0, \; g_{i - \pi^e} = \partial g / \partial (i - \pi^e) < 0 \qquad (15)$$

where r^e = expected rate of profit, i = nominal rate of interest, π^e = expected rate of price inflation, and $i - \pi^e$ = expected real rate of interest. Next, let us introduce the following assumptions.

$$r^e = r^e(u, \omega) \; ; \; r_u^e = \partial r^e / \partial u > 0, \; r_\omega^e = \partial r^e / \partial \omega < 0 \qquad (16)$$

$$\pi^e = \pi^c \tag{17}$$

Eq. (16) is an 'animal spirit function', which implies that the expected rate of profit positively correlates with the current rate of the capacity utilization of capital stock that represents the influence of effective demand on expected profitability, and it negatively correlates with the current real wage rate that represents the influence of real cost on expected profitability.[5] Eq. (17) means that the 'inflation climate' π^c is adopted in place of the 'expected rate of inflation' π^e in our model. Substituting equations (16) and (17) into Eq. (15), we obtain Eq. (14).

As for the government expenditure, we adopt the following simple assumption because the study of the fiscal stabilization policy is not the theme of the present paper.[6]

$$h = G / K = \text{constant} > 0 \tag{18}$$

5. Specifications of monetary policy rule, equilibrium condition of money market, and formation of inflation climate

We can close our model by specifying the monetary policy rule, equilibrium condition of money market, and how the inflation climate is formed. In this paper, we adopt the following specifications.

$$\dot{i} = \begin{cases} \varepsilon(\dot{p} / p - \bar{\pi}) & \text{if} \quad i > 0 \\ \max[0, \varepsilon(\dot{p} / p - \bar{\pi})] & \text{if} \quad i = 0 \end{cases} \tag{19}$$

$$M / p = \phi(i, \pi^c)Y \; ; \; \phi_i = \partial\phi / \partial i < 0, \; \phi_{\pi^c} = \partial\phi / \partial\pi^c < 0 \tag{20}$$

$$\dot{\pi}^c = \gamma\{\theta(\bar{\pi} - \pi^c) + (1-\theta)(\dot{p} / p - \pi^c)\} \; ; \; \gamma > 0, \quad 0 \leqq \theta \leqq 1 \tag{21}$$

where ε is a positive parameter, $\bar{\pi}$ is the target rate of inflation that is adopted by the central bank, and M is nominal money supply. It is assumed that the central bank announces the target rate of inflation to the public.

Eq. (19) is a version of the monetary policy of the central bank by means of the 'Taylor rule' that is originated in Taylor(1993). In fact, this is a simplified version of the Taylor interest rate monetary policy rule which was formulated by Asada(2009), and we can consider that this is a kind of the inflation targeting monetary policy rule.[7]

Eq. (20) is the 'LM equation' that describes the equilibrium condition of the money

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[5] Similar hypothesis was adopted by Bhaduri and Marglin(1990) and Marglin and Bhaduri(1990).

[6] See Asada(2006) as for the study of the macroeconomic stabilization policy by means of fiscal policy in the similar analytical framework.

[7] In this rule, the nonnegative constraint of the nominal rate of interest is explicitly considered.

market. The right hand side of this equation is the Keynesian money demand function. Since $y = Y / K = uy^p$, we can rewrite this equation as

$$M /(pK) = \phi(i, \pi^c) uy^p. \tag{22}$$

Differentiating Eq. (22) with respect to time, we have the following expression.

$$\mu = (\dot{p} / p) + g - \eta_i (\dot{i} / i) - \eta_{\pi^c} (\dot{\pi}^c / \pi^c) + (\dot{u} / u) \ ; \ \mu = \dot{M} / M, \ \ g = \dot{K} / K \tag{23}$$

where $\eta_i = \left| \dfrac{\partial\phi / \partial i}{\phi / i} \right| = -\dfrac{\partial\phi / \partial i}{\phi / i}$ and $\eta_{\pi^c} = \left| \dfrac{\partial\phi / \partial\pi^c}{\phi / \pi^c} \right| = -\dfrac{\partial\phi / \partial\pi^c}{\phi / \pi^c}$ are elasticities of the

money demand with respect to the changes of the nominal rate of interest and the inflation climate respectively. We can consider that Eq. (23) is a dynamic version of the 'LM equation'.

Eq. (21) is a formalization of a formation hypothesis of the 'inflation climate', which is a mixture of the 'forward looking' and the 'backward looking' or 'adaptive' formations. The parameter θ is the weight of the 'forward looking' element. If the public strongly believes that the actual rate of inflation is governed by the target rate of inflation $\bar{\pi}$ that is announced by the central bank, we shall have $\theta = 1$. On the other hand, we shall have $\theta = 0$ if the public does not believe the central bank's announcement at all. Therefore, we can consider that the parameter θ reflects the 'degree of credibility' of the inflation targeting by the central bank.

6. Derivation of the fundamental dynamical equations and analysis of the long run equilibrium solution

Now, we obtain the following five dimensional system of nonlinear differential equations, which constitutes a system of fundamental dynamical equations in our model.

(i) $\quad \dot{\omega} = \omega\kappa[(1-\kappa_p)\beta_w(e-\bar{e}) - (1-\kappa_w)\beta_p(u-\bar{u})] = f_1(\omega, u, e)$

(ii) $\quad \dot{u} = \alpha[\{(1-\tau_w)(\omega/a)-1\}u + \{g(u, \omega, i-\pi^c) + h\}/y^p] = f_2(\omega, u, i, \pi^c)$

(iii) $\quad \dot{e} = e[\{f_2(\omega, u, i, \pi^c)/u\} + g(u, \omega, i-\pi^c) - n] = f_3(\omega, u, e, i, \pi^c)$

(iv) $\quad \dot{i} = \begin{cases} \varepsilon[\kappa\{\kappa_p\beta_w(e-\bar{e}) + \beta_p(u-\bar{u})\} + \pi^c - \bar{\pi}] = f_4(u, e, \pi^c) & if \quad i > 0 \\ \max[0, f_4(u, e, \pi^c)] & if \quad i = 0 \end{cases}$

（ⅴ） $\dot{\pi}^c = \gamma[\theta(\bar{\pi} - \pi^c) + (1-\theta)\kappa\{\kappa_p\beta_w(e-\bar{e}) + \beta_p(u-\bar{u})\}] = f_5(u,e,\pi^c)$ （24）

Apart from the above five equations, we have the following additional equation from equations (5), (14) and (23).

$$\mu = \kappa\{\kappa_p\beta_w(e-\bar{e}) + \beta_p(u-\bar{u})\} + \pi^c + g(u,\omega,i-\pi^c) - \eta_i(\dot{i}/i)$$

$$-\eta_{\pi^c}(\dot{\pi}^c/\pi^c) + (\dot{u}/u) = f_6(\omega,u,e,i,\pi^c) \qquad (25)$$

However, the system (24) is an independent closed system that is enough to determine the dynamics of five endogenous variables (ω,u,e,i,π^e). The only role of Eq. (25) is to determine the dynamic of the growth rate of nominal money supply μ endogenously, and there is no feedback mechanism from Eq. (25) to the system of equations(24).

If we neglect the nonnegative constraint of the nominal rate of interest, we can determine the long run equilibrium solution $(\omega^*,u^*,e^*,i^*,\pi^c{}^*,\mu^*)$ of the system of equations (24) and (25) such that $\dot{\omega} = \dot{u} = \dot{e} = \dot{i} = \dot{\pi}^c = 0$ by the following system of equations.

（ⅰ） $\omega^* = (\dfrac{a}{1-\tau_w})(1 - \dfrac{n+h}{\bar{u}y^p})$ （ⅱ） $u^* = \bar{u}$ （ⅲ） $e^* = \bar{e}$

（ⅳ） $g(\bar{u},\omega^*,i^*-\bar{\pi}) = n$ （ⅴ） $(\dot{p}/p)^* = (\dot{w}/w)^* = \pi^c{}^* = \bar{\pi}$

（ⅵ） $\mu^* = \bar{\pi} + n$ （26）

Let us assume as follows.

Assumption 1. $\bar{u}y^p > n+h$

It is easy to see that we have $\omega^* > 0$ under **Assumption 1.** Next, let us consider the determination of the equilibrium nominal rate of interest i^*.

The equilibrium real rate of interest ρ^* is uniquely determined by solving the equation $g(\bar{u},\omega^*,\rho^*) = n$. Then, we can determine i^* as

$i^* = \rho^* + \bar{\pi}.$ （27）

If $i^* < 0$, the economically meaningful long run equilibrium does not exist. We have $i^* > 0$ if and only if the following inequality is satisfied.

$\bar{\pi} > -\rho^*$ （28）

This inequality is always satisfied if the target rate of inflation $\bar{\pi}$ that is selected by the central bank is sufficiently large, even in case of $\rho^* < 0$. The deflationary-biased central bank may, however, fail to satisfy this inequality. In this paper, we *assume* that the inequality (28) is in fact satisfied so that the economically meaningful long run

equilibrium point exists uniquely.

Incidentally, Eq. (26)(vi) means that the equilibrium value of the growth rate of nominal money supply μ^* is determined by the target rate of inflation $\bar{\pi}$ and the 'natural' rate of growth n, and not the other way round.

7. Dynamic stability/instability of the long run equilibrium point and the existence of cyclical fluctuations

Next, let us study the dynamic stability/instability of the economically meaningful long run equilibrium point, the existence and the uniqueness of which were studied in the previous section.

The Jacobian matrix of the five dimensional dynamical system (24) *at the long run equilibrium point* becomes as follows.

$$
J = \begin{bmatrix}
0 & f_{12} & f_{13} & 0 & 0 \\
f_{21} & f_{22} & 0 & f_{24} & f_{25} \\
f_{31} & f_{32} & 0 & f_{34} & f_{35} \\
0 & f_{42} & f_{43} & 0 & f_{45} \\
0 & f_{52} & f_{53} & 0 & f_{55}
\end{bmatrix}
\tag{29}
$$

where $f_{12} = -\omega^*\kappa(1-\kappa_w)\beta_p < 0,\quad f_{13} = \omega^*\kappa(1-\kappa_p)\beta_w > 0,$

$f_{21} = \alpha[\{(1-\tau_w)\bar{u}/a\} + (g_\omega/y^p)],\quad f_{22} = (\alpha/y^p)[-\{(n+h)/\bar{u}\} + g_u],$

$f_{24} = \alpha g_{i-\pi^c}/y^p < 0,\quad f_{25} = -\alpha g_{i-\pi^c}/y^p = -f_{24} > 0,\quad f_{31} = \bar{e}[(f_{21}/\bar{u}) + g_\omega],$

$f_{32} = \bar{e}[(f_{22}/\bar{u}) + g_u],\quad f_{34} = \bar{e}[(f_{24}/\bar{u}) + g_{i-\pi^c}] < 0,$

$f_{35} = \bar{e}[(f_{25}/\bar{u}) - g_{i-\pi^c}] = -f_{34} > 0,\quad f_{42} = \varepsilon\kappa\beta_p > 0,\quad f_{43} = \varepsilon\kappa\kappa_p\beta_w > 0,\quad f_{45} = \varepsilon > 0,$

$f_{52} = \gamma(1-\theta)\kappa\beta_p \geqq 0,\quad f_{53} = \gamma(1-\theta)\kappa\kappa_p\beta_w \geqq 0,\quad f_{55} = -\gamma\theta \leqq 0.$

The characteristic equation of this system becomes as follows.

$$
\Gamma(\lambda) = |\lambda I - J| = \lambda^5 + b_1\lambda^4 + b_2\lambda^3 + b_3\lambda^2 + b_4\lambda + b_5 = 0
\tag{30}
$$

where
$$
b_1 = -traceJ = -f_{22} - f_{55},
\tag{31}
$$

$b_2 = $ sum of all principal second-order minors of J

$$= \begin{vmatrix} 0 & f_{12} \\ f_{21} & f_{22} \end{vmatrix} + \begin{vmatrix} 0 & f_{13} \\ f_{31} & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 0 & f_{55} \end{vmatrix} + \begin{vmatrix} f_{22} & 0 \\ f_{32} & 0 \end{vmatrix} + \begin{vmatrix} f_{22} & f_{24} \\ f_{42} & 0 \end{vmatrix} + \begin{vmatrix} f_{22} & f_{25} \\ f_{52} & f_{55} \end{vmatrix}$$

$$+ \begin{vmatrix} 0 & f_{34} \\ f_{43} & 0 \end{vmatrix} + \begin{vmatrix} 0 & f_{35} \\ f_{53} & f_{55} \end{vmatrix} + \begin{vmatrix} 0 & f_{45} \\ 0 & f_{55} \end{vmatrix}$$

$$= \underset{-}{- f_{12}} \underset{+}{f_{21}} - \underset{+}{f_{13}} f_{31} - \underset{-}{f_{24}} \underset{+}{f_{42}} + \underset{-or0}{f_{22}} \underset{+}{f_{55}} - \underset{+or0}{f_{25}} f_{52} - \underset{-}{f_{34}} \underset{+}{f_{43}} - \underset{+}{f_{35}} \underset{+or0}{f_{53}}, \tag{32}$$

$b_3 = -($ sum of all third-order minors of J), $\tag{33}$

$b_4 = $ sum of all fourth-order minors of J, $\tag{34}$

$b_5 = -\det J.$ $\tag{35}$

It is worth to note that the Liénard-Chipart expression of the Routh-Hurwitz conditions for stable roots implies that a set of conditions

$$b_j > 0 \quad \text{for all} \quad j \in \{1, 2, \cdots, 5\} \tag{36}$$

is a set of *necessary* (but not sufficient) conditions for the local asymptotic stability of the equilibrium point of the dynamical system (24).[8] This means that the equilibrium point of this system becomes dynamically unstable if we have $b_j < 0$ for at least one of $j \in \{1, 2, \cdots, 5\}$.

Now, we can define the 'wage-led economy' and the 'profit-led economy' formally as follows.

Definition.

The economy in our model is called 'wage-led' if we have $\dfrac{\partial(c + g + h)}{\partial \omega} > 0$ at the equilibrium point, and it is called 'profit-led' if we have $\dfrac{\partial(c + g + h)}{\partial \omega} < 0$ at the equilibrium point.

It is quite easy to prove the following result.

Lemma.

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[8] See Gandolfo(1996) Chap. 16.

We have $|g_\omega| < (1-\tau_w)\bar{u}y^p/a$ $(f_{21}>0)$ if and only if the economy is 'wage-led', and

we have $|g_\omega| > (1-\tau_w)\bar{u}y^p/a$ $(f_{21}<0)$ if and only if the economy is 'profit-led'.

Now, we can prove the following proposition concerning the *sufficient* conditions for local *instability* of the equilibrium point.

Proposition 1.
(1) Suppose that $g_u > (n+h)/\bar{u}$ and α is sufficiently large. Then, the equilibrium point of the system (24) is locally unstable.
(2) Suppose that θ is sufficiently close to 0 and γ is sufficiently large. Then, the equilibrium point of the system (24) is locally unstable.
(3) Suppose that ε is sufficiently small, the economy is 'wage-led' $(f_{21}>0)$, and furthermore suppose that $f_{31}>0$. Then, the equilibrium point of the system (24) is

locally unstable for all sufficiently large values of $\beta_w > 0$ when $\beta_p > 0$ is fixed

arbitrarily.
(4) Suppose that ε is sufficiently small and the economy is 'profit-led' $(f_{21}<0)$. Then, the equilibrium point of the system (24) is locally unstable for all sufficiently large

values of $\beta_p > 0$ when $\beta_w > 0$ is fixed arbitrarily.

Proof.
In case of (1), we have $b_1 < 0$. In case of (2), (3) and (4), we have $b_2 < 0$. In all of these cases, one of the necessary conditions for local stability is violated. □

Next, we shall prove another proposition concerning the wage-led economy under the following set of assumptions.

Assumption 2.
(1) The economy is 'wage-led' $(f_{21}>0)$ and furthermore $f_{31}>0$.
(2) The sensitivity parameter of Taylor rule ε is sufficiently large.
(3) The sensitivity of investment with respective to the capacity utilization g_u is so small that we have $f_{22}<0$ and $f_{32}<0$.
(4) The sensitivity of investment with respect to the expected real rate of interest

$\left| g_{i-\pi^c} \right|$ is sufficiently small.

(5) The 'credibility' parameter θ is sufficiently close to 1 (including the case of $\theta = 1$).

Proposition 2.

Suppose that **Assumption 2** is satisfied and $\beta_p > 0$ is fixed arbitrarily. Then, (i) the equilibrium point of the system (24) is locally asymptotically stable for all sufficiently small values of $\beta_w > 0$, (ii) it is locally unstable for all sufficiently large values of $\beta_w > 0$, and (iii) cyclical fluctuations around the equilibrium point occur at some intermediate values of $\beta_w > 0$.

Proof. See **Appendix A.**

Next, let us consider the properties of the profit-led economy under the following assumption.

Assmption 3.
(1) The economy is 'profit-led' ($f_{21} < 0$).
(2) The speed of adjustment in the goods market α is sufficiently large.
(3) All of (3), (4), (5) in **Assumption 2** are satisfied.

Proposition 3.

Suppose that **Assumption 3** is satisfied and $\beta_w > 0$ is fixed arbitrarily. Then, (i) the equilibrium point of the system (24) is locally asymptotically stable for all sufficiently small values of $\beta_p > 0$, (ii) it is locally unstable for all sufficiently large values of

$\beta_p > 0$, and (iii) cyclical fluctuations around the equilibrium point occur at some

intermediate values of $\beta_p > 0$.

Proof. See **Appendix B**.

Proposition 2 implies that relatively low (high) values of β_w compared with β_p has

a stabilizing (destabilizing) effect under some conditions in case of the 'wage-led' economy, while **Proposition 3** implies that we have the opposite conclusion under some conditions in case of the 'profit-led' economy.

8. Numerical simulations

In this section, we present the results of some numerical simulations which support the analytical conclusions in the previous section. Throughout this section, we adopt the following parameter values.[9]

$$\alpha = 10, \quad \gamma = 0.3, \quad \kappa_p = 0.4, \quad \tau_w = 0.2, \quad y^p = 0.39,$$

$$h = 0.2, \quad \bar{u} = 0.95, \quad \bar{e} = 0.9 \tag{37}$$

First, let us consider the case of the 'wage-led' economy with the following additional specifications of the parameter values and the functional forms.

$$\theta = 0.8, \quad a = 0.8, \quad n = 0.09, \quad \kappa_w = 0.5, \quad \bar{\pi} = 0.025, \quad \beta_p = 0.22 \tag{38}$$

$$c = (1 - \tau_w)\omega u y^p / a = 0.39\omega u \tag{39}$$

$$g = 0.28\{0.7u^{0.08} - 0.37\omega - 0.7(i - \pi^c)\} \tag{40}$$

The parameter values ε and β_w are not yet specified. In fact, this numerical example corresponds to the 'wage-led' economy, because we have $f_{21} \cong 0.128 > 0$ at the equilibrium point in this case (cf. **Lemma** in the previous section).

The long run equilibrium solution of this system becomes as follows.

$$\omega^* \cong 0.217, \quad u^* = \bar{u} = 0.95, \quad e^* = \bar{e} = 0.9, \quad i^* \cong 0.447,$$

$$\pi^c* = (\dot{p}/p)^* = \bar{\pi} = 0.025 \tag{41}$$

Figures 1 – 4 are the results of the simulations of this system with the alternative specifications of the parameter values ε and β_w under the following initial conditions.

$$\omega(0) = \omega^* - 0.1, \quad u(0) = u^* - 0.1, \quad e(0) = e^* + 0.1,$$

$$i(0) = i^* + 0.05, \quad \pi^c(0) = \pi^c * -0.1 \tag{42}$$

Next, let us consider the 'profit-led' economy with the following specifications.

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[9] The purpose of this section is not to present the quantitatively realistic numerical examples but only to visualize and illustrate the qualitative conclusions which were obtained analytically in the previous section.

$$\theta = 0.95, \quad a = 3, \quad n = 0.02, \quad \kappa_w = 0.05, \quad \bar{\pi} = 0.03, \quad \beta_w = 0.22 \tag{43}$$

$$c = (1 - \tau_w)\omega u y^p / a = 0.104\omega u \tag{44}$$

$$g = 0.28\{0.85u^{0.08} - 0.51\omega - 0.5(i - \pi^c)\} \tag{45}$$

The parameter values ε and β_p are not yet specified. In fact, this numerical example corresponds to the 'profit-led' economy, because we have $f_{21} \cong -10.544 < 0$ at the equilibrium point in this case (cf. **Lemma** in the previous section).

The long run equilibrium solution of this system becomes as follows.

$$\omega^* \cong 1.523, \quad u^* = \bar{u} = 0.95, \quad e^* = \bar{e} = 0.9, \quad i^* \cong 0.026,$$

$$\pi^c* = (\dot{p}/p)^* = \bar{\pi} = 0.03 \tag{46}$$

Figures $5 - 8$ are the results of the numerical simulations of this system with the alternative specifications of the parameter values ε and β_p under the following initial conditions.

$$\omega(0) = \omega^* - 0.1, \quad u(0) = u^* - 0.1, \quad e(0) = e^* + 0.1,$$

$$i(0) = i^* + 0.35, \quad \pi^c(0) = \pi^c * - 0.1 \tag{47}$$

Figures 1 and 5 represent the base line cases of the 'wage-led' and the 'profit-led' economies respectively. In both cases, the limit cycles around the equilibrium points occur.

Comparison of Figures $1 - 3$ suggests that the *increase* of the speed of wage adjustment (β_w) at the given speed of price adjustment (β_p) tends to *destabilize* the system as well as *shorten* the period of the business cycle in case of the 'wage-led' economy. On the other hand, comparison of Figures $5 - 7$ suggests that the *increase* of β_p at given β_w tends to *destabilize* the system although there is no obvious tendency to affect the period of the business cycle in case of the 'profit-led' economy. These observations are consistent with Propositions 2 and 3 in the previous section.
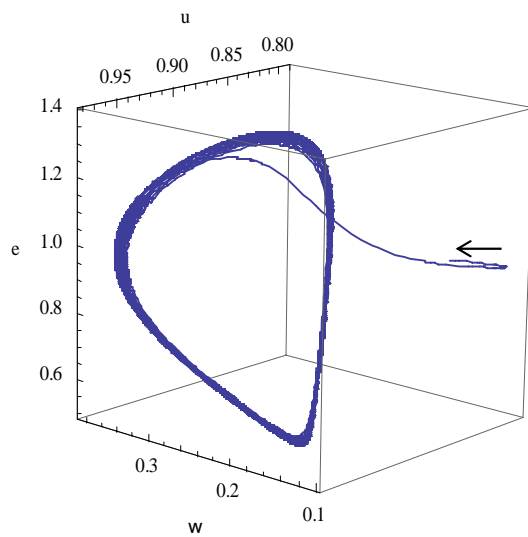
Incidentally, comparison of Figures 3 and 4 as well as comparison of Figures 7 and 8 suggests that the monetary authority(central bank) can *stabilize* the unstable macroeconomic systems by choosing the sufficiently *high* speed of response (ε) of inflation targeting in a Taylor monetary policy rule in both of the 'wage-led' and the 'profit-led' economies.

(a) ω

(b) u
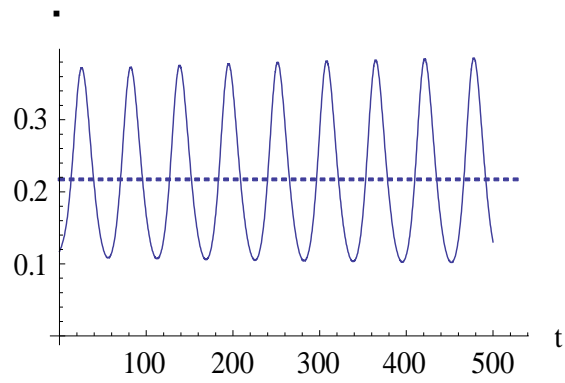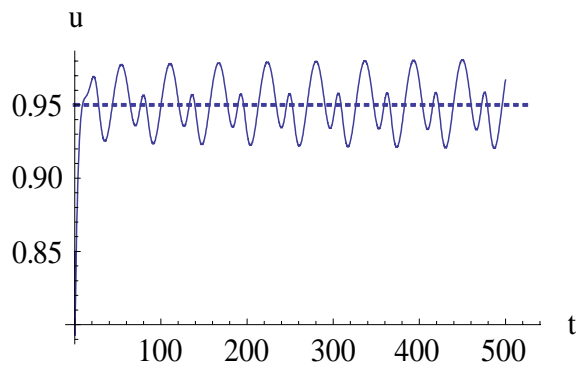
(c) ω, u, e
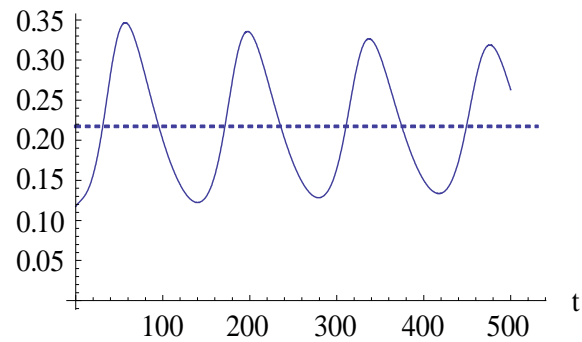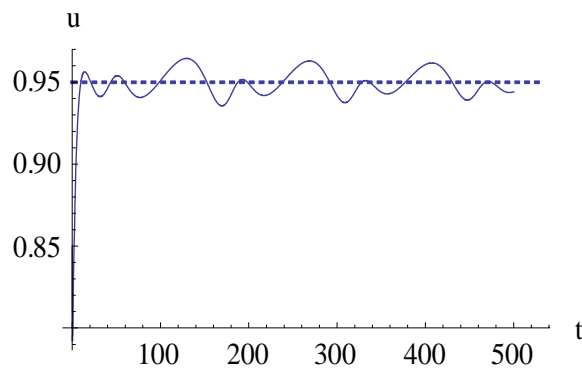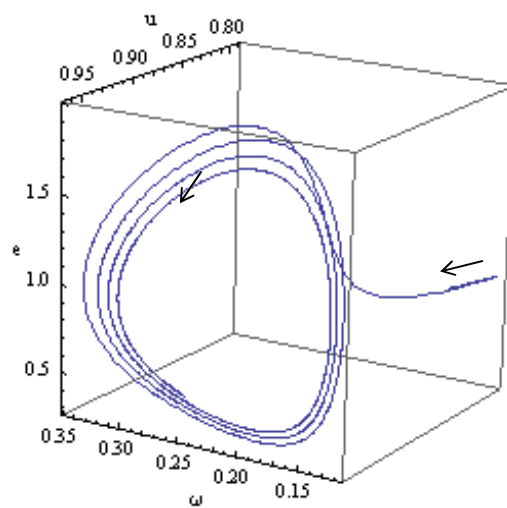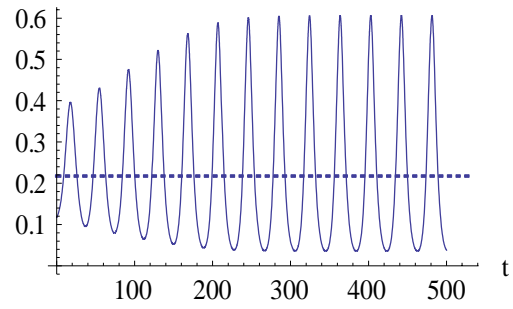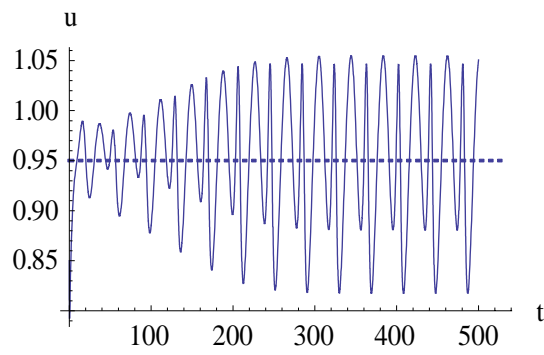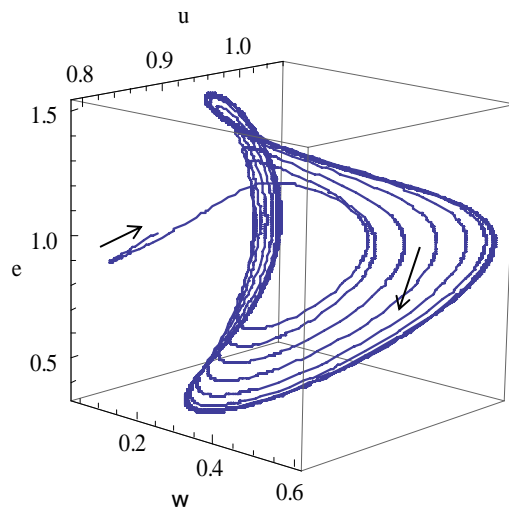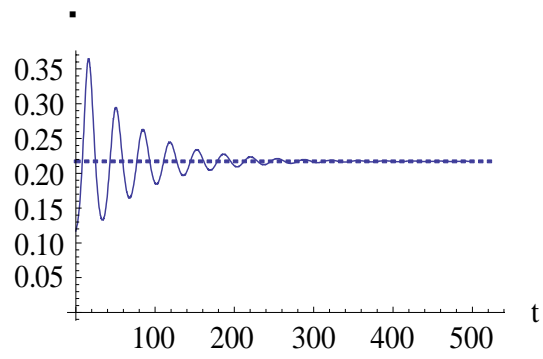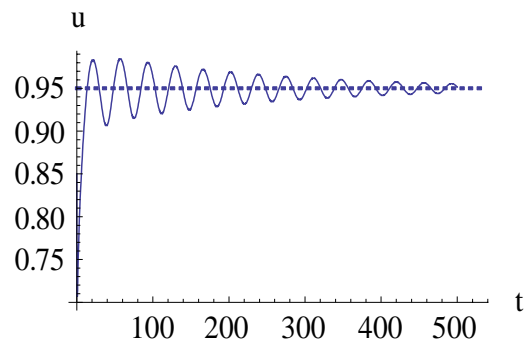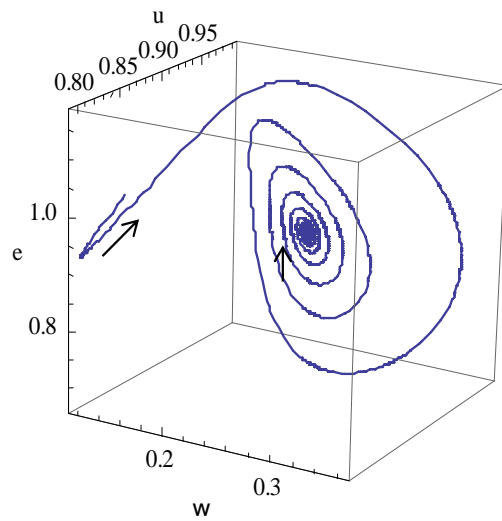
Figure 1. The 'wage-led' economy $(f_{21} \cong 0.128 > 0)$ with $\varepsilon = 0.4$, $\beta_w = 0.22$.

(a) ω

(b) u

(c) $\omega, \quad u, \quad e$

Figure 2. The 'wage-led' economy $(f_{21} \cong 0.128 > 0)$ with $\varepsilon = 0.4, \quad \beta_w = 0.06$.

(a) ω

(b) u

(c) $\omega, \quad u, \quad e$

Figure 3. The 'wage-led' economy $(f_{21} \cong 0.128 > 0)$ with $\varepsilon = 0.4, \quad \beta_w = 0.55.$

(a) ω
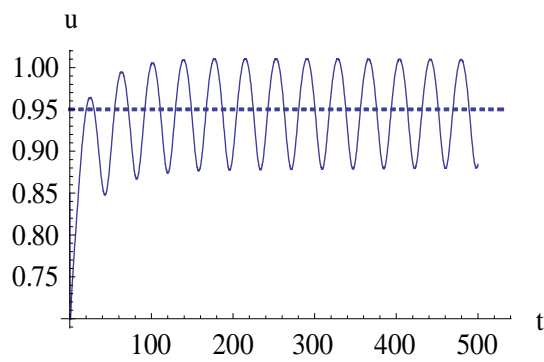

(b) u

(c) $\omega, \quad u, \quad e$

Figure 4. The 'wage-led' economy $(f_{21} \cong 0.128 > 0)$ with $\varepsilon = 0.55, \quad \beta_w = 0.55$.

(a) ω

(b) u
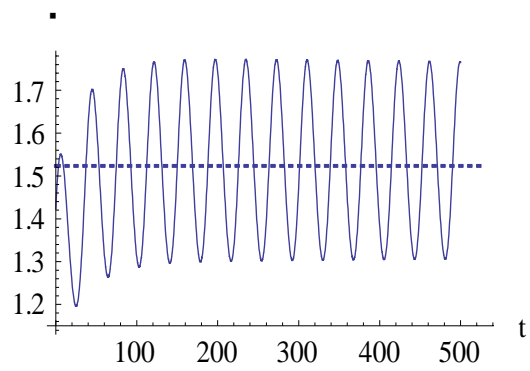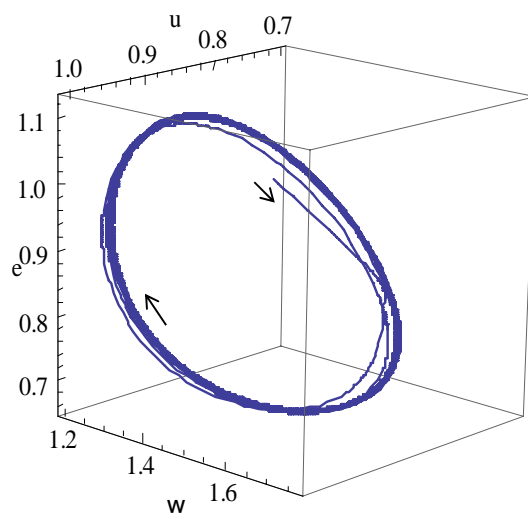
(c) ω, u, e

Figure 5. The 'profit-led' economy $(f_{21} \cong -10.544 < 0)$ with $\varepsilon = 0.25$, $\beta_p = 0.16$.

(a) ω
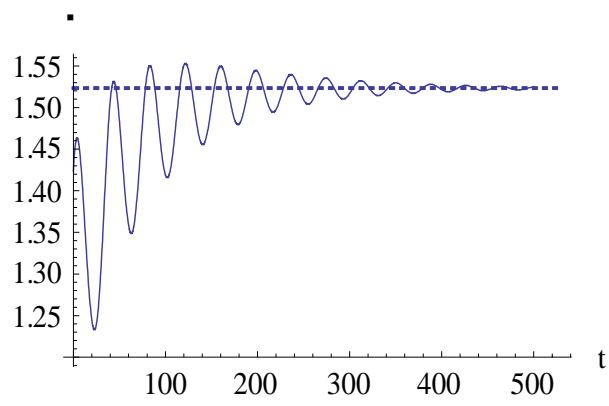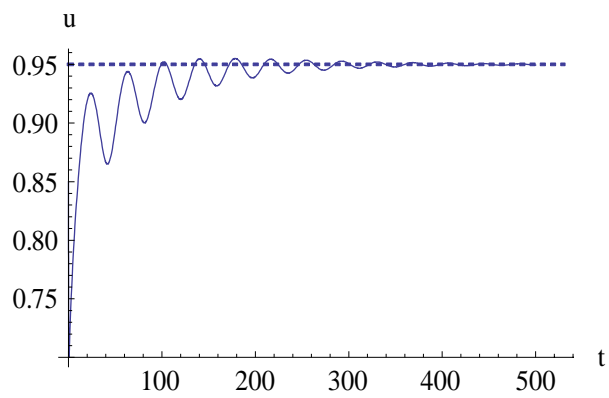
(b) u
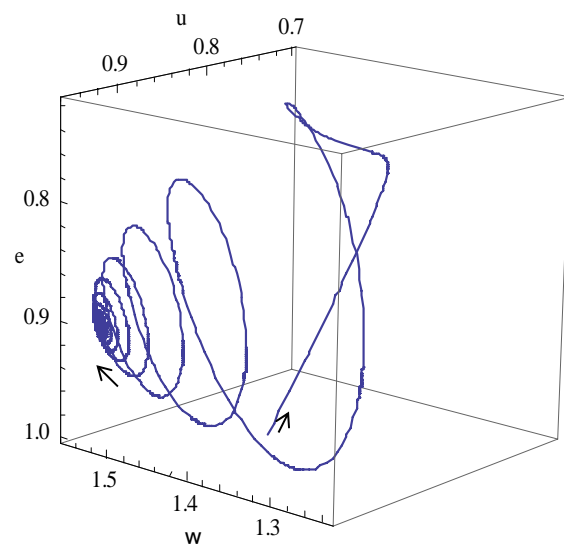
(c) $\omega, \quad u, \quad e$

Figure 6. The 'profit-led' economy $(f_{21} \cong -10.544 < 0)$ with $\varepsilon = 0.25, \quad \beta_p = 0.10$.

(a) ω

(b) u

(c) $\omega, \quad u, \quad e$
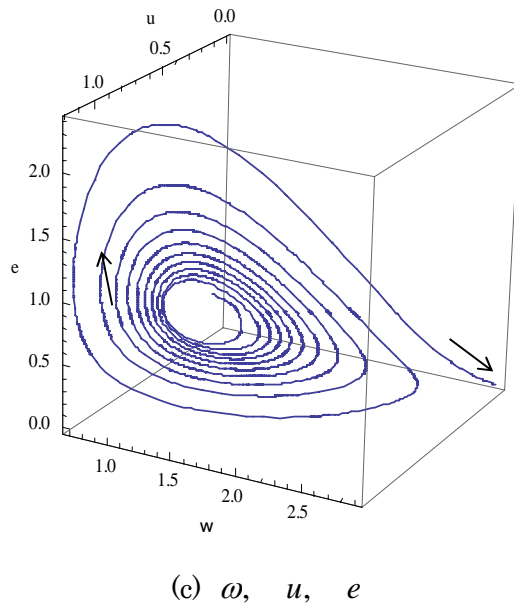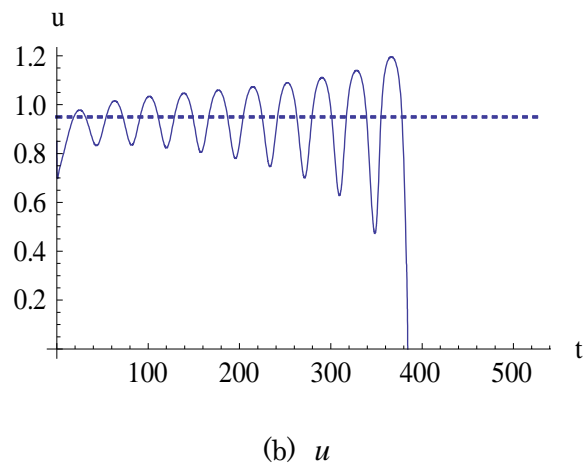
Figure 7. The 'profit-led' economy ($f_{21} \cong -10.544 < 0$) with $\varepsilon = 0.25$, $\beta_p = 0.18$.

(a) ω

(b) u

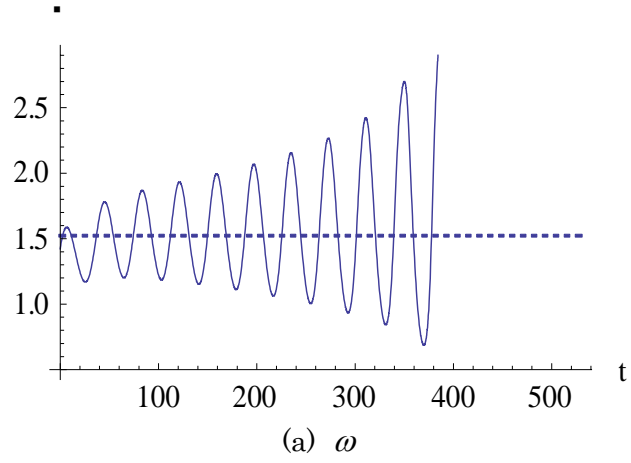(c) ω, u, e

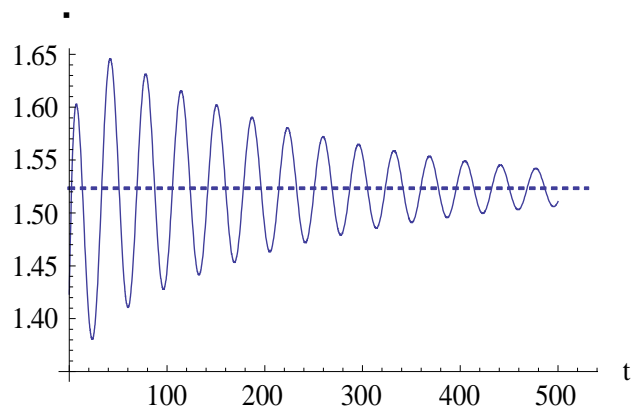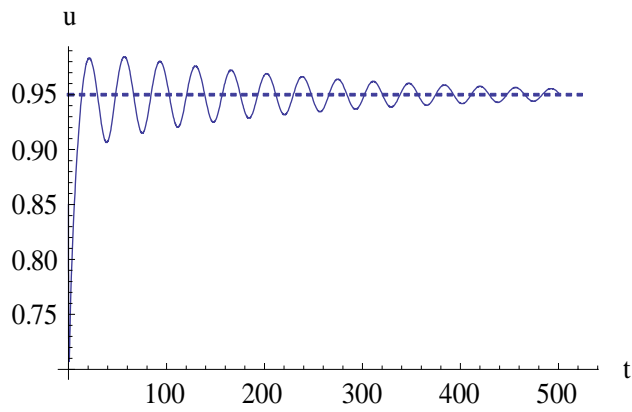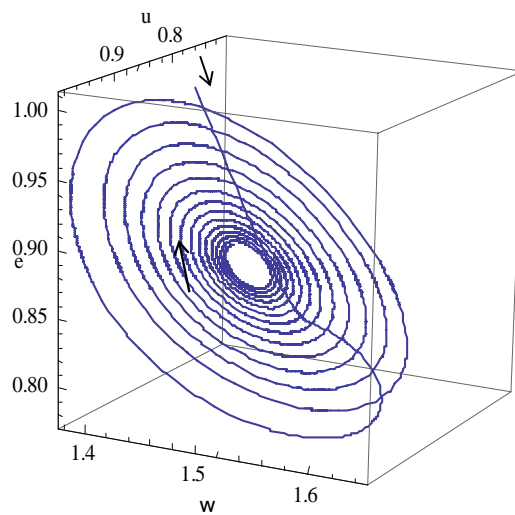**Figure 8.** The 'profit-led' economy $(f_{21} \cong -10.544 < 0)$ with $\varepsilon = 0.48$, $\beta_p = 0.18$.

9. Economic interpretation of the analytical and numerical results

In this final section, we shall try to give an economic interpretation of the results which were obtained both analytically and numerically in this paper.

Proposition 1 (1) means that the equilibrium point becomes dynamically unstable if the sensitivity of investment with respect to the changes of the rate of capacity utilization of capital stock (g_u) is sufficiently strong because of the following destabilizing positive feedback mechanism.

" Increase(decrease) of the rate of capacity utilization $\Rightarrow$ rise(fall) of the rate of investment $\Rightarrow$ further increase(further decrease) of the rate of capacity utilization due to the increase(decrease) of the effective demand."

Proposition 1 (2) means that the equilibrium point also becomes unstable if the credibility of the central bank's inflation targeting (θ) is so small that the formation of inflation climate is highly backward looking(adaptive) and the speed of adjustment of inflation climate (γ) is sufficiently high because of the following destabilizing positive feedback mechanism, which is called 'Mundell effect'(cf. Asada 2006 and Asada, Chen, Chiarella and Flaschel 2006).

" Increase(decrease) of the inflation climate $\Rightarrow$ decrease(increase) of the expected rate of interest $\Rightarrow$ rise(decrease) of the rate of investment $\Rightarrow$ increase(decrease) of the actual rate of inflation because of the increase (decrease) of the effective demand $\Rightarrow$ further increase(further decrease) of inflation climate."

Proposition 1 (3) and **Proposition 2** assert under some conditions that in a 'wage-led' economy, (1) relatively *high* speed of wage adjustment in the labor market (β_w) combined with relatively *low* speed of price adjustment in the goods market (β_p) is a *destabilizing* factor of the system, (2) relatively *low* β_w combined with relatively *high* β_p is a *stabilizing* factor of the system, and (3) cyclical fluctuations occur at some intermediate speeds of adjustment of wages and prices. In particular, we can express the destabilizing positive feedback mechanism of the wage-led economy in **Proposition 1** (3) schematically as follows.

" Increase(decrease) of the real wage rate $\Rightarrow$ rise(fall) of the rate of employment and

the rate of capacity utilization due to the increase(decrease) of effective demand $\Rightarrow$ both of nominal wage rate and price level rise(fall), but the further increase(decrease) of the real rage rate is induced because of the relatively high speed of wage adjustment compared with the speed of price adjustment."

On the other hand, **Proposition 1** (4) and **Proposition 3** assert under some conditions that in a 'profit-led' economy, (1) relatively *high* β_w combined with relatively *low* β_p is a *stabilizing* factor of the system, (2) relatively *low* β_w combined with relatively *high* β_p is a *destabilizing* factor of the system, and (3) cyclical fluctuations occur at some intermediate speeds of adjustment of wages and prices. In particular, we can express the destabilizing positive feedback mechanism of the profit-led economy in **Proposition 1** (4) as follows.

" Increase(decrease) of the real wage rates $\Rightarrow$ fall(rise) of the rate of employment and the rate of capacity utilization due to the decrease(increase) of effective demand $\Rightarrow$ both of nominal wage rate and price level fall(rise), but the further increase(decrease) of the real wage rate is induced because of the relatively low speed of wage adjustment compared with the speed of price adjustment."

In section 8 of this paper, we presented some numerical simulations which support the above conclusions which were derived analytically.

Incidentally, in a popular textbook interpretation of Macroeconomics, usually the negative correlation between the real wage rate and employment is derived by using a postulate that "the real wage rate is equal to the marginal product of labor", which was called the 'first postulate of classical economics' by Keynes(1936). This postulate depends on the behavior of the price taking perfectly competitive firms which try to maximize their profit subject to diminishing returns neglecting the constraint of effective demand. In other words, the textbook interpretation of the negative correlation between the real wage rate and employment depends only on the supply side condition of the perfectly competitive firms. In our model, however, the 'first postulate of classical economics' does not apply, because in our model the perfect competition is not assumed and the constant returns are assumed.

Unlike the 'classical' model, in our model both of positive correlation and negative correlation between the real wage rate and employment are possible due to alternative

demand side conditions rather than the supply side conditions. In fact, in our model the *positive* correlation between these variables applies in case of the 'wage-led' economy, while the *negative* correlation applies in case of the 'profit-led' economy because of the different effects of the changes of the real wage rate on the effective demand.[10] We cannot say *a priori* which case applies in the real economy. At least theoretically, both cases are possible.[11] Incidentally, our analysis in this paper suggests that the appropriate monetary policy by the central bank can stabilize the macroeconomic system even if there are some destabilizing factors due to wage or price adjustment which are peculiar to the 'wage-led' or the 'profit-led' economies.

Postscript and Acknowledgment

Appendix A : Proof of Proposition 2.

(i) First, let us consider the case of $\theta = 1$. In this case, the Jacobian matrix in Eq. (29) in the text becomes as follows.

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[10] The same fact were already pointed out by Bhaduri and Marglin(1990), Marglin and Bhaduri(1990), and Bhaduri(2008). See also Chen, Chiarella, Flaschel and Hung(2005) and Asada, Flaschel, Jaeger and Proaño(2007).

[11] Harada and Egawa(2003) presented an empirical analysis of the Japanese economy under the 'deflationary depression' in the 1990s, which supports the hypothesis that "the crash of the nominal wage rigidity and the price deflation due to the deflationary-biased monetary policy of BOJ(Bank of Japan) aggravated the 'great stagnation' of the Japanese economy in the 1990s." If their empirical analysis is correct, we can conclude that the Japanese economy in this period was the 'profit-led' economy.

$$J = \begin{bmatrix} 0 & f_{12} & f_{13} & 0 & 0 \\ f_{21} & f_{22} & 0 & f_{24} & f_{25} \\ f_{31} & f_{32} & 0 & f_{34} & f_{35} \\ 0 & f_{42} & f_{43} & 0 & f_{45} \\ 0 & 0 & 0 & 0 & -\gamma \end{bmatrix} \tag{A1}$$

Then, the characteristic equation in Eq. (30) in the text becomes

$$\Gamma(\lambda) = |\lambda I - J| = |\lambda I - J_4|(\lambda + \gamma) = 0, \tag{A2}$$

where

$$J_4 = \begin{bmatrix} 0 & f_{12} & f_{13} & 0 \\ f_{21} & f_{22} & 0 & f_{24} \\ f_{31} & f_{32} & 0 & f_{34} \\ 0 & f_{42} & f_{43} & 0 \end{bmatrix}. \tag{A3}$$

The characteristic equation (A2) has a negative real root $\lambda_5 = -\gamma$, and other four roots are determined by the following equation.

$$\Gamma_4(\lambda) = |\lambda I - J_4| = \lambda^4 + a_1 \lambda^3 + a_2 \lambda^2 + a_3 \lambda + a_4 = 0 \tag{A4}$$

where

$$a_1 = -traceJ_4 = -\underset{-}{f_{22}} > 0 \tag{A5}$$

because of **Assumption 2** (3),

a_2 = sum of all principal second-order minors of J_4

$$= \begin{vmatrix} 0 & f_{12} \\ f_{21} & f_{22} \end{vmatrix} + \begin{vmatrix} 0 & f_{13} \\ f_{31} & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} f_{22} & 0 \\ f_{32} & 0 \end{vmatrix} + \begin{vmatrix} f_{22} & f_{24} \\ f_{42} & 0 \end{vmatrix} + \begin{vmatrix} 0 & f_{34} \\ f_{43} & 0 \end{vmatrix}$$

$$= -\underset{-}{f_{12}}\underset{+}{f_{21}} - \underset{+}{f_{13}}\underset{+}{f_{31}} - \underset{-}{f_{24}}\underset{+}{f_{42}} - \underset{-}{f_{34}}\underset{+}{f_{43}}$$

$$= \kappa[\{\omega^*(1-\kappa_w)\underset{+}{f_{21}} - \underset{-}{f_{24}}\varepsilon\}\beta_p + \{-\omega^*(1-\kappa_p)\underset{+}{f_{31}} - \underset{-}{f_{34}}\varepsilon\kappa_p\}\beta_w] \tag{A6}$$

because of **Assumption 2** (1),

$a_3 = -($ sum of all principal third-order minors of $J_4)$

$$= -\begin{vmatrix} f_{22} & 0 & f_{24} \\ f_{32} & 0 & f_{34} \\ f_{42} & f_{43} & 0 \end{vmatrix} - \begin{vmatrix} 0 & f_{13} & 0 \\ f_{31} & 0 & f_{34} \\ 0 & f_{43} & 0 \end{vmatrix} - \begin{vmatrix} 0 & f_{12} & 0 \\ f_{21} & f_{22} & f_{24} \\ 0 & f_{42} & 0 \end{vmatrix} - \begin{vmatrix} 0 & f_{12} & f_{13} \\ f_{21} & f_{22} & 0 \\ f_{31} & f_{32} & 0 \end{vmatrix}$$

$$= f_{43}(-f_{24}\,f_{32}+f_{22}\,f_{34})+f_{13}(-f_{32}\,f_{21}+f_{22}\,f_{31})$$

$$= \kappa\beta_w\{\varepsilon(-f_{24}\,f_{32}+f_{22}\,f_{34})+\omega*(1-\kappa_p)(-f_{32}\,f_{21}+f_{22}\,f_{31})\}$$

$$= \kappa\beta_w\overline{e}\{\varepsilon(-f_{24}\,g_u+f_{22}\,g_{i-\pi^c})+\omega*(1-\kappa_p)(-f_{21}\,g_u+f_{22}\,g_\omega)\}>0 \qquad (A7)$$

because of **Assumption 2** (2) and (3),

$$a_4 = \det J_4 = \begin{vmatrix} 0 & f_{12} & f_{13} & 0 \\ f_{21} & f_{22} & 0 & f_{24} \\ f_{31} & f_{32} & 0 & f_{34} \\ 0 & f_{42} & f_{43} & 0 \end{vmatrix} = \omega*\kappa^2\varepsilon \begin{vmatrix} 0 & -(1-\kappa_w)\beta_p & (1-\kappa_p)\beta_w & 0 \\ f_{21} & f_{22} & 0 & f_{24} \\ f_{31} & f_{32} & 0 & f_{34} \\ 0 & \beta_p & \kappa_p\beta_w & 0 \end{vmatrix}$$

$$= \omega*\kappa^2\varepsilon \begin{vmatrix} 0 & 0 & \{1-\kappa_p+(1-\kappa_w)\kappa_p\}\beta_w & 0 \\ f_{21} & f_{22} & 0 & f_{24} \\ f_{31} & f_{32} & 0 & f_{34} \\ 0 & \beta_p & \kappa_p\beta_w & 0 \end{vmatrix}$$

$$= \omega*\kappa^2\varepsilon(1-\kappa_w\kappa_p)\beta_w \begin{vmatrix} f_{21} & f_{22} & f_{24} \\ f_{31} & f_{32} & f_{34} \\ 0 & \beta_p & 0 \end{vmatrix} = -\omega*\kappa^2\varepsilon(1-\kappa_w\kappa_p)\beta_w\beta_p \begin{vmatrix} f_{21} & f_{24} \\ f_{31} & f_{34} \end{vmatrix}$$

$$= \omega*\kappa^2\varepsilon(1-\kappa_w\kappa_p)\beta_w\beta_p(f_{24}\,f_{31}-f_{21}\,f_{34})$$

$$= \omega*\kappa^2\varepsilon(1-\kappa_w\kappa_p)\beta_w\beta_p\overline{e}(f_{24}\,g_\omega-f_{21}\,g_{i-\pi^c})$$

$$= -\omega*\kappa^2\varepsilon(1-\kappa_w\kappa_p)\beta_w\beta_p\overline{e}\alpha(1-\tau_w)(\overline{u}/a)\,g_{i-\pi^c}>0. \qquad (A8)$$

It is well known that the Routh-Hurwitz conditions for stable roots of the characteristic equation (A4) are given by the following set of inequalities (cf. mathematical appendix of Asada, Chiarella, Flaschel and Franke 2003).

$$a_j>0 \ \ \text{for all} \ \ j\in\{1,2,3,4\}, \quad \Phi = a_1a_2a_3 - a_1^2a_4 - a_3^2>0 \qquad (A9)$$

Inequalities (A5), (A7) and (A8) mean that the conditions $a_1>0$, $a_3>0$, and $a_4>0$ are always satisfied irrespective of the value of the parameter $\beta_w>0$ under **Assumption 2** (1), (2), (3). On the other hand, Eq. (A6) means that we have $a_2>0$ for all sufficiently small values of $\beta_w>0$. By the way, we can write that

$$\Phi(\beta_w) = \{a_1 a_2(\beta_w) - a_3(\beta_w)\}a_3(\beta_w) - a_1^2 a_4(\beta_w), \quad \Phi(0) = 0 \tag{A10}$$

because $a_3(0) = a_4(0) = 0$. Furthermore, we have

$$\Phi'(0) = \frac{\partial \Phi(0)}{\partial \beta_w} = a_1 a_2(0) a_3' - a_1^2 a_4' = \varepsilon(A\varepsilon + B) \tag{A11}$$

where

$$a_2(0) = \{\omega^*(1-\kappa_w)\underset{+}{f_{21}} - \underset{-}{f_{24}}\,\varepsilon\}\kappa\beta_p > 0, \tag{A12}$$

$$a_3' = \frac{\partial a_3}{\partial \beta_w} = \kappa\bar{e}\varepsilon(-\underset{-}{f_{24}}\,\underset{+}{g_u} + \underset{-}{f_{22}}\,\underset{-}{g_{i-\pi^c}}) > 0, \tag{A13}$$

$$a_4' = \frac{\partial a_4}{\partial \beta_w} = \omega^*\kappa^2\varepsilon(1-\kappa_w\kappa_p)\beta_p\bar{e}(\underset{-}{f_{24}}\,\underset{-}{g_\omega} - \underset{+}{f_{21}}\,\underset{-}{g_{i-\pi^c}}) > 0, \tag{A14}$$

$$A = -a_1\underset{+}{f_{24}}\,\kappa^2\beta_p\bar{e}(-\underset{-}{f_{24}}\,\underset{+}{g_u} + \underset{-}{f_{22}}\,\underset{-}{g_{i-\pi^c}}) > 0. \tag{A15}$$

Eq. (A11) and inequality (A15) mean that we have $\Phi'(0) > 0$ for all sufficiently large values of $\varepsilon > 0$. Therefore, all of the Routh-Hurwitz conditions of the four dimensional system (A9) are satisfied for all sufficiently small values of $\beta_w > 0$ under **Assumption 2** (1), (2), (3). This means that all of the roots of the characteristic equation (30) in the text have negative real parts under **Assumption 2** (1), (2), (3) in case of $\theta = 1$. By continuity, however, all of the roots of Eq. (30) still have negative real parts even if $\theta < 1$, as long as θ is sufficiently close to 1. This completes the proof of **Proposition 2** (i).

(ii) First, let us consider the case of $\theta = 1$ and $\left| g_{i-\pi^c} \right|$ is so small that we have

$-\omega^*(1-\kappa_p)\underset{+}{f_{31}} - \underset{-}{f_{34}}\,\varepsilon\kappa_p < 0$ (note that we have $\lim_{g_{i-\pi^c} \to 0} f_{34} = 0$). In this case, we

have $a_2 < 0$ for all sufficiently large values of $\beta_w > 0$ under **Assumption 2** (1), which means that one of the necessary conditions of the local stability is violated for all sufficiently large values of $\beta_w > 0$. By continuity, this conclusion also applies to the case of $\theta < 1$, as long as θ is sufficiently close to 1. This completes the proof of **Proposition 2** (ii).

(iii) It follows from **Proposition 2** (i) and (ii) that there exists at least one 'bifurcation

point' $\beta_w^0 > 0$, at which the real part of at least one root becomes zero, by continuity.

Incidentally, we have

$$\Gamma(0) = \left| -J \right| = -\det J = \gamma(\det J_4) > 0 \tag{A16}$$

in case of $\theta = 1$ from Eq. (30) in the text, equations (A1), (A3), and (A8). Therefore, by continuity, we have $\Gamma(0) = -\det J > 0$ even if $\theta < 1$, as long as θ is sufficiently close to 1. This means that the characteristic equation $\Gamma(\lambda) = 0$ does not have the real root such that $\lambda = 0$, so that at least one pair of pure imaginary roots exist at the 'bifurcation point'. If the characteristic equation has only one pair of pure imaginary roots at $\beta_w = \beta_w^0$, such a point is a 'Hopf bifurcation' point, and the existence of the closed orbits is ensured at some range of the parameter values which are sufficiently close to β_w^0 (cf. Gandolfo 1996 Chap. 25). If the characteristic equation has two pairs of pure imaginary roots at $\beta_w = \beta_w^0$, such a point is not Hopf bifurcation point, and the existence of the closed orbits is not necessarily ensured. Even in this case, however, the existence of the cyclical fluctuations at some range of the parameter values which are sufficiently close to β_w^0 is ensured, because of the existence of two pairs of complex roots. □

Appendix B : Proof of Proposition 3.

(i) If $f_{21} < 0$, we automatically have $f_{31} < 0$. Considering this fact, first, let us consider the case of $\theta = 1$. In this case, the characteristic equation (30) in the text has a negative real root $\lambda_5 = -\gamma$, and other four roots are determined by Eq. (A4), where

$$a_1 = -\underset{-}{f_{22}} > 0 \tag{B1}$$

because of **Assumption 2** (3),

$$a_2 = \kappa[\{\omega*(1-\kappa_w)\underset{-}{f_{21}} - \underset{-}{f_{24}}\,\varepsilon\}\beta_p + \{-\omega*(1-\kappa_p)\underset{-}{f_{31}} - \underset{-}{f_{34}}\,\varepsilon\kappa_p\}\beta_w \tag{B2}$$

because of **Assumption 3** (1),

$$a_3 = \kappa\beta_w\bar{e}\{\varepsilon(-\underset{-}{f_{24}}\,\underset{+}{g_u} + \underset{-}{f_{22}}\,\underset{-}{g_{i-\pi^c}}) + \omega*(1-\kappa_p)(-\underset{-}{f_{21}}\,\underset{+}{g_u} + \underset{-}{f_{22}}\,\underset{-}{g_\omega}) > 0 \tag{B3}$$

because of **Assumption 3** (1) and **Assumption 2** (3),

$$a_4 = \omega^* \kappa^2 \varepsilon (1 - \kappa_w \kappa_p) \beta_w \beta_p \bar{e} \alpha (1 - \tau_w)(\bar{u}/a) \underset{-}{g_{i-\pi^e}} > 0. \tag{B4}$$

Now, we have the following properties.

$$\lim_{\beta_p \to 0} a_2 = \{-\omega^*(1-\kappa_p)\underset{-}{f_{31}} - \underset{-}{f_{34}} \varepsilon \kappa_p\}\beta_w > 0 \tag{B5}$$

$$\lim_{\beta_p \to 0} \Phi = \lim_{\beta_p \to 0} (a_1 a_2 a_3 - a_1^2 a_4 - a_3^2) = \lim_{\beta_p \to 0} \{\underset{+}{(a_1} \underset{+}{a_2} - \underset{+}{a_3})\underset{+}{a_3}\} \tag{B6}$$

$$E = \lim_{\beta_p \to 0} (a_1 a_2 - a_3)$$

$$= \kappa \beta_w [\underset{-}{f_{22}}\{\omega^*(1-\kappa_p)\underset{-}{f_{31}} + \underset{-}{f_{34}} \varepsilon \kappa_p\} - \bar{e}\varepsilon(-\underset{-}{f_{24}} \underset{+}{g_u} + \underset{-}{f_{22}} \underset{-}{g_{i-\pi^c}})$$

$$- \bar{e}\omega^*(1-\kappa_p)(-\underset{-}{f_{21}} \underset{+}{g_u} + \underset{-}{f_{22}} \underset{-}{g_\omega})] \tag{B7}$$

Since $\lim_{g_{i-\pi^c} \to 0} f_{24} = \lim_{g_{i-\pi^c} \to 0} f_{34} = 0,$ we obtain

$$\lim_{g_{i-\pi^c} \to 0} E = \kappa \beta_w \omega^*(1-\kappa_p)\{\underset{-}{f_{22}}(\underset{-}{f_{31}} - \bar{e} \underset{-}{g_\omega}) + \bar{e} \underset{-}{f_{21}} \underset{+}{g_u})$$

$$= \kappa \beta_w \omega^*(1-\kappa_p)\{\underset{-}{f_{22}} \bar{e}(\underset{-}{f_{21}}/\bar{u}) + \bar{e} \underset{-}{f_{21}} \underset{+}{g_u}\}$$

$$= \kappa \beta_w \omega^*(1-\kappa_p)\bar{e} f_{21}(\alpha)\{\underset{-}{f_{22}}(\alpha) + \underset{+}{g_u}\} > 0 \tag{B8}$$

if α is sufficiently large, because we have $\underset{-}{f_{22}}(\alpha) + \underset{+}{g_u} < 0$ if α is sufficiently

large. Equations (B6) – (B8) mean that we have $\Phi > 0$ for all sufficiently small

values of $\beta_p > 0$ if α is sufficiently large and $\left|g_{i-\pi^c}\right|$ is sufficiently small, by

continuity. In this case, all of the Routh-Hurwitz conditions for stable roots (A9) are

satisfied. This proves the local stability of the equilibrium point for all sufficiently

small values of $\beta_p > 0$ under **Assumption 3** (1), **Assumption 3** (2), and **Assumption 2**

(3) in case of $\theta = 1$. By continuity, however, the local stability is also ensured even

in case of $\theta < 1$, as long as θ is sufficiently close to 1. This completes the proof of

Proposition 3 (i).

(ii) First, let us consider the case of $\theta = 1$ and $\left|g_{i-\pi^c}\right|$ is so small that we have

$\omega * (1 - \kappa_w) \underset{-}{f_{21}} - \underset{-}{f_{24}} \, \varepsilon < 0$ (note that we have $\displaystyle \lim_{g_{i-\pi^e} \to 0} f_{24} = 0$). In this case, we have

$a_2 < 0$ for all sufficiently large values of $\beta_p > 0$ under **Assumption 3** (1), which means that one of the necessary conditions of the local stability is violated for all sufficiently large values of $\beta_p > 0$. By continuity, this conclusion also applies to the case of $\theta < 1$, as long as θ is sufficiently close to 1. This completes the proof of **Proposition 3** (ii).

(iii) The method of the proof of **Proposition 3** (iii) is almost the same as that of **Proposition 2** (iii). □

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