EFFECTS OF INTEGRATION AND UPSTREAM MARKET STRUCTURES

Chuan Yang ^{and} Yasuo Kawashima

Faculty of Economics

Chuo University

May, 2011

Abstract

This paper shows that vertical integration can lower equilibrium prices in a downstream market, whether an upstream market is monopolized or not. However, there is not necessarily a trade-off between competitiveness and market efficiency. In fact, the integration enables an upstream monopolist to exploit leverage in a market and to exclude a rival from that market, but firms other than a monopolist can not drive a rival from a market. Thus, upstream market structure matters. It will be shown when integration promotes efficiency and when it does not, and also that situations of "keiretsu", found in Japanese industries, emerge.

Acknowledgement

Financial support from Chuo University (Research Grant, 0382) and the Japan Ministry of Education, Culture, Sports, Science and Technology (Grant-in-Aid for Scientific Research (C),20604001) is highly appreciated. We would like to thank Professors Akio Matsumoto, Toichiro Asada and Masahiro Yabuta of Chuo University for their suggestions and M. Hosoe of Kumamoto Gakuin University for their kind advice.

JEL classification: L2, L4, L5

Keywords: vertical integration, difference in productivity, cost effect, productivity effect, upstream market structure.

Address: 742-1, Higashinakano, Hachioji-shi, Tokyo 192-0393,Japan fax:+81 426 74 3425 e-mail: yasuokaw@tamacc.chuo-u.ac.jp

1 Introduction

Vertical integration is a typical means to achieve market foreclosure. There is the fear, however, that vertical integration makes a market inefficient by reducing competition through market foreclosure. On the other hand, if it is competitively neutral or improves market efficiencies, the integration is likely to lower market prices through efficiency gains of the merger. Thus, it is crucially important and controversial not only for the firm's management strategies, but also for courts and regulators to determine whether the merger is competition-neutral or not.

There have thus been two lines of thought about effects of vertical integration; one is that it is competitively neutral or promotes market efficiencies through its efficiency gains; see, for example, Bork (1978) and Chen (2001). Chen (2001) in particular makes an important contribution on the subject; he develops an equilibrium theory of vertical integration and shows that the merger lowers downstream prices and that it can do so even if market foreclosure arises. The other line of thouight is that vertical integration has an anticompetitive effect on a market; see, for example, Salinger (1988,1991), Ordover, Saloner and Salop (1990), Hart and Tirole (1990), Ma (1997), Riordan (1998), Choi and Yi (2000), Church and Gandale (2000). However, Chen and Riordan (2007) show that the vertical integration and the exclusive contract lead to market foreclosure. If the integration has an anticompetitive effect, market price is increased because it confers more monopolistic power on the integrated firm.

Following Hart and Tirole (1990), Chen and Riordan (2007) made major contributions on the analysis of effects of vertical integration in vertically related markets using the contract theory. Their analysis focuses on the collusive behavior of firms in the vertically related markets because a final good is differentiated. Thus, downstream prices are increased by the collusion of downstream firms. By contrast, the present paper focuses on a market where a final good is homogeneous and then analyzes effects of integration on the basis of competitive behavior of firms without paying attention to incentives for integration. Firms compete à la Cournot in the upstream and the downstream markets and we will show important features of this in vertically related markets. The characteristics of the present model are that downstream firms differ in their productivity and that upstream market structure has an important influence on the analysis. Even if the merger is competitively neutral in the sense that it does not result in foreclosing rival firms, market efficiencies are enhanced because post-merger equilibrium price goes down.¹ Thus, our model shows that there are no trade-offs between competitiveness and

¹These observations are consistent with what Hortacsu and Syverson (2007) reported in their empirical analysis of cements and ready-mixed industries in the US.

market efficiency.

We model these markets as multi-stage games with perfect and complete information. Upstream firms supply inputs in the upstream market and downstream firms demand them. Downstream firm i, i = 1, 2, produces one unit of a consumer good with the use of the α_i units of inputs and sells a consumer good in the downstream market. Note that α_i is a fixed constant and $\alpha_1 \neq \alpha_2$.² Backward induction is used to analyze these games. Given the total demand for their outputs, the downstream firms compete to supply it. Once equilibrium outputs of downstream firms are determined, demand for inputs to be supplied by the upstream firms is derived from equilibrium outputs of the downstream firms. With known total demand for inputs by the downstream firms, the upstream firms compete to supply it in a market where demand for inputs is a function of input prices. Thus, equilibrium prices in the upstream and the downstream market are determined in these markets. A detailed version of our model is shown in Figures 1 and 2 in Sections 2 and 3 respectively.

In what follows, we will distinguish two types of games; a Monopoly-Duopoly game (hereafter, MD game) and a Double-Duopoly game (hereafter, DD game). The MD game has an upstream monopolist and two downstream firms, whereas the DD game has two independent firms in both markets. We shall argue that in these games, the effect of the integration depends crucially upon the difference in productivities (α_1/α_2) between the downstream firms. Introduction of the difference in productivity enables us to analyze some important features of vertically related markets. Vertical integration has two distinct types of effects on a market.³ One is a cost-reducing effect (hereafter, cost effect) which reduces the downstream price. The other is the productivity effect, which is due to the fact that the downstream division of the integrated firm may have superior or inferior production facilities relative to a rival firm. The impact of vertical integration will be examined on the basis of these cost and productivity effects. Downstream equilibrium prices may go down and market foreclosure can arise, depending upon the relative strength of these two effects.

The foreclosure brought about by the integration depends crucially upon upstream market structure. Although the vertical integration enables the upstream monopolist to exploit leverage too effectively for the inefficient rival to stay in the downstream market, an efficient competitor can still be viable. However, the duopolist may fail to foreclose a rival firm even if the rival

²Chen and Riordan (2007) assumes that $\alpha_i = 1$ for both downstream firms.

 $^{^{3}}$ Chen (2001) also identifies two effects of the integration: the efficiency and the collusive effects. The former is our cost effect and the latter, which is different from the productivity effect, comes solely from the fact that downstream products are differentiated.

has lower-productivity facilities. Thus, the strength of the effect of vertical integration depends upon upstream market structure. Moreover, the model can investigate if the integrated upstream duopolist can maximize profits by not supplying inputs to an upstream market. The model may also explain why "keiretsu" emerges in the DD game.⁴ These results obtained in this paper are in stark contrast with those in Chen (2001) and Chen and Riordan (2007) in which the presence of upstream rivals makes no difference to their analyses. Note that the present model does not consider incentives for the merger.

Our paper is organized as follows. Section 2 sets forth the effects of the integration in the MD game. The question of possible foreclosure resulting from integration by a monopolist is examined. The effect of integration on the downstream prices is investigated to determine if market efficiency is affected. Section 3 explores the DD game to investigate when a rival firm can continue to operate despite integration. Another facet of this investigation is whether the upstream division of the integrated firm supplies the upstream market. These investigations reveal the importance of upstream market structure. Our model also investigates the appearance of "keiretu". We end with conclusions and a discussion of limitations of the model in Section 4.

⁴Salinger (1988) also reached the same conclusions under some assumptions.

2 Basic Ingredients and Upstream Monopoly

Consider a vertically related market in which two firms D1 and D2 supply a consumer good. The consumer good is produced by using inputs which are provided by an upstream monopolist U. The former market, supplied by D1 and D2, is called the downstream market and the latter the upstream market which is monopolized by the monopolist U. This game is called the MD game defined above. The demand for the consumer good is given by

$$P = a - x = a - (x_1 + x_2), \tag{1}$$

where P represents the price of consumer good, x_i the quantity of output by Di, i = 1, 2, and a is a constant.

To simplify our analysis, assume that α_i units of inputs are translated into a unit of output (or a consumer good) by Di, where the α_i is a positive constant.⁵ It follows that

$$x_i = \frac{1}{\alpha_i} y_i, \ i = 1, \ 2,$$
 (2)

where y_i is the quantity of inputs for D*i*. These are the production functions of D1 and D2. To proceed with our analysis, it will be assumed that productivity of D2 is higher than that of D1, and that the ratio of their productivities is not larger than 4. Specifically, let $\alpha = \alpha_1/\alpha_2$ and $A_{\alpha} = \{\alpha | 1 < \alpha \leq 4\}$. Formally, our assumption can be expressed as

Assumption 1. $\alpha \in A_{\alpha}$.

Together with this, it is also assumed that

$$ac_i = \alpha_i p, \quad i = 1, 2, \tag{3}$$

where ac_i stands for the unit cost of the consumer good produced by Di and p for the price of inputs. This means that Di using α_i units of inputs produces one unit of the consumer good at a cost $\alpha_i p$.

To simplify our analysis, assume that the marginal cost c of U is a positive constant. To proceed with our analysis, the following assumption is made:

Assumption 2. $a > 4\alpha_1 c$.

⁵Words "inputs" and "outputs" are named from the viewpoint of downstream firms.

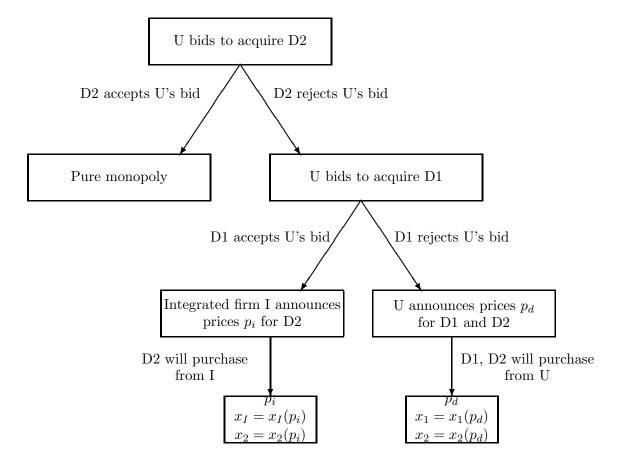


Figure 1: The Monopoly Duopoly game

Several interesting integrated market structures emerge depending upon the number of independent firms in the upstream and the downstream market. Our games structures are depicted in Fig. 1. We consider our games with the following stages. First, U bids to acquire the efficient firm D2. When D2 accepts U's bid, a vertical integration takes place and a downstream market changes into a monopoly. In fact, it will be shown that integration of U and D2 results in a pure monopoly in view of Assumptions 1 and 2. In stage 2 if D2 rejects U's bid, U can bid to acquire the less efficient firm D1. If D1 accepts U's bid an integrated firm, called I, is formed. This subgame is called the Partial-Monopoly game (i.e., PM game) because I is the upstream monopolist and has a downstream competitor at the same time. The upstream division of I announces monopoly quantities for D2. If D1 does not accept U's bid, all firms are independent. This subgame is the benchmark game against which the PM game is evaluated and is called the MD game. U announces inputs prices p_d .

In stage 3, the downstream division of I and D2 simultaneously choose outputs x_i given a demand for consumer good, and inputs are purchased to produce their demanded outputs. In the MD game, D1 and D2 simultaneously purchase inputs from U and produce a consumer good at Nash equilibrium quantities x_i . In what follows, consider first the MD game and then the PM game. In this section, we focus on analyzing the conditions under which the vertical integration causes equilibrium prices in the downstream market to go down or not, together with the question if the independent firm can be viable, and also if it exits from the market after vertical integration by U.

After the MD game, we will consider a one-sided vertical integration in a Double-Duopoly (i.e., DD) game in which there are two independent firms in both, the upstream and the downstream, markets. When one of the upstream firms merges with one of the downstream firms, the other firms remain independent. This new game is called the One-sided Integration game (hereafter, OI game). The problem examined will be whether the independent firms can stay in the market in the post-merger game. These games will be examined in the next section and structure for these games will be shown in the next section.

To summarize, the following games will be considered and analyzed:

- (1) MD game where there are an upstream monopolist and downstream duopolists,
- (2) PM game where the monopolist merges with one of the downstream duopolists,
- (3) DD game where there are duopolists in both markets,
- (4) OI game where one of the upstream duopolists merges with one of the downstream duopolists.

The MD and the PM games will be examined in this section, while the DD and the OI games will be taken up in the next section.

Consider first the MD game in which an upstream monopolist supplies inputs to D1 and D2. Given a demand for a consumer good, the downstream firms play a Cournot game in the market. Unit costs for them are given by ac_i , i = 1, 2. Their profits are expressed as

$$\pi_i = (P - ac_i)x_i = (a - \alpha_i p - x_1 - x_2)x_i, \ i = 1, \ 2.$$

The objective of each firm is to maximize profits, which requires

$$\frac{\partial \pi_1}{\partial x_1} = a - \alpha_1 p - 2x_1 - x_2 = 0,$$

$$\frac{\partial \pi_2}{\partial x_2} = a - \alpha_2 p - x_1 - 2x_2 = 0.$$

Solving these for x_i yields

$$\hat{x}_1 = \frac{a - (2\alpha_1 - \alpha_2)p}{3},$$

 $\hat{x}_2 = \frac{a + (\alpha_1 - 2\alpha_2)p}{3},$

where outputs of both firms are assumed positive and $\hat{x}_2 > \hat{x}_1$ because of Assumption 1. Thus, total equilibrium output \hat{X} is given by

$$\hat{X} = \hat{x}_1 + \hat{x}_2^* = \frac{2a - (\alpha_1 + \alpha_2)p}{3}.$$

On the other hand, total demand \hat{Y} for inputs by these firms is expressed as

$$\hat{Y} = \alpha_1 \hat{x}_1 + \alpha_2 \hat{x}_2 = \alpha_1 \frac{a - (2\alpha_1 - \alpha_2)p}{3} + \alpha_2 \frac{a + (\alpha_1 - 2\alpha_2)p}{3}$$

= $\frac{a(\alpha_1 + \alpha_2) - 2(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)p}{3}.$

Now we turn to the formal statement of our analysis of the MD game:

Lemma 1. In the Monopoly-Duopoly game, the equilibrium prices in the upstream and the downstream markets are given by the following expressions respectively

$$p_d^* = \frac{a(\alpha_1 + \alpha_2) + 2(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)c}{4(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} > c,$$

and

$$P_D = \frac{a(5\alpha_1^2 - 2\alpha_1\alpha_2 + 5\alpha_2^2) + 2(\alpha_1^3 + \alpha_2^3)c}{12(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} > \alpha_1 p_d^*,$$

where p_d^* and P_D stand for equilibrium prices in the upstream and the downstream markets.

Proof. The upstream monopolist U maximizes profits, given a demand for inputs, which are given by

$$\pi_m = (p-c)\hat{Y} = (p-c)\frac{a(\alpha_1 + \alpha_2) - 2(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)p}{3}.$$

Then, differentiating π_m with respect to p and solving for p,

$$p_d^* = \frac{a(\alpha_1 + \alpha_2) + 2(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)c}{4(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} > c.$$

Note that equilibrium outputs of D1 and D2 are given by \hat{x}_1 , \hat{x}_2 and that $\hat{x}_2 > \hat{x}_1$ because of Assumption 1. Then, it is enough to show that $\hat{x}_1 > 0$ at $p = p_d^*$. Substituting p_d^* into output \hat{x}_1 of D1,

$$\begin{aligned} x_1^* &= \frac{a - (2\alpha_1 - \alpha_2)p_b^*}{3} = \frac{1}{3} \left(\frac{a(2\alpha_1^2 - 5\alpha_1\alpha_2 + 5\alpha_2^2) - 2c(\alpha_1 - \alpha_2)^2(2\alpha_1 - \alpha_2)}{4(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \right) \\ &> \frac{1}{3} \left(\frac{4\alpha_1 c(2\alpha_1^2 - 5\alpha_1\alpha_2 + 5\alpha_2^2) - 2c(\alpha_1 - \alpha_2)^2(2\alpha_1 - \alpha_2)}{4(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \right) \\ &= \frac{c}{6} \left(\frac{2\alpha_1^3 - 5\alpha_1^2\alpha_2 + 6\alpha_1\alpha_2^2 + \alpha_2^3}{\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2} \right), \end{aligned}$$

where the inequality above is due to Assumption 2 and the denominator is positive under Assumption 1. The sign of x_1^* depends upon the numerator, which is expressed as

$$2\alpha_1^3 - 5\alpha_1^2\alpha_2 + 6\alpha_1\alpha_2^2 + \alpha_2^3 = (2\alpha^3 - 5\alpha^2 + 6\alpha + 1)\alpha_2^3 > 0,$$

which is positive for $\alpha = \alpha_1/\alpha_2 \in A_{\alpha}$. Then, outputs of the downstream firms are positive.

Next, we will show that p_d^* is larger than c. In fact, the difference between them is

$$p_d^* - c = \frac{a(\alpha_1 + \alpha_2) + 2(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)c}{4(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} - c = \frac{a(\alpha_1 + \alpha_2) - 2(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)c}{4(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}$$

This difference will be positive if the numerator is positive, because the denominator is positive for $\alpha_1 > \alpha_2$. It follows that the numerator is written as

$$a(\alpha_1 + \alpha_2) - 2(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)c > 4\alpha_1c(\alpha_1 + \alpha_2) - 2(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)c = 2c(\alpha_1^2 + 3\alpha_1\alpha_2 - \alpha_2^2)$$
$$= 2\alpha_2^2c(\alpha^2 + 3\alpha - 1) > 0, \ for \ \alpha \in A_\alpha,$$

where the inequality is due to Assumption 2. Thus, p_d^* is larger than c. It follows from these results that U can make positive profits by supplying inputs to this market. Noting that the demand for the consumer good is given by (1), it follows from p_d^* derived above that

$$P_D = \frac{a(5\alpha_1^2 - 2\alpha_1\alpha_2 + 5\alpha_2^2) + 2(\alpha_1^3 + \alpha_2^3)c}{12(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} > \alpha_1 c.$$

The proof will be given after we first show that $P_D - \alpha_1 p_d^* > 0$.

Finally, consider if the downstream firms can be viable. Noting that $\alpha_1 p_d^*$ is the average cost of D1, the difference between P_D and $\alpha_1 p_d^*$ is given by

$$\begin{split} P_D - \alpha_1 p_d^* &= \frac{a(5\alpha_1^2 - 2\alpha_1\alpha_2 + 5\alpha_2^2) + 2(\alpha_1^3 + \alpha_2^3)c}{12(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} - \alpha_1 \frac{a(\alpha_1 + \alpha_2) + 2(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)c}{4(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)c} \\ &= \frac{1}{12(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} (a(2\alpha_1^2 - 5\alpha_1\alpha_2 + 5\alpha_2^2) + 2c(-2\alpha_1^3 + 3\alpha_1^2\alpha_2 - 3\alpha_1\alpha_2^2 + \alpha_2^3)) \\ &= \frac{1}{12(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} (a(2\alpha^2 - 5\alpha + 5)\alpha_2^2 + 2c(-2\alpha^3 + 3\alpha^2 - 3\alpha + 1)\alpha_2^3)) \\ &= \frac{1}{12(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \left(\frac{a}{2\alpha_2 c} + \frac{(-2\alpha^3 + 3\alpha^2 - 3\alpha + 1)}{(2\alpha^2 - 5\alpha + 5)} \right) (2\alpha^2 - 5\alpha + 5)2\alpha_2^3 c \\ &= \frac{\alpha_2 c(2\alpha^2 - 5\alpha + 5)}{6(\alpha^2 - \alpha + 1)} \left(\frac{a}{2\alpha_2 c} + \frac{(-2\alpha^3 + 3\alpha^2 - 3\alpha + 1)}{(2\alpha^2 - 5\alpha + 5)} \right) \\ &= \frac{\alpha_2 c(2\alpha^2 - 5\alpha + 5)}{6(\alpha^2 - \alpha + 1)} (\frac{a}{2\alpha_2 c} + f(\alpha)) \\ &> \frac{\alpha_2 c(2\alpha^2 - 5\alpha + 5)}{6(\alpha^2 - \alpha + 1)} (\frac{4\alpha_1 c}{2\alpha_2 c} + f(\alpha)) \\ &= \frac{\alpha_2 c(2\alpha^2 - 5\alpha + 5)}{6(\alpha^2 - \alpha + 1)} (2\alpha + f(\alpha)), \end{split}$$

where $f(\alpha) = \frac{(-2\alpha^3 + 3\alpha^2 - 3\alpha + 1)}{(2\alpha^2 - 5\alpha + 5)}$. Noting that $2\alpha_1^2 - 5\alpha_1\alpha_2 + 5\alpha_2^2 = (2\alpha^2 - 5\alpha + 5)\alpha_2^2$ is positive for $\alpha \in A_\alpha$, the sign of $(P_D - \alpha_1 p_d^*)$ is determined by that of the second term $(2\alpha + f(\alpha))$. Solving $(2\alpha + f(\alpha) = 0)$ for α , we have one real solution; $\alpha = -0.1263$. Thus, the second term is positive for $\alpha \in A_\alpha$. Then, $(P_D - \alpha_1 p_d^*) > 0$ for $\alpha \in A_\alpha$. Then, profits π_1^* of D1 are positive because $\pi_1^* = (P_D - \alpha_1 p_d^*)x_1^*$, in which x_1^* and $(P_D - \alpha_1 p_d^*)$ are positive for $\alpha \in A_\alpha$.

Finally, we show that $(P_D - \alpha_1 c) > 0$. Noting that $p_d^* > c$, it is easy to show that $(P_D - \alpha_1 c) > (P_D - \alpha_1 p_d^*) > 0$. Then, equilibrium price P_D enables D1 and D2 to reap positive profits and all firms in vertically related markets can be viable for $\alpha \in A_{\alpha}$.

Next, we study the PM game in which the upstream monopolist U merges with D1, but D2 remains independent. However, if the the upstream monopolist merges with D2, which has superior production technology, downstream market structure changes into a pure monopoly. This will be shown in the proof of *Lemma* 2. This is the reason why U merges with D1.

Note also that D1 and U form the integrated firm I and the marginal cost of inputs for the upstream division of I is c. Thus, the profits of the two firms, I and D2, are given by

$$\pi_I = \pi_d + \pi_u = (P - \alpha_1 c)x_I + \alpha_2 (p - c)x_2 = (a - \alpha_1 c - (x_I + x_2))x_I + \alpha_2 (p - c)x_2,$$

$$\pi_2 = (P - \alpha_2 p)x_2 = (a - \alpha_2 p - (x_I + x_2))x_2,$$

where π_d and π_u stand for profits of the integrated firm I in the downstream and upstream markets. The first order conditions for the maximization of profits are

$$\frac{\partial \pi_I}{\partial x_I} = \frac{\partial \pi_d}{\partial x_I} = a - \alpha_1 c - 2x_I - x_2 = 0,$$

$$\frac{\partial \pi_2}{\partial x_2} = a - \alpha_2 p - x_I - 2x_2 = 0,$$

where x_I stands for output of I and x_2 for that of D2. The Nash equilibrium outputs of these firms are expressed as

$$x_I^* = \frac{a - 2\alpha_1 c + \alpha_2 p}{3},\tag{4}$$

$$x_2^* = \frac{a + \alpha_1 c - 2\alpha_2 p}{3}.$$
 (5)

D2 has to purchase inputs from I and its demand is given by $\alpha_2 x_2^*$, which is

$$\alpha_2 x_2^* = Y = \frac{\alpha_2 (a + \alpha_1 c - 2\alpha_2 p)}{3}.$$

Solving this equation for p, we have the inverse demand p for inputs:

$$p = \frac{-3Y + \alpha_2(a + \alpha_1 c)}{2\alpha_2^2}.$$

Firm I faces demand for its products derived above. Its profit is

$$\pi_{I} = \pi_{d} + \pi_{u} = (P - \alpha_{1}c)x_{I}^{*} + (p - c)Y = (P - \alpha_{1}c)x_{I}^{*} + (\frac{-3Y + \alpha_{2}(a + \alpha_{1}c)}{2\alpha_{2}^{2}} - c)Y$$
$$= \frac{-5Y^{2} + 4Y(\alpha_{1} - \alpha_{2})\alpha_{2}c + \alpha_{2}^{2}(a - \alpha_{1}c)^{2}}{4\alpha_{2}^{2}}.$$

It follows that the condition for optimality is

$$\frac{d\pi_I}{dY} = -\frac{5Y}{2\alpha_2^2} + (-1 + \frac{\alpha_1}{\alpha_2})c = 0.$$

U supplies inputs to D2, which are given by

$$Y^* = \frac{2(\alpha_1 - \alpha_2)\alpha_2 c}{5}.$$

Given the inverse demand p for input, price charged for this input by the supplier is

$$p_i^* = \frac{-3Y^* + \alpha_2(a + \alpha_1 c)}{2\alpha_2^2} = \frac{5a - \alpha_1 c + 6\alpha_2 c}{10\alpha_2} = \frac{5a - (\alpha_1 - 6\alpha_2)c}{10\alpha_2}.$$
 (6)

It would be of some interest to check whether the equilibrium price in the upstream market is higher than the provider's marginal cost c. Subtracting c from p_i^* , we have

$$p_i^* - c = \frac{5a - (\alpha_1 - 6\alpha_2)c}{10\alpha_2} - c = \frac{5a - (\alpha_1 + 4\alpha_2)c}{10\alpha_2} > 0,$$

which will be positive under our Assumptions 1 and 2. Thus, I has cost advantage over its competitor in the downstream market.

We can now summarize our analysis above as:

Lemma 2. Equilibrium prices in the upstream and the downstream market in the Partial-Monopoly game are given by

$$p_{i}^{*} = \frac{5a - (\alpha_{1} - 6\alpha_{2})c}{10\alpha_{2}},$$

$$P_{I} = \frac{5a + c(3\alpha_{1} + 2\alpha_{2})}{10},$$

where P_I is higher than $\alpha_2 p_i^*$. Moreover, output of the independent firm D2 is given by

$$x_2^* = \frac{2(\alpha_1 - \alpha_2)c}{5} > 0.$$

Proof. Substituting (6) into (4) and (5) yields,

$$x_{I}^{*} = \frac{a - 2\alpha_{1}c + \alpha_{2}p_{i}^{*}}{3} = \frac{a - 2\alpha_{1}c + \alpha_{2}\frac{5a - (\alpha_{1} - 6\alpha_{2})c}{10\alpha_{2}}}{3}$$
$$= \frac{5a - (7\alpha_{1} - 2\alpha_{2})c}{10},$$

and

$$x_2^* = \frac{a + \alpha_1 c - 2\alpha_2 p_i^*}{3} = \frac{a + \alpha_1 c - 2\alpha_2 \frac{5a - (\alpha_1 - 6\alpha_2)c}{10\alpha_2}}{3} = \frac{2(\alpha_1 - \alpha_2)c}{5} > 0,$$

where the inequality comes from Assumption 1 and x_2^* is equal to zero at $\alpha = \alpha_1/\alpha_2 = 1$. Then, it follows from (1), x_I^* and x_2^* that we have

$$P_I = a - x_I^* - x_2^* = \frac{5a + c(3\alpha_1 + 2\alpha_2)}{10}.$$
(7)

As noted before, I has cost advantage over its downstream competitor D2. However, it follows from Assumption 1 that the competitor has more efficient technology. Then, consider whether this cost advantage is enough for firm I to drive D2 out of the market. Using P_I and p_i^* derived above, we get:

$$P_I - \alpha_2 p_i^* = \frac{2c(\alpha_1 - \alpha_2)}{5} > 0.$$

where the sign comes from Assumption 1 and the equality holds at $\alpha = 1$. Thus, both I and D2 can make positive profits and they can be viable.

Comparisons between P_I and P_D are much more complicated. It follows from *Lemmas* 1 and 2 that

$$P_{I} - P_{D} = \frac{5a + (3\alpha_{1} + 2\alpha_{2})c}{10} - \frac{a(5\alpha_{1}^{2} - 2\alpha_{1}\alpha_{2} + 5\alpha_{2}^{2}) + 2(\alpha_{1}^{3} + \alpha_{2}^{3})c}{12(\alpha_{1}^{2} - \alpha_{1}\alpha_{2} + \alpha_{2}^{2})}$$

$$= \frac{5a(\alpha_{1}^{2} - 4\alpha_{1}\alpha_{2} + \alpha_{2}^{2}) + 2(4\alpha_{1}^{3} - 3\alpha_{1}^{2}\alpha_{2} + 3\alpha_{1}\alpha_{2}^{2} + \alpha_{2}^{3})c}{60(\alpha_{1}^{2} - \alpha_{1}\alpha_{2} + \alpha_{2}^{2})}$$

$$= \frac{5a(\alpha^{2} - 4\alpha + 1) + 2(4\alpha^{3} - 3\alpha^{2} + 3\alpha + 1)\alpha_{2}c}{60(\alpha^{2} - \alpha + 1)}.$$

For notational simplicity, we now use new definitions:

$$f(\alpha) = P_I - P_D,$$

$$g(\alpha) = \frac{4\alpha^3 - 3\alpha^2 + 3\alpha + 1}{\alpha^2 - 4\alpha + 1}$$

Using these definitions, the differences in equilibrium prices are rewritten as

$$P_{I} - P_{D} = f(\alpha)$$

$$= \frac{2\alpha_{2}c}{60(\alpha^{2} - \alpha + 1)} \{ \frac{5a}{2\alpha_{2}c} (\alpha^{2} - 4\alpha + 1) + (4\alpha^{3} - 3\alpha^{2} + 3\alpha + 1) \}$$

$$= \frac{\alpha_{2}c}{30(\alpha^{2} - \alpha + 1)} (\alpha^{2} - 4\alpha + 1) \{ \frac{5a}{2\alpha_{2}c} + \frac{4\alpha^{3} - 3\alpha^{2} + 3\alpha + 1}{\alpha^{2} - 4\alpha + 1} \}$$

$$= \frac{\alpha_{2}c}{30(\alpha^{2} - \alpha + 1)} (\alpha^{2} - 4\alpha + 1) \{ \frac{5a}{2\alpha_{2}c} + g(\alpha) \},$$

where $(\alpha^2 - \alpha + 1)$ is positive for $\alpha \in A_{\alpha}$. It then follows that $f(\alpha) = 0$ when the second and/or the last terms in equation above is equal to zero. Then, consider first the following equation:

$$\frac{5a}{2\alpha_2 c} + g(\alpha) = 0, \tag{8}$$

which in turn implies that $f(\alpha) = 0$.

In view of Assumptions 1 and 2,

$$\frac{5a}{2\alpha_2c} > \frac{5 \times 4\alpha_1c}{2\alpha_2c} > \frac{5 \times 4\alpha_2c}{2\alpha_2c} = 10.$$

Next, consider the following equation:

$$10 + g(\alpha) = 10 + \frac{4\alpha^3 - 3\alpha^2 + 3\alpha + 1}{\alpha^2 - 4\alpha + 1} = 0.$$

The approximate values of solutions to the equation are given by $\alpha = -4.142$, 0.320, and 2.072. Note also that the function $g(\alpha)$ is less than -2.5 for $\alpha > 1$ because g(1) = -2.5, and that $g(\alpha)$ is negative, monotonically decreasing and approaching $-\infty$ as α increases in the open interval (1, 3.732). In the analysis to follow, the solution which is appropriate here is $\alpha = 2.072$ because $\alpha > 1$ (Assumption 1). It follows from the properties of $g(\alpha)$ and $\frac{5a}{2\alpha_2c} > 10$ that the solution α^* to equation (8) is larger than 2.072, but less than 3.732.

Finally, together with Lemmas 1 and 2, we can formally state:

Proposition 1. The Partial-Monopoly game yields lower equilibrium prices than the Monopoly-Duopoly game if differences in productivity are relatively small. However, if productivity of the partial monopolist I becomes low enough, it provides a higher equilibrium price than the Monopoly-Duopoly game does. Formally, we have

$$P_I \le P_D, \quad for \quad 1 < \alpha \le \alpha^*,$$

 $P_I > P_D, \quad for \quad 4 \ge \alpha > \alpha^*$

where $2.072 < \alpha^* < 3.732$.

Proof. Consider the following:

$$P_I - P_D = \frac{\alpha_2 c}{30(\alpha^2 - \alpha + 1)} (\alpha^2 - 4\alpha + 1) \{ \frac{5a}{2\alpha_2 c} + g(\alpha) \},$$

where $\frac{\alpha_2 c}{30(\alpha^2 - \alpha + 1)}$ is positive for $\alpha \in A_{\alpha}$ and $(\alpha^2 - 4\alpha + 1)$ is negative for $0.2679 < \alpha < 3.7320$. However, the sign of the last term $\{\frac{5a}{2\alpha_2 c} + g(\alpha)\}$ is indeterminate. As mentioned above,

$$\frac{5a}{2\alpha_2 c} > 10.$$

Next, consider (8):

$$10 + g(\alpha) = 10 + \frac{4\alpha^3 - 3\alpha^2 + 3\alpha + 1}{\alpha^2 - 4\alpha + 1} = 0,$$

where the solution to the equation above which we consider here is 2.072. It follows from the properties of $g(\alpha)$ that

$$\frac{5a}{2\alpha_2 c} + g(\alpha) = 0$$

has a unique, positive solution α^* in the open interval (2.072, 3.732).

Then, we encounter two cases depending upon whether the quadratic term $(\alpha^2 - 4\alpha + 1)$ is negative or not. It is easy to show that the quadratic term is positive and $g(\alpha) > 0$ for $\alpha > 3.732$. This means that $\frac{5a}{2\alpha_2 c} + g(\alpha) > 0$ for $\alpha > 3.732$. Then, it follows from (8) that $P_I > P_D$ for $\alpha > 3.732$. It also follows from the property of $g(\alpha)$ that we have

$$\begin{split} g(\alpha) &+ \frac{5a}{2\alpha_2 c} \geq 0, \quad for \quad \alpha \leq \alpha^*, \\ g(\alpha) &+ \frac{5a}{2\alpha_2 c} < 0, \quad for \ 3.732 > \alpha > \alpha^* \end{split}$$

As $(\alpha^2 - 4\alpha + 1) < 0$ for α in the open interval, (1, 3.732), it follows from (8) and the discussion above that

$$P_I \leq P_D, \quad for \quad \alpha \leq \alpha^*,$$

$$P_I > P_D, \quad for \quad 3.732 > \alpha > \alpha^*.$$

This completes the proof.

This proposition clearly shows one of the essential features of vertically related markets. The vertical integration generally causes equilibrium prices to increase. However, our investigation shows that this is not necessarily true when the integration occurs in the MD game. The integration has two distinct effects on equilibrium prices in the downstream market: the cost effect and the productivity effect. The partial monopolist I (or integrated firm) can get access to inputs at lower costs (i.e., the marginal costs), while the un-integrated, independent firm D2 is supplied inputs at market price. Decreases in production costs result in lower equilibrium price and *vice versa*. Then, this effect may be called the cost effect.⁶ This is equivalent to the cancellation of the double marginalization.⁷

 $^{^{6}}$ Our cost effect is called the efficiency effect in Chen (2001).

⁷See, for example, Spengler (1950).

Considering the technological aspects, productivity of the partial monopolist I exerts strong influence on the cost effect. For example, when productivity of the monopolist I is high, it strengthens the cost effect. By the same token, however, the cost effect is weakened when productivity is low. This new effect may be called the productivity effect, which comes from the *difference in productivity* between the partial monopolist I and the downstream firm D2.

When productivity of the monopolist I is high enough in the sense that $\alpha \leq 1$, the cost effect is strengthened by the productivity effect so that equilibrium prices in the downstream market become too low for the independent firm to make positive profits. When productivity of the monopolist I becomes lower (or productivity of D2 becomes higher) so that α becomes larger than 1, the productivity effect works against the cost effect caused by the low productivity, and equilibrium prices go up. However, the productivity effect is dominated by the cost effect so that equilibrium prices are lower in the PM game than in the MD game. To be precise, equilibrium prices become lower after integration as long as $\alpha \leq \alpha^*$. However, if the productivity of the integrated firm is so low that $\alpha > \alpha^*$, the productivity effect dominates the cost effect so that downstream prices become so high that they are higher in the PM game than in the MD game. These results are mainly due to our modeling which focuses on the difference in productivity of downstream firms.

3 One-sided Vertical Integration in Double-Duopoly Game

There is a long history of whether the vertical integration matters because it may cause market foreclosure of rival firms. Two controversial issues have been examined by those concerned with antitrust proceedings and with regulation. One issue is whether the vertical integration is anticompetitive or not, and the other is whether it promotes market efficiencies. In this section effects of one-sided vertical integration are examined in the DD game.

The DD game is a benchmark game against which the OI game (one-sided integration game) is evaluated. This game structure is depicted in Fig. 2. In what follows, it will be considered whether the vertical integration by one of the upstream duopolist causes decreases in downstream prices, and whether it results in excluding a competitor from the market. From the viewpoint of antitrust, the integration by efficient firms are taken seriously by authorities. In fact, this type of merger is considered to pave the way for the monopolization of a market. Thus, there are concerns that merger may lead to reduced competition. This is the reason why the merger of efficient firms is examined in what follows.

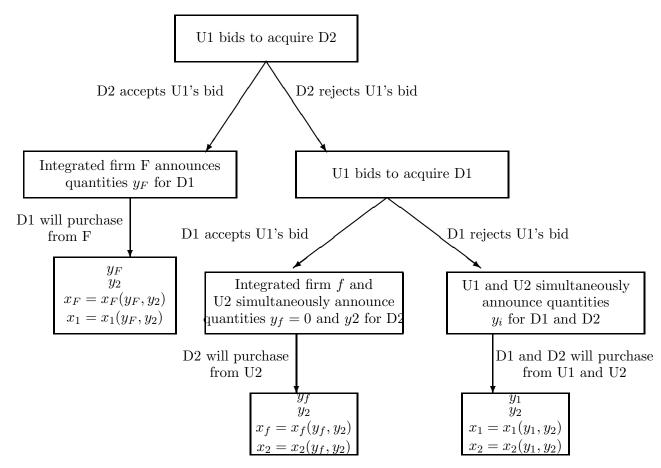


Figure 2: The Double-Duopoly game

Game structure of the present model is as follows: In stage 1, upstream firm U1 can bid to acquire the efficient downstream firm D2. When D2 accepts U1's bid, one-sided vertical integration occurs, and the integrated firm is called F. If D2 rejects the bid, the vertically related markets are a DD game. In stage 2, after integration, the upstream division of F and upstream firm U2 simultaneously announce quantities y_i at Nash equilibrium. When D2 rejects U1's bid, U1 can bid to acquire the less efficient firm D1.

In stage 3, in the OI game, the downstream division of F and D2 produce outputs at Nash quantities x_i for a given demand for the consumer good. If D1 accepts U1's bid, the U1-D1 integration occurs, which is called f. The integrated firm f and the independent firm U2 simultaneously announce quantities y_i at Nash equilibrium in the upstream market. If both D1 and D2 reject U1's bid, the vertically related markets are the DD game. In the U1-D1 merger setting, the upstream division of f and U2 simultaneously announce Nash equilibrium quantities y_i .

In stage 4, in the vertically integrated setting, the downstream division of f and the independent firm D2 purchase inputs to produce consumer goods and simultaneously choose Nash equilibrium quantities of outputs $x_i(y_i)$. In the DD game, D1 and D2 purchase inputs in the upstream market and produce outputs at Nash quantities $x_i = x_i(y_i)$ for a given demand for the consumer good.

Consider the DD game, where upstream firms U1 and U2 supply their products to downstream firms. U1 and U2 produce inputs at constant marginal costs (c_1 and c_2), where $c_1 \leq c_2$ without loss of generality. Demand for upstream products is derived from outputs by D1 and D2, whose products are produced according to the demand function (1) and sold. Their production functions are given by (2). To solve for the equilibrium in this game, we use backward induction. It follows that firms in the downstream market play a simultaneous-move (or a static) game, given the demand function (1), and maximize their respective profits.

To proceed with our analysis, assume that all firms are constrained to compete \dot{a} la Cournot and that

Assumption 3. $a > 4\alpha_1 c_2$.

This new assumption replaces Assumption 2 in the MD game.

Using eqs. (1) and (3), profits of firm *i* are given by

$$\pi_i = (P - ac_i)x_i = (a - \alpha_i p - (x_1 + x_2))x_i, \quad i = 1, 2,$$

where fixed costs are assumed away because they will not play any role in the analysis to follow.

The objective of each firm is to maximize π_i , which requires

$$\frac{\partial \pi_1}{\partial x_1} = a - \alpha_1 p - 2x_1 - x_2 = 0$$

and

$$\frac{\partial \pi_2}{\partial x_2} = a - \alpha_2 p - x_1 - 2x_2 = 0.$$

Solving these equations for equilibrium outputs yields

$$\begin{aligned} x_1^* &= \frac{a - (2\alpha_1 - \alpha_2)p}{3}, \\ x_2^* &= \frac{a + (\alpha_1 - 2\alpha_2)p}{3}, \end{aligned}$$

where the outputs of both firms are assumed positive. Total equilibrium output X^* is

$$X^* = x_1^* + x_2^* = \frac{2a - (\alpha_1 + \alpha_2)p}{3}$$

Demand for inputs is derived from outputs in the downstream market. In view of the production function of firms in this market, total equilibrium demand Y for inputs is given by

$$Y = \alpha_1 x_1^* + \alpha_2 x_2^* = \frac{a(\alpha_1 + \alpha_2) + 2p(\alpha_1 \alpha_2 - \alpha_1^2 - \alpha_2^2)}{3}.$$

Solving this for p yields inverse demand for inputs, which is given by

$$p = \frac{3Y - a(\alpha_1 + \alpha_2)}{2(\alpha_1 \alpha_2 - \alpha_1^2 - \alpha_2^2)}.$$
(9)

Given the demand for inputs, the two firms play a simultaneous-move game in the upstream market. Under these assumptions, upstream firms supply inputs to the upstream market. Then,

$$Y = y_1 + y_2.$$

It follows from this and (9) that profit of U1 is

$$\pi_{U1} = (p - c_1)y_1 = \left(\frac{3Y - a(\alpha_1 + \alpha_2)}{2(\alpha_1\alpha_2 - \alpha_1^2 - \alpha_2^2)} - c_1\right)y_1 = \left(\frac{3(y_1 + y_2) - a(\alpha_1 + \alpha_2)}{2(\alpha_1\alpha_2 - \alpha_1^2 - \alpha_2^2)} - c_1\right)y_1.$$

where c_1 is the constant marginal cost of U1 and y_i is supply of the inputs by Ui, i = 1, 2. Differentiating profits with respect to y_1 yields

$$\frac{\partial \pi_{U1}}{\partial y_1} = \frac{6y_1 + 3y_2 - a(\alpha_1 + \alpha_2)}{2(\alpha_1 \alpha_2 - \alpha_1^2 - \alpha_2^2)} - c_1 = 0.$$

Similarly, we have

$$\pi_{U2} = (p - c_2)y_2 = \left(\frac{3Y - a(\alpha_1 + \alpha_2)}{2(\alpha_1\alpha_2 - \alpha_1^2 - \alpha_2^2)} - c_2\right)y_2 = \left(\frac{3(y_1 + y_2) - a(\alpha_1 + \alpha_2)}{2(\alpha_1\alpha_2 - \alpha_1^2 - \alpha_2^2)} - c_2\right)y_2,$$

where c_2 is the constant marginal cost of U2. Differentiating π_{U2} with respect to y_2 and equating the derivative to zero gives

$$\frac{\partial \pi_{U2}}{\partial y_2} = \frac{3y_1 + 6y_2 - a(\alpha_1 + \alpha_2)}{2(\alpha_1 \alpha_2 - \alpha_1^2 - \alpha_2^2)} - c_2 = 0.$$

Solving these equations for y_1 and y_2 , we get the equilibrium values of inputs supplied by upstream firms:

$$y_1^* = \frac{a(\alpha_1 + \alpha_2) + 2(c_2 - 2c_1)(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}{9}$$
$$y_2^* = \frac{a(\alpha_1 + \alpha_2) + 2(c_1 - 2c_2)(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}{9},$$

which are both positive. In fact, it follows from Assumption 1, 2 and $c_1 \leq c_2$ that the numerator of y_1^* is reduced to

$$2(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(c_2 - 2c_1) + a(\alpha_1 + \alpha_2)$$

$$> 2(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(c_2 - 2c_1) + 4\alpha_1c_1(\alpha_1 + \alpha_2)$$

$$> 2(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(-2c_1) + 4\alpha_1c_1(\alpha_1 + \alpha_2)$$

$$= c_1(-4\alpha_1^2 + 4\alpha_1\alpha_2 - 4\alpha_2^2 + 4\alpha_1^2 + 4\alpha_1\alpha_2)$$

$$= 4\alpha_2^2c_1(2\alpha - 1) > 0,$$

where the inequality is due to Assumption 1.

The numerator of y_2^* is reduced to

$$2(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(c_1 - c_2 - c_2) + a(\alpha_1 + \alpha_2)$$

>
$$2(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(-2c_2) + 4\alpha_1c_2(\alpha_1 + \alpha_2)$$

=
$$4c_2(-\alpha_1^2 + \alpha_1\alpha_2 - \alpha_2^2 + \alpha_1^2 + \alpha_1\alpha_2)$$

=
$$4c_2(2\alpha_1\alpha_2 - \alpha_2^2)$$

=
$$4\alpha_2^2c_2(2\alpha - 1) > 0,$$

where again the inequality is due to Assumption 1.

Then, the total demand for inputs is

$$Y^* = y_1^* + y_2^* = \frac{2(\alpha_1\alpha_2 - \alpha_1^2 - \alpha_2^2)(c_1 + c_2) + 2a(\alpha_1 + \alpha_2)}{9}.$$

The equilibrium price p_N^* of inputs is given by substituting Y^* into (9), and it is

$$p_N^* = \frac{a(\alpha_1 + \alpha_2) + 2(c_1 + c_2)(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}{6(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}.$$

It is easy to show that price p_N^* is higher than c_2 . In fact,

$$p_N^* - c_2 = \frac{a(\alpha_1 + \alpha_2) + 2(c_1 - 2c_2)(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}{6(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)},$$

where the denominator is positive under Assumption 1 and the numerator is positive because it is the same as that of y_2^* in the duopoly game.

Next, consider if x_1^* is positive at $p = p_N^*$. Substituting p_N^* into x_1^* yields

$$\begin{aligned} x_1^* &= \frac{a - (2\alpha_1 - \alpha_2)p_N^*}{3} = \frac{a - (2\alpha_1 - \alpha_2)\frac{a(\alpha_1 + \alpha_2) + 2(c_1 + c_2)(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}{6(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}}{3} \\ &= \frac{6a(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2) - (2\alpha_1 - \alpha_2)(a(\alpha_1 + \alpha_2) + 2(c_1 + c_2)(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}{18(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2))}. \end{aligned}$$

To simplify our calculations, consider the following:

$$6a(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2) - (2\alpha_1 - \alpha_2)(a(\alpha_1 + \alpha_2) + 2(c_1 + c_2)(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2))$$

$$= a(4\alpha_1^2 - 7\alpha_1\alpha_2 + 7\alpha_2^2) - 2(2\alpha_1 - \alpha_2)(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(c_1 + c_2)$$

$$> 4\alpha_1c_2(4\alpha_1^2 - 7\alpha_1\alpha_2 + 7\alpha_2^2) - 8\alpha_1(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)c_2 = 4\alpha_1c_2(2\alpha_1^2 - 5\alpha_1\alpha_2 + 5\alpha_2^2)$$

$$= 4\alpha_1c_2(2\alpha^2 - 5\alpha + 5)\alpha_2^2 > 0,$$

where the last inequality is due to Assumption 1. It then follows from these calculations that x_1^* is positive. Similar calculations show that x_2^* is positive.

In view of the demand function for products in the downstream market, it is easy to show that the equilibrium price P_N in the downstream market is

$$P_N = a - x_1^* - x_2^* = \frac{a + (\alpha_1 - \alpha_2)p_N^*}{3} = \frac{a(7\alpha_1^2 - 4\alpha_1\alpha_2 + 7\alpha_2^2) + 2(\alpha_1^3 + \alpha_2^3)(c_1 + c_2)}{18(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}, \quad (10)$$

which is larger than $\alpha_1 p_N^*$. In fact, the difference is reduced to

$$P_N - \alpha_1 p_N^* = \frac{a(4\alpha_1^2 - 7\alpha_1\alpha_2 + 7\alpha_2^2) - 2(2\alpha_1^3 - 3\alpha_1^2\alpha_2 + 3\alpha_1\alpha_2^2 - \alpha_2^3)(c_1 + c_2)}{18(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\ = \frac{a(4\alpha^2 - 7\alpha + 7) - 2\alpha_2(2\alpha^3 - 3\alpha^2 + 3\alpha - 1)(c_1 + c_2)}{18(\alpha^2 - \alpha + 1)}.$$

It is easy to show that the denominator of the difference is positive, but the sign of the numerator is not obvious. However, it will be shown that it is also positive. From *Assumptions* 1 and 2, the numerator is given by

$$\begin{aligned} a(4\alpha^2 - 7\alpha + 7) - 2\alpha_2(2\alpha^3 - 3\alpha^2 + 3\alpha - 1)(c_1 + c_2) \\ > & 4\alpha_1c_2(4\alpha^2 - 7\alpha + 7) - 4\alpha_2c_2(2\alpha^3 - 3\alpha^2 + 3\alpha - 1) \\ > & 4\alpha_2c_2(4\alpha^2 - 7\alpha + 7) - 4\alpha_2c_2(2\alpha^3 - 3\alpha^2 + 3\alpha - 1) \\ = & 4\alpha_2c_2((4\alpha^2 - 7\alpha + 7) - (2\alpha^3 - 3\alpha^2 + 3\alpha - 1)) \\ = & 4\alpha_2c_2(-2\alpha^3 + 7\alpha^2 - 10\alpha + 8) > 0, \end{aligned}$$

where it should be noted that $c_1 \leq c_2$. Then, P_N is larger than $\alpha_1 p_N^*$. It follows from these arguments that the independent firms can make positive profits and can stay in the up- and the down-stream markets.

We can now establish:

Lemma 3. In Double-Duopoly game the equilibrium prices in the upstream and downstream markets are respectively given by

$$p_N^* = \frac{a(\alpha_1 + \alpha_2) + 2(c_1 + c_2)(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}{6(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)},$$

$$P_N = \frac{a(7\alpha_1^2 - 4\alpha_1\alpha_2 + 7\alpha_2^2) + 2(\alpha_1^3 + \alpha_2^3)(c_1 + c_2)}{18(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)}.$$

As we focus on effects of merger on the downstream market, consider the DD game: Efficient firm D2 integrates with upstream firm U1, and the downstream division of the integrated firm F is superior in technologies to the downstream rival D1. It follows that F can get access to inputs at lower costs than D1. This game is called OI game. In the previous section, effects of vertical integration were examined in the game in which there is an upstream monopolist. Now, we will focus on effects of the vertical integration on a market in a DD game. To examine the features of the integration, consider a game in which efficient D2 merges with the more productive U1. Figure 3 depicts this OI game.

F may be able to make profits by supplying outputs produced by using inputs not only from the upstream division, but also from an upstream market. However, assume that the integrated firm does not purchase inputs from the upstream market because prices of inputs are higher than the marginal costs of inputs. Thus, profits of F and D1 are expressed as

$$\pi_F = (P - \alpha_2 c_1) x_F + (p - c_1) y_1 = (a - \alpha_2 c_1 - x_F - x_1) x_F + (p - c_1) y_1,$$

$$\pi_1 = (P - \alpha_1 p) x_1 = (a - \alpha_1 p - x_F - x_1) x_1,$$

where x_F stands for output of F produced by through-puts and p for input price.

The first order conditions for maximum profits are

$$\frac{\partial \pi_F}{\partial x_F} = a - \alpha_2 c_1 - 2x_F - x_1 = 0,$$

$$\frac{\partial \pi_1}{\partial x_1} = a - \alpha_1 p - x_F - 2x_1 = 0.$$

Solving these equations for x_F and x_1 , we have

$$\begin{aligned} x_F^* &= \frac{a - 2\alpha_2 c_1 + \alpha_1 p}{3}, \\ x_1^* &= \frac{a + \alpha_2 c_1 - 2\alpha_1 p}{3}, \end{aligned}$$

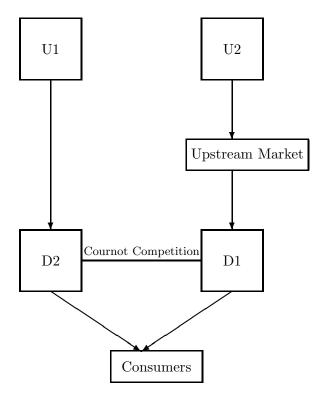


Figure 3: Market Structure of the OI Game

which are analogous to eqs. (4) and (5). As noted above, derived demand d for inputs by D1 is given by

$$d = \alpha_1 x_1^* = \alpha_1 \frac{a + \alpha_2 c_1 - 2\alpha_1 p}{3}.$$

Solving this for p yields inverse demand for the inputs, and is given by

$$p = \frac{(a + \alpha_2 c_1)\alpha_1 - 3d}{2\alpha_1^2}.$$
(11)

Given this demand for the inputs, the upstream division of F and U2 supply the inputs to the downstream firms, and their profits are

$$\begin{aligned} \pi_F &= (a - \alpha_2 c_1 - x_F^* - x_1^*) x_F^* + (p - c_1) y_1 = (a - \alpha_2 c_1 - x_F^* - x_2^*) x_F^* + (\frac{(a + \alpha_2 c_1)\alpha_1 - 3d}{2\alpha_1^2} - c) y_1 \\ &= (a - \alpha_2 c_1 - x_F^* - x_1^*) x_F^* + (\frac{(a + \alpha_2 c_1)\alpha_1 - 3(y_1 + y_2)}{2\alpha_1^2} - c_1) y_1, \\ \pi_{U2} &= (p - c_2) y_2 = (\frac{(a + \alpha_2 c_1)\alpha_1 - 3d}{2\alpha_1^2} - c_2) y_2 = (\frac{(a + \alpha_2 c_1)\alpha_1 - 3(y_1 + y_2)}{2\alpha_1^2} - c_2) y_2, \\ d &= y_1 + y_2. \end{aligned}$$

Substituting x_F^* , x_2^* , and p derived above into π_F and π_{U2} , the first order condition for

maximum profit yields

$$\frac{\partial \pi_F}{\partial y_1} = -\frac{5y_1 + 2(y_2 + \alpha_1(\alpha_1 - \alpha_2)c_1)}{2\alpha_1^2} \le 0, \text{ for } y_1, y_2 \ge 0,$$

where the inequality is due to Assumption 1. It follows from the Kuhn-Tucker conditions that optimal supply \hat{y}_1 of inputs of F is equal to 0. Thus, the integrated firm F produces throughputs and does not supply inputs to the upstream market. It follows from these arguments that U2 is the sole supplier of inputs, which are demanded by D1. U2 maximizes its profits, given the derived demand for inputs. Then, we have

$$\frac{\partial \pi_{U2}}{\partial y_2} = \frac{-6y_2 + \alpha_1(a + \alpha_2 c_1 - 2\alpha_1 c_2)}{2\alpha_1^2} = 0,$$

where $(a + \alpha_2 c_1 - 2\alpha_1 c_2)$ is positive because of Assumption 3. Solving this equation for y_2 , we have

$$\hat{y}_2 = \frac{\alpha_1(a + \alpha_2 c_1 - 2\alpha_1 c_2)}{6} > 0,$$

where the inequality is due to Assumption 3. Substituting this into (11) and noting that $d = y_2$, equilibrium price \hat{p} of inputs is determined and then substituting this \hat{p} into x_F^*, x_1^* yields equilibrium outputs \hat{x}_F, \hat{x}_1 of the two firms.

Now we can summarize our arguments above as:

Lemma 4. If the more productive D2 in a downstream market merges with the more efficient upstream firm U1, the one-sided vertically merged firm can maximize profits by not supplying inputs to an upstream market. Equilibrium outputs, prices and inputs of firms in the post-merger game are given by

$$\hat{y}_1 = 0, \hat{y}_2 = \frac{\alpha_1(a + \alpha_2c_1 - 2\alpha_1c_2)}{6}, \hat{p} = \frac{a + \alpha_2c_1 + 2\alpha_1c_2}{4\alpha_1}, \hat{x}_F = \frac{5a - 7\alpha_2c_1 + 2\alpha_1c_2}{12}, \hat{x}_1 = \frac{a + \alpha_2c_1 - 2\alpha_1c_2}{6}, \hat{P} = \frac{5a + 5\alpha_2c_1 + 2\alpha_1c_2}{12}.$$

Proof. In the post-merger game, U2 maximizes profits by supplying products for the given demand (11) for inputs. Note also that $d = y_2$ because the upstream division does not supply inputs to the upstream market. Then, profits of U2 are given by

$$\pi_{U2} = (p - c_2)y_2 = \left(\frac{\alpha_1(a + \alpha_2 c_1) - 3d}{2\alpha_1^2} - c_2\right)y_2.$$

Solving the first order condition for y_2 yields

$$\hat{y}_2 = \frac{\alpha_1(a + \alpha_2 c_1 - 2\alpha_1 c_2)}{2\alpha_1^2}.$$

Substituting this into (11), we have

$$\hat{p} = \frac{a + \alpha_2 c_1 + 2\alpha_1 c_2}{4\alpha_1}.$$

It is easy to check that $\hat{p} > c_2$ under Assumption 3. In fact,

$$\hat{p} - c_2 = \frac{a + \alpha_2 c + 2\alpha_1 c_2}{4\alpha_1} - c_2 = \frac{a + \alpha_2 c_1 - 2\alpha_1 c_2}{4\alpha_1} > 0.$$

As \hat{y}_2 is positive, profits of U2 are positive and U2 can supply inputs to the upstream market. Substituting \hat{p} into eqs. (4) and (5) yields

$$\hat{x}_F = \frac{a - 2\alpha_2 c_1 + \alpha_1 \hat{p}}{3} = \frac{5a - 7\alpha_2 c_1 + 2\alpha_1 c_2}{12} > 0,$$

$$\hat{x}_1 = \frac{a + \alpha_2 c_1 - 2\alpha_1 \hat{p}}{3} = \frac{a + \alpha_2 c_1 - 2\alpha_1 c_2}{6} > 0.$$

Finally, together with (1), equilibrium price in the downstream market is

$$\hat{P} = a - \hat{x}_F - \hat{x}_2 = \frac{5a + 5\alpha_2c_1 + 2\alpha_1c_2}{12}.$$

It is easy to show that \hat{P} is larger than $\alpha_1 \hat{p}$. In fact, per-units profits $(\hat{P} - \alpha_1 \hat{p})$ are

$$\hat{P} - \alpha_1 \hat{p} = \frac{5a + 5\alpha_2 c_1 + 2\alpha_1 c_2}{12} - \alpha_1 \frac{a + \alpha_2 c_1 + 2\alpha_1 c_2}{4\alpha_1} = \frac{a + \alpha_2 c_1 - 2\alpha_1 c_2}{6} > 0,$$

where the sign comes from Assumption 3. This means that firm F can reap positive profits and sell outputs in the downstream market because equilibrium output \hat{x}_1 of firm D1 is positive. \Box

It is of some interest to note that the vertical integration does not exclude a rival firm from the market in spite of the fact that the integrated firm is more competitive because the downstream division (firm 2) is more productive in view of *Assumption* 1 and the upstream division (U1) has advantages in costs. Thus, these results are in a striking contrast with the effects of the vertical integration by the upstream monopolist, who can foreclose a rival from the market even if the downstream division does not have advantages in production technologies (see, for example, *Lemma* 2). In other words, even if the cost effect and the efficiency effect work for the integrated firm, a downstream rival can be viable in the OI game. On the other hand, the cost effect is so strong in the PM game that a rival has to exit from the market even if the efficiency effect is inactive. However, as was shown above, even if both the cost and efficiency effects do work for the duopolist, the integration does not enable the upstream duopolist to exclude a rival from the market. Thus, the strength of the cost effect depends upon upstream market structure.

Moreover, the firm can reap the maximum profits by not supplying inputs to a market because $\hat{y}_1 = 0$ in *Lemma* 4. As the downstream division does not purchase inputs in the upstream market, this result is similar to that from Salinger (1988) model, where the integrated firms do not participate in the upstream market.⁸ This reminds us of "keiretu" in Japanese manufacturing industries. For example, Toyota has purchased inputs (or parts) solely from its subcontractors and so has Nissan. The subcontractors of Toyota have not supplied their products to Nissan and *vice versa*, though the situation has recently been changing. Our results above may be similar to this situation. The model explains the occurrence of "keiretu", in which parties to the contract are not allowed to trade with firms which are not parties to the contract. If firm D1 integrates with upstream firm U1, it will be shown that the integrated firm may maximize profits by not supplying inputs to the upstream market.⁹

There are two important controversial questions on effects of the vertical merger on a market: Is it anti-competitive and does it promote market efficiency? The present model can answer these questions. We first examine if a vertical merger is anti-competitive. Our *Lemma* 4 shows that the merger is not anti-competitive in the DD game because rival firms are not excluded by the merger. However, it follows from *Lemma* 2 that the vertical merger enables the upstream monopolist to drive a rival out of the downstream market when an integrated upstream monopolist has equally productive facilities (*i.e.*, $\alpha = 1$). Thus, the answer to the first issue depends crucially upon upstream market structure. However, Chen and Riordan (2007) concluded in its *Proposition* 2 that the answer is independent of upstream market structure.

It is also possible to examine whether the one-sided vertical merger improves market efficiency. If the vertical merger improves efficiency in a market, post-merger market prices decline. Usually, it has been considered that the merger confers more monopoly power to the integrated firm and enables the firm to control a market. Then, the merger results in aggravation of the market efficiency. However, it is not easy to show which is correct. More precisely, we can state:

Proposition 2. One-sided vertical integration of D2 and U1 is neutral in competitiveness, but

⁸Chen and Riordan (2007) observed that vertical integration enables the integrated firm to preempt an upstream independent firm.

 $[\]frac{9}{\partial y_1} > 0$ for positive y_1 , y_2 . If the best response functions of two firms intersect in the first quadrant, then $\hat{y}_1 > 0$ and the upstream division can maximize profits by supplying inputs to the upstream market. However, calculations similar to previous ones will show that $\hat{y}_1 = 0$.

it does promote market efficiency for $\alpha \leq 2.618$.

Proof. As shown in Lemma 4 and the argument above, U2 can reap positive profits by supplying inputs in the upstream market. Moreover, it will be shown that D1 can be viable in the downstream market. Profits of D1 are given by $\hat{x}_1 \times (\hat{P} - \alpha_1 \hat{p})$, where \hat{x}_1 is positive in view of Lemma 4, which shows per-unit profits $(\hat{P} - \alpha_1 \hat{p})$ to be positive..

Thus, the independent firm D1 can be viable in the post-merger game. The one-sided merger is neutral from the viewpoint of the competitiveness of the market.

Noting that $4\alpha_2 \ge \alpha_1 > \alpha_2$ and $c_2 \ge c_1$, it follows from *Lemmas* 3 and 4 that we can compare equilibrium prices in pre- and post-merger game:

$$\begin{split} P_N - \hat{P} &= \frac{a(7\alpha_1^2 - 4\alpha_1\alpha_2 + 7\alpha_2^2) + 2(\alpha_1^3 + \alpha_2^3)(c_1 + c_2)}{18(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} - \frac{5a + 5\alpha_2c_1 + 2\alpha_1c_2}{12} \\ &= \frac{-(\alpha_1^2 - 7\alpha_1\alpha_2 + \alpha_2^2)a + (\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(-6\alpha_1c_2 - 15\alpha_2c_1 + 4\alpha_2(c_1 + c_2))}{36(1 - \alpha + \alpha^2)} \\ &= \frac{-(\alpha_1^2 - 7\alpha_1\alpha_2 + \alpha_2^2)a + (\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(-6\alpha_1c_2 - 15\alpha_2c_1 + 4\alpha_2(c_1 + c_2) + 4\alpha_1(c_1 + c_2))}{36(1 - \alpha + \alpha^2)} \\ &= \frac{-(\alpha_1^2 - 7\alpha_1\alpha_2 + \alpha_2^2)a + (\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(-11\alpha_2c_1 + 4\alpha_1c_1 + 4\alpha_2c_2 - 2\alpha_1c_2)}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\ &> \frac{-(\alpha_1^2 - 7\alpha_1\alpha_2 + \alpha_2^2)a + (\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(4\alpha_1c_1 - 4\alpha_1c_1 - 7\alpha_2c_1 + 4\alpha_2c_2 - 2\alpha_1c_2))}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\ &> \frac{-(\alpha_1^2 - 7\alpha_1\alpha_2 + \alpha_2^2)a + (\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(4\alpha_1c_1 - 4\alpha_1c_1 - 7\alpha_1c_2 + 4\alpha_2c_2 - 2\alpha_1c_2))}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\ &> \frac{-(\alpha_1^2 - 7\alpha_1\alpha_2 + \alpha_2^2)a + (\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(4\alpha_1c_1 - 4\alpha_1c_1 - 7\alpha_1c_2 + 4\alpha_2c_2 - 2\alpha_1c_2))}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\ &= \frac{-(\alpha_1^2 - 7\alpha_1\alpha_2 + \alpha_2^2)a + (\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(-9\alpha_1c_2 + 4\alpha_2c_2)}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\ &= \frac{-(\alpha_1^2 - 7\alpha_1\alpha_2 + \alpha_2^2)a + (\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(-9\alpha_1c_2 + 4\alpha_2c_2)}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\ &\geq \frac{-(\alpha_1^2 - 7\alpha_1\alpha_2 + \alpha_2^2)a + (\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(-9\alpha_1c_2 + \alpha_1c_2)}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\ &\geq \frac{-(\alpha_1^2 - 7\alpha_1\alpha_2 + \alpha_2^2)a + (\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)(-9\alpha_1c_2 + \alpha_1c_2)}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\ &= \frac{\alpha_1c_2(3\alpha_1\alpha_2 - 4\alpha_1^2 - 4\alpha_2^2 - 8\alpha_1^2 + 8\alpha_1\alpha_2 - 8\alpha_2^2)}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\ &= \frac{\alpha_1c_2(36\alpha_1\alpha_2 - 12\alpha_1^2 - 12\alpha_2^2)}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\ &= \frac{\alpha_1c_2(36\alpha_1\alpha_2 - 12\alpha_1^2 - 12\alpha_2^2)}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\ &= \frac{\alpha_1c_2(36\alpha_1\alpha_2 - 12\alpha_1^2 - 12\alpha_2^2)}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\ &= \frac{\alpha_1c_2(36\alpha_1\alpha_2 - \alpha_2^2)}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\ &= \frac{\alpha_1c_2(36\alpha_1\alpha_2 - \alpha_2^2)}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\ &= \frac{\alpha_1c_2(36\alpha_1\alpha_2 - \alpha_2)}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\ &= \frac{\alpha_1c_2(36\alpha_1\alpha_2 - \alpha_2)}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\ &= \frac{\alpha_1c_2(36\alpha_1\alpha_2 - \alpha_2)}{36(\alpha_1^2 - \alpha_1\alpha_2 + \alpha_2^2)} \\ &= \frac{\alpha_1c_2(36\alpha_1\alpha_2 - \alpha_2)}{36(\alpha_1$$

where the first two inequalities are due to $\alpha_1 > \alpha_2$, the third inequality is due to $4\alpha_2 \ge \alpha_1$, the fourth inequality is due to $a > 4\alpha_1c_2$ and $(3\alpha - \alpha^2 - 1) \ge 0$ for $1 < \alpha \le 2.618$. The calculations above show that equilibrium price is higher in the pre-merger game than in the post-merger game; $P_N > \hat{P}$ for $1 < \alpha \le 2.618$ to be precise. This means that the merger promotes market efficiency.

AS was shown in *Proposition* 1, in the Integrated Monopoly game the vertical integration causes equilibrium downstream prices to go down. Together with *Proposition* 2, whether the upstream market is a monopoly or not, the answer to the second question is that the integration can result in higher market efficiency. Note, however, that the integrated *duopolist* can not exclude a rival firm out of the market. This is in striking contrast with merger by a *monopolist*, who can drive a rival firm out of a market, but only if the monopolist has superior productivity. Vertical integration has two distinct effects: the cost effect and the productivity effect. The productivity effect is weaker in the DD game so that the rival can be viable even if the rival has inferior productivity in the DD game. Thus. the exclusion of a downstream rival firm can arise only if upstream market structure is a monopoly.

Even if a downstream firm can stay in the market, the integrated firm will have incentives to monopolize a market. There are several means to exclude rivals from a market. One of the typical means is a price squeeze by the integrated firm. It is shown in Yang and Kawashima (2011) that the integrated firm does not have incentives to monopolize the market by a price squeeze in the Partial-Monopoly game (or PM game), but that it does in the One-sided Integration game (or OI game).

4 Conclusions

The proposed model has been applied to examine conditions under which the vertical integration *per se* causes market foreclosure and when it does not. The integration has two distinct effects on a market: (1) the cost effect which comes from the fact that the integration enables the integrated firm to get access to inputs at lower costs, and (2) the productivity effect which comes from the fact that the integrated firm owns a downstream division. Both the cost and productivity effects exert influences on market prices. If an upstream monopolist merges with an equally efficient downstream firm, the monopolist can exploit leverage in the downstream market and exclude the rival from the market. The cost effect is strong enough for the integrated monopolist to exclude the rival from a market. On the other hand, when the merged firm has advantages in costs and production relative to an independent firm in the duopoly game, market foreclosure does not take place because the cost effect of the duopolist is not strong and hence the integration does not exclude the independent from the market.

From the viewpoint of welfare, the vertical integration results in lower downstream prices even if foreclosure occurs. Even if integration is neutral in competitiveness in a market because the number of rivals is not reduced, it nevertheless promotes market efficiency by lowering prices. Thus, there is no trade-off between anti-competitiveness and market efficiency.

In a one-sided vertical integration game, the upstream division of the merged firm provides throughputs only to the downstream division. This is similar to "keiretu" of Japanese industries. This is a game in which an efficient upstream firm and an efficient downstream firm merge. Moreover, as mentioned in Footnote 5, our results are independent of the pair of firms which merge.

There are several limitations to our model. For example, it was assumed that the upstream and the downstream divisions of the integrated firm have constant returns to scale. If the upstream division exhibits increasing returns to scale, it is probable that the division can maximize profits by supplying inputs to the upstream market. The model assumes a linear demand function. This enabled us to make our model tractable. However, when we try other demand functions, it may be very difficult to solve the two-stage games even with complete and perfect information because it is necessary to derive an upstream demand from downstream equilibrium outputs. It is easy to solve for downstream equilibrium, but it is hard to solve for upstream equilibrium. Until now we have not found other demand functions which enable us to explicitly solve the two-stage games, hence this remains a subject for future investigation.

References

- Bork, R.H.(1978) The Antitrust Paradox: A Policy at War with Itself. New York: Basic Books.
- [2] Chen, Y.(2001) "On Vertical Mergers and Their Competitive Effects", Rand Journal of Economics, 32(4), pp.667-685.
- [3] Chen, Y. and Riordan, M.H.(2007) "Vertical Integration, Exclusive Dealing, and ex post Cartelization", Rand Journal of Economics, 38(1), pp. 1-21.
- [4] Choi, J.P. and Yi, S.-S.(2000) "Vertical Foreclosure and the Choice of Input Specification," Rand Journal of Economics, 31,pp. 717-743.
- [5] Church, J. and Gandal, N.(2000) "Systems Competition, Vertical Merger, and Foreclosure," Journal of Economics and Management Strategy, 9, pp. 25-51.
- [6] Hart, O. and Tirole, J.(1990) "Vertical Integration and Market Foreclosure", Brookings Papers on Economic Activity, "Microeconomics":Brookings Institution, Washington, pp. 205-286.
- [7] Hortacsu, A. and Syverson, C.(2007) "Cementing Relationships: Vertical Integration, Foreclosure, Productivity, and Prices", *Journal of Political Economy*, 115, pp. 250-301.
- [8] Ma, C.A.(1997) "Option Contracts and Vertical Foreclosure", Journal of Economics and Management Strategy, 6, pp. 725-753.
- [9] Ordover, J., Saloner G., and Salop, S.(1990) "Equilibrium Vertical Foreclosure", American Economic Review, 80(1), pp. 127-142.
- [10] Riordan, M.H.(1998) "Anticompetitive Vertical Integration by a Dominant Firm", American Economic Review, 88,pp. 1232-1248.
- [11] Salinger, M.A.(1988) "Vertical Mergers and Market Foreclosure", Quarterly Journal of Economics, 103(2), pp. 345-356.
- [12] Salinger, M.A.(1991) "Vertical Mergers in Multi-Product Industries and Edgeworth E fs Paradox of Taxation", *Journal of Industrial Economics*, 40, pp. 545-556.
- [13] Spengler, J.(1950) "Vertical Integration and Antitrust Policy", Journal of Political Economy, 58(4), pp.347-352.

[14] Yang, C. and Kawashima, Y. (2011), "Is Price Squeeze Optimal Strategy for Integrated Firms ?", mimeo.