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*Faculty of Economics, Chuo University. Financial support from Chuo University Grant for Special Research 2002-2003 is gratefully acknowledged. This paper presents a theoretical base for analysis of Environmental Kuznets Curve (EKC) in static as well as dynamic frameworks. This work is in preparation for a statistical and econometric verification of EKC, which will be presented in a future joint paper with K. Nakamura and R. Isa.

Simple Theoretical Analysis of The Environmental Kuznets Curve

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Abstract

Theoretical analysis of Environmental Kuznets Curve (EKC) is performed by extending the static model of Andreoni-Levinson (2001) to its dynamic version. The prototype model is also revised towards featuring some different aspects of pollution abatement and production process. Then the contributors which constitute various interrelations between pollution and income are examined. Theoretical possibility of an inverted U-shaped EKC is tested, which shows initial deterioration but later environmental improvement along the economic development path.

Even if environmental aggravation is seen with continuous growth of consumption or income, it becomes clear that the expenditure on pollution abatement increases, and then an environmental improvement can be found. Such a policy becomes possible because people's consciousness to pollution control is relatively high so that people ask for an environmental improvement as part of improvement in the living standard. Such an induced governmental control on pollution must lead to a feasible technological progress in pollution abatement. Thus, strengthening of environmental policy reflects people's environmental consciousness.

Keywords: Environmental Kuznets Curve, dynamic optimal control, environmental consciousness, environmental policy

JEL Classification: O130

1. Introduction

It is very natural to ask whether it is possible to attain economic prosperity without deterioration of environment. As far as some environment-related indicators such as SO_2, NO_2 as well as garbage per capita are concerned, at an early stage of economic development and in the process of rapid industrialization, almost all economies have experienced their dramatic increase, leading to terrible environmental problems. In the course of time, governments have changed their policy stance to incorporate pollution control. For example, after the mess of environmental problems in the 1960's, particularly related to air and water pollution, the Japanese government enacted a new fundamental law of countermeasure for pollution and established an Environmental Protection Authority (EPA). Corresponding to a changing pattern of people's preference between income growth and environment, they might support these reforms. In fact, after a series of reforms on environmental policies or systems in the early 1970's, we have experienced halved economic growth but remarkable improvement in health and environment.

What kind of relation is there between environmental improvement and economic growth? And what is the major reason to cause such a changing pattern of environmental policies? The main purpose of this paper is to give answers to these questions mainly from theoretical viewpoints. It seems natural to accept that the environmental situation in general deteriorates with economic growth at low-income levels, but it will reach a turning point and then further growth leads to environmental improvement. The inverted U-shaped relationship between income and the environment is known as the environmental Kuznets curve (EKC). Grossman and Krueger (1993) have found that for some air and water pollutants, the inverted U-shaped patterns could be empirically observed. A large number of papers, including Cole, Rayner and Bates (1997), based on empirical studies have followed the Grossman and Krueger paper and the common conclusions are that a meaningful EKC can be observed only for local pollutants like NO_2 and CO but for global pollutants like CO_2, and for some pollutants like CFCs, no EKC relation can be confirmed. Possibly, there may be no general relation between pollutants and economic growth.

There is considerable literature on EKC based on theoretical viewpoints, including Andreoni and Levinson (2001), Levinson (2002) and Lieb (2002). Their analytical frameworks are basically static ones, which incorporate a specific type of utility function and pollution abatement function.

On the other hand, Selden and Song (1995), Stokey (1998) and Kelly (2003) have given dynamic model frameworks for EKC. Selden and Song (1995), a revised version of Foster (1973), have shown that even at an early stage of economic development, rapid increases in pollution abatement can occur due to some contributors, including technological change of abatement and consumers' demand for pollution abatement. They have argued that although capital accumulation can slow reduction in pollution, high marginal efficacy of pollution abatement can reduce pollution as a whole, leading to an inverted U-shaped EKC. Kelly (2003) has proved that in a dynamic model with pollution stock externality, the conditions for an inverted U-shaped EKC can emerge which require convexity of cost function of pollution control and normality of environmental goods. Pollution control provides benefit, but it costs additional spending to the society. Consequently, it may be argued that an inverted U-shaped EKC occurs when marginal costs of pollution control rise by less than marginal benefits.

Although the studies concerning EKC have given consistent explanations, there is no clear understanding that ties static frameworks to dynamic ones. Our primary purpose in this paper is to show the possibility of inconsistency of the conditions for EKC to have inverted U-shape both from static as well as dynamic viewpoints. Moreover, in the dynamic model framework, we shall compare different cases with various behavior or policies towards pollution abatement and economic growth.

The plan of this paper is as follows. In Section 2, a basic version of the EKC model is demonstrated which contains the temporal optimization controls of consumption and pollution, and proves an inverted U-shaped EKC to exist in a static framework. To give a static model on which we base some different types of dynamic models, we shall feature a framework of utility functions as well as pollution functions, which are employed by Andreoni and Levinson (2002). In Section 3, the basic model is developed towards a dynamic version and it is proved that as far as the dynamic optimal control of consumption and pollution is concerned, the necessary conditions in the static version of the model for existence of the inverted U-shaped EKC would be contradicted in the dynamic one. In Section 4, two extensions of the dynamic model are investigated; one is a stock-related pollution case and the other a variable cost case of pollution abatement. In the latter case, no inverted U-shaped EKC emerges. However, in the former case, an inverted U-shaped EKC may occur mainly because the pollution abatement to curb environmental deterioration eventually serves for improvement in the consumer's welfare. In Section 5, after remarking on the relationship between pollution control and economic development, the importance of strengthening the policy on environmental quality is examined.

2. A Basic Static Model

At the beginning of the arguments concerning EKC, it is useful to mention a simple formula developed by Andreoni and Levinson (2001) and Levinson (2002). With this simple static formula, they have proved that the inverted U-shaped EKC occurs corresponding to a consumer's preference between consumption and pollution. The steps to compose their model are given as follows. The utility function *U* of consumer is

(1) $\quad = \quad (C, \quad),$

with consumption *C* and pollution *P*. Let the utility function be specified as

(1)' $\quad = C - \quad .$

Pollution function is given by

(2) $\quad = \quad (C, E) \quad ,_{C} > 0, \quad _{E} < 0,$

where *E* denotes the effort to abate pollution and (2) is assumed to be specified as

(2)' $\quad = C - C^{\alpha} E^{\beta}.$

In (2)', α and β are positive parameters and the second term represents pollution abatement. β can be interpreted as a parameter concerning the efficiency of investment for pollution abatement. It is clear in (2)' that a larger β will lead to a decrease in pollution.

The marginal rate of substitution (MRS) is given by $\quad = \frac{\beta}{\alpha} \frac{C}{E}$ because from (1)' and (2)', U is simply $C^{\alpha} E^{\beta}$. Moreover, for a simple manipulation, we shall assume that the price of consumption goods and the abatement cost of pollution are both unity. Then we have the consumer's budget constraint with income, *M*:

(3) $\quad = C + E.$

Consumer is assumed to be able to control both *C* and *E*. Accordingly, maximization behavior of consumer in this static framework will lead to an optimal consumption, *C** as well as an optimal effort of pollution abatement, *E** given by

(4) $\mathrm{C}^* = \frac{\alpha}{\alpha+\beta}$, $\mathrm{E}^* = \frac{\beta}{\alpha+\beta}$.

By substituting (4) into the specified pollution function, we have the reduced form of pollution function as

(5) $$*(\quad) = \frac{\alpha}{\alpha+\beta} \quad - \left(\frac{\alpha}{\alpha+\beta}\right)^{\alpha}\left(\frac{\beta}{\alpha+\beta}\right)^{\beta} \quad {}^{\alpha+\beta}.$$

It is possible to confirm the shape of EKC from (5) because it shows a direct relationship between *P* and *M*. Differentiation of (5) with respect to *M* leads to

(6) $$\begin{aligned} \frac{\quad *}{\quad} &= \frac{\alpha}{\alpha+\beta} - \left(\frac{\alpha}{\alpha+\beta}\right)^{\alpha}\left(\frac{\beta}{\alpha+\beta}\right)^{\beta}(\alpha+\beta) \quad {}^{\alpha+\beta-1} \\ \frac{{}^{2}\ *}{\quad{}^{2}} &= -(\alpha+\beta)(\alpha+\beta-1)\left(\frac{\alpha}{\alpha+\beta}\right)^{\alpha}\left(\frac{\beta}{\alpha+\beta}\right)^{\beta} \quad {}^{\alpha+\beta-2} \end{aligned}.$$

Hence it can be verified that as long as $\alpha+\beta$ is greater than unity in (6), the second derivative becomes negative and the first derivative will be negative for a sufficiently large income, meaning that an inverted U-shaped EKC exists as depicted in Figure 1.

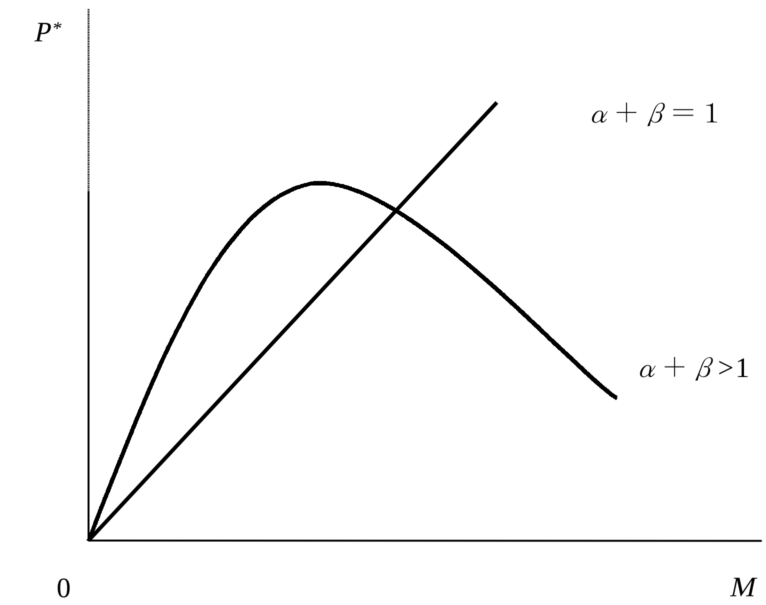

Figure 1. An Inverted U-shaped EKC

The inverted U-shaped performance is only induced by parameters affecting the marginal rate of substitution between consumption and effort to reduce pollution. It is clear that this depends on a specification of the pollution function. For example, if □ is sufficiently large, it will lead to a large pollution abatement and environmental improvement irrespective of income.

Lieb (2002) gives the static model analysis of EKC and proves that satiation in consumption leads to the downturn of the EKC because, with satiation, an increment of income does not increase utility and hence it should be devoted to pollution abatement. This means that, without satiation, pollution increases monotonically. His research has made some good points. However it should be noted that his model is not dynamic but static. Hence it does not show a dynamic optimal relationship between income and pollution. Moreover it seems difficult to accept that, after satiation in consumption, EKC proves to be downward sloping. We do not think that the major reason of pollution abatement comes from the consumer behavior towards their utility maximization.

3. Dynamic Optimization Models and the EKC

Main tasks in this section are to develop the dynamic counterparts of EKC model from the static versions, and to show their dynamic properties. Moreover, attention will be paid to the dynamic conditions for possible optimal policy for transition towards steady state. It is assumed that the consumers have the same utility function and pollution function specified in the former section. For the dynamic version, the production function is replaced by capital stock, K, in the budget constraint (3);

(7) $\quad = \quad (\; , \;) \; , \quad > 0, \qquad < 0, \qquad < 0 \, .$

The capital accumulation dynamics is defined by

(8) $\quad \dot{} \; = \quad - E - C$

where it is assumed that there is no capital depreciation and that both the real price of consumption goods and the cost of effort for pollution abatement are unity.

Accordingly, it is natural to accept that the static model given in the former section will be extended towards the dynamic optimization version, given by

(9) $$\rightarrow \int_0^{\infty} \quad {}^{-\rho} \quad = \int_0^{\infty} (C^{\alpha}E^{\beta}) \ {}^{-\rho}$$

$$\dot{} = \ (\ , C - C^{\alpha}E^{\beta}) - E - C, \quad (0) = \ _0$$

where society should control C as well as E for the evolution of capital accumulation. A problem of pollution control has been already studied by Foster (1973) in the context of the neo-classical growth model. Whereas in the Foster's model, pollution is additively separable in the capital stock and effort on pollution abatement, and is assumed not to affect production, the dynamic version (9) does not include K explicitly in the pollution function and P in the production function.

The current-value Hamiltonian is given by

(10) $$H = C^{\alpha}E^{\beta} + \lambda \quad (\ , C - C^{\alpha}E^{\beta}) - E - C \ .$$

The static and dynamic necessary conditions for optimality are given by

(11) $$\frac{\partial H}{\partial C} = \alpha C^{\alpha-1}E^{\beta} + \lambda \ \frac{\partial}{\partial}(1 - \alpha C^{\alpha-1}E^{\beta}) - 1 \ = 0,$$

(12) $$\frac{\partial H}{\partial E} = \beta C^{\alpha}E^{\beta-1} + \lambda \ \frac{\partial}{\partial}(-\beta C^{\alpha}E^{\beta-1}) - 1 \ = 0,$$

and

(13) $$\dot{\lambda} = -\frac{\partial H}{\partial} + \rho\lambda = - \ \lambda\frac{\partial}{\partial} \ + \rho\lambda = \lambda \ \rho - \frac{\partial}{\partial} \ ,$$

where (13) is the dynamic condition.

To give a clearer image of the dynamic process, we shall assign some specific properties to production function, (7). The first one is the case where M_P=0 and the second one is related to M given by a separable form. The case of M_P=0 occurs when any pollutants, even if they are emitted from production process, never affect the production level. In the second case, it is assumed that production decreases in proportion to the deterioration of pollution.

3-1. A Simple case: $\frac{\partial}{\partial} = 0$, then $E = \frac{\beta}{\alpha}C$ and $= C - \left(\frac{\beta}{\alpha}\right)^{\beta} C^{\alpha+\beta}$.

In this case, the dynamic equations are summarized by the following system:

$$(14)\quad \begin{cases} \dot{C} = \dfrac{C}{\alpha+\beta-1}\left[\rho - \quad '(\quad)\right] \\ \dot{} \ = \quad (\quad) - \left(\dfrac{\alpha+\beta}{\alpha}\right)C \end{cases}.$$

where M' is identical with M_K in (7) because M_P=0. The steady state of the system (14) is given by (C^*, K^*), which satisfies

$$(15)\quad * = \quad '^{-1}(\rho)\ , C^* = \left(\frac{\alpha}{\alpha+\beta}\right)\ (\quad '^{-1}(\rho)\ .$$

It is easy to find that the Jacobi matrix of (14) evaluated at the steady state (15) is

$$(16)\quad = \begin{bmatrix} 0 & -\dfrac{C}{\alpha+\beta-1}\quad '' \\ -\dfrac{\alpha+\beta}{\alpha} & ' \end{bmatrix}.$$

where M'' is identical with M_{KK} in (7) and negative. Therefore, the trace and the determinant of the Jacobi matrix (16) are given by

$$(17)\begin{cases} \qquad = \quad ' > 0 \\ D \qquad = -\dfrac{\alpha+\beta}{\alpha(\alpha+\beta-1)}C \quad '' > 0 \qquad \alpha+\beta > 1\ . \\ \qquad\qquad < 0 \qquad \alpha+\beta < 1 \end{cases}$$

In the case of $\alpha+\beta>1$, the equilibrium point of the system (14) is unstable. For the optimality of dynamic control towards the equilibrium point, the equilibrium point should be a stable (saddle) point. This implies that $\alpha+\beta<1$ must be assumed. In an ordinary optimization framework, the sum of parameters, □ and □, must be less than one. This means that there never can be the inverted U-shaped EKC along the optimal path towards the long-run equilibrium where income continues to increase. As far as the simple model is concerned, it may be said that the 'inverted U-shaped' case will not be dependent on how consumers react in their decision making.

Figure 2 illustrates a relation between environmental deterioration and economic growth. The phase angles are given by pairs of arrows in the first quadrant of Figure 2. In Figure 2, we assume that the investment for pollution abatement is not sufficiently efficient so that the overall pollution is increasing in the process of economic growth. This is denoted by $P(C)$ in the second quadrant in Figure 2. The first quadrant in Figure 2 illustrates the phase diagram of (14) when the system has a stable saddle point denoted by E. The fourth quadrant in Figure 2 denotes the production function. By taking the relations among *P, C* and *K* depicted by the first, second and fourth quadrant in Figure 2 into consideration, it is proved that an optimal control towards a steady state, from A to E, provides a corresponding relation between pollution and growth, denoted by the path from A to E in the third quadrant in Figure 2.

Although the simple case can not lead to the inverted U-shaped EKC, it may be possible to show that whenever a society turns its policy stance towards more environment-friendly development, introducing a damping effort on pollution can lead to a sharp decline in consumption as well as pollution. This is shown by the path from a to E via b and c in Figure 2. This can be a semblable pattern of U-shaped EKC.

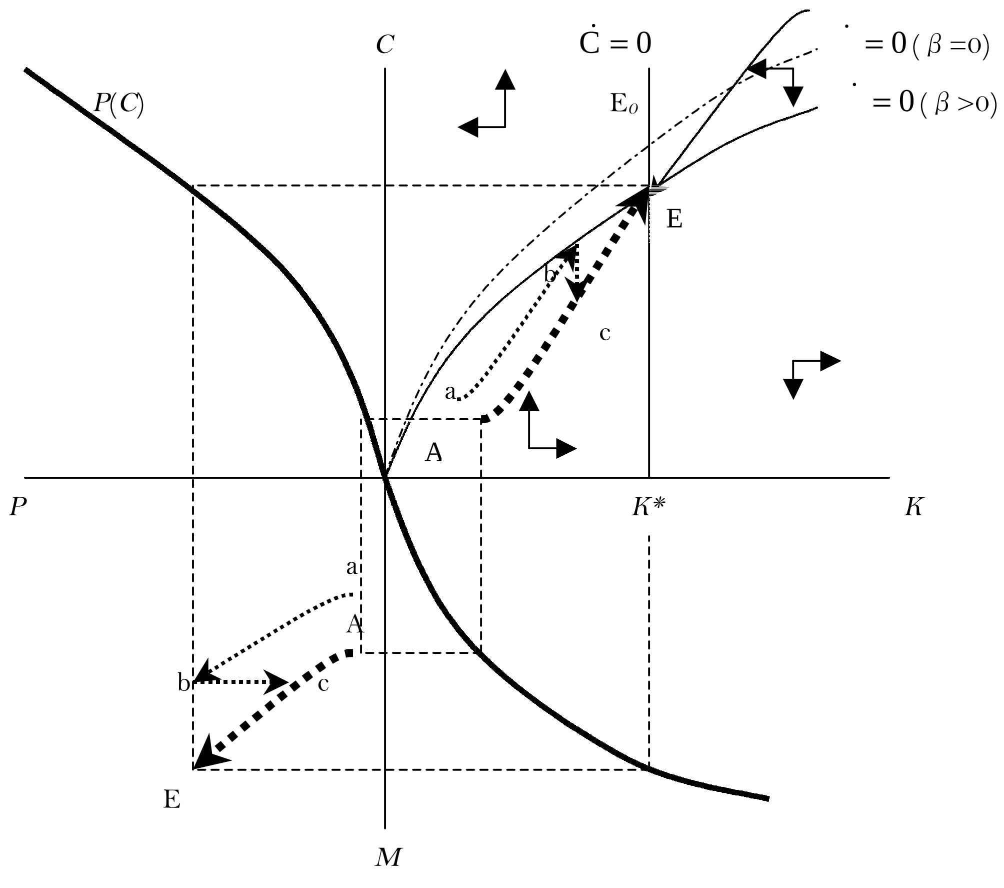

Figure 2. The phase diagram (A simple case)

3-2. Separable production function case: $= {}^{\gamma} - \quad , 0 < \gamma < 1,\ 0 <$

Under these restrictions, the production function is separable, increasing at a diminishing rate in capital stock, and decreasing at a constant rate in pollution. We shall assume that compared to the effect of pollution on the overall welfare, its direct effect on production is very small, indicated by a sufficiently small *b*. Clearly, it must be ensured that $\frac{\partial}{\partial} = \gamma \ {}^{\gamma-1} > 0$ and $\frac{\partial}{\partial} = - \ < 0$. Substituting the latter equation into (11) and (12) leads to

(18) $\mathrm{E} = (1+ \ )\frac{\beta}{\alpha}\mathrm{C}$.

In this case, the dynamic equations (14) change to:

$$(19)\quad \begin{cases} \dot{\mathrm{C}} = \dfrac{1+ \ (1-\Lambda \mathrm{C}^{\alpha+\beta-1}) \ \mathrm{C}}{(\alpha+\beta-1)\ 1(+ \ )} \ \rho - \gamma \ {}^{\gamma-1} \\ \dot{} = {}^{\gamma} - \left((1-\dfrac{\Lambda}{\alpha}\mathrm{C}^{\alpha+\beta-1}) + \dfrac{\alpha+\beta}{\alpha}\right)\mathrm{C} \end{cases},$$

where $\Lambda \equiv \alpha\left((1+ \ )\frac{\beta}{\alpha}\right)^{\beta}$. In this case, it is easy to show that the dynamic property of the system is slightly different from the simple case where $\frac{\partial}{\partial} = 0$. For the Jacobi matrix of (19) at the steady state, the trace and the determinant are given by

$$(20)\quad \begin{cases} \quad = \dfrac{\partial}{\partial} > 0 \\ \mathrm{D} \quad = -\dfrac{1+ \ (1-\Lambda \mathrm{C}^{\alpha+\beta-1}) \quad (1-(\alpha+\beta)\dfrac{\Lambda}{\alpha}\mathrm{C}^{\alpha+\beta-1}) + \dfrac{\alpha+\beta}{\alpha}}{(\alpha+\beta-1)\ 1(+ \ )}\mathrm{C}\dfrac{\partial^2}{\partial \ {}^2} \end{cases}.$$

Because *Trace J* is positive, the condition for the eigenvalues of characteristic equation to be anticlastic, meaning that *Determinant J* should be negative is necessary for the steady state to be stable saddle point. The sign of *Determinant J* depends on the terms in the numerator as well as in the denominator. For a sufficiently small *b*, however, this can be positive irrespective of $1-\Lambda \mathrm{C}^{\alpha+\beta-1}$. Hence, we reach a conclusion similar to that in the former simple case that the system has the stable optimal path only in the case of the sum of parameters, □ and □, less than one. In this case, under no circumstances does the existence of a controllable optimal policy lead to the inverted U-shaped EKC. As shown in Section 2, this is not compatible with the static optimization case.

As for the case of large *b*, the conditions for the stable dynamic system are slightly different. Noting the non-negativity of the pollution level along the optimizing process clearly lead to the condition given by (21)

$$(21) \qquad = C\left(1-\frac{\Lambda}{\alpha}C^{\alpha+\beta-1}\right) = \quad (C) \geq 0 .$$

What we would like to know is the possibility of the stable saddle point case even if □+ □ is greater than unity. To obtain a sufficient condition for this, it is convenient to assume □+ □ − 1 > 0 as well as $\alpha \;\; = 1$, that is $\beta \;\; > 0$. As long as these conditions are incorporated into analysis, the second term in the numerator of *Determinant J* of (20) can be negative but the first term is positive when *b* is sufficiently large[1]. Therefore, it is likely that when the effect of pollution on production, *b*, is large enough, the system can have an optimal control policy related to consumption as well as investment for pollution abatement. It is of interest to consider the meaning of the largeness of □. Differentiating (21) with respect to *C* yields

$$(22) \qquad \frac{}{C} = 1-(\alpha+\beta)\frac{\Lambda}{\alpha}C^{\alpha+\beta-1}, \quad \frac{2}{C^2} = -(\alpha+\beta)\,(\alpha+\beta-1)\frac{\Lambda}{\alpha}C^{\alpha+\beta-2} .$$

Hence, it is easy to see that the conditions, $1-(\alpha+\beta)\frac{\Lambda}{\alpha}C^{\alpha+\beta-1} < 0$ and $\alpha+\beta-1>0$ yield an unambiguous relationship between consumption and pollution. Then, equation (22) implies that pollution is decreasing at a diminishing rate in consumption because $dP/dC>0$ and $d^2P/dC^2<0$. The initial consumption in the under-developed economies must be so small that the first derivative in (22) will be positive; implying that any increase in consumption should entail environmental deterioration in the embryonic stage of the economic development. In the process of capital accumulation, however, an increase in *K* or *C*, and a decrease in *P* together can occur because people are presumed to prefer pollution abatement to consumption as consumption increases.

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[1] As for the second term of numerator of *determinant J* in (20), we need to assume $1-(\alpha+\beta)\frac{\Lambda}{\alpha}C^{\alpha+\beta-1}$ to be negative.

We shall offer some of these conclusions as a précis. A social planner should manage and coordinate the expenditure for pollution abatement adequately towards the optimal growth path. In the optimal process, when pollution has a negative impact on production and people think environment as more important, the policy that holds down pollution in the process of economic growth becomes the optimal policy. This may lead to an inverted U-shaped EKC along A'-B'-C' depicted by the second quadrant in Figure 3, corresponding to an optimization path from A to E via B depicted by the first quadrant in Figure 3. Point B can be a turning point after which the policy to abate pollution may be preferable to growth-oriented one for the society. This is the case for a sufficiently large *b* and □[2].

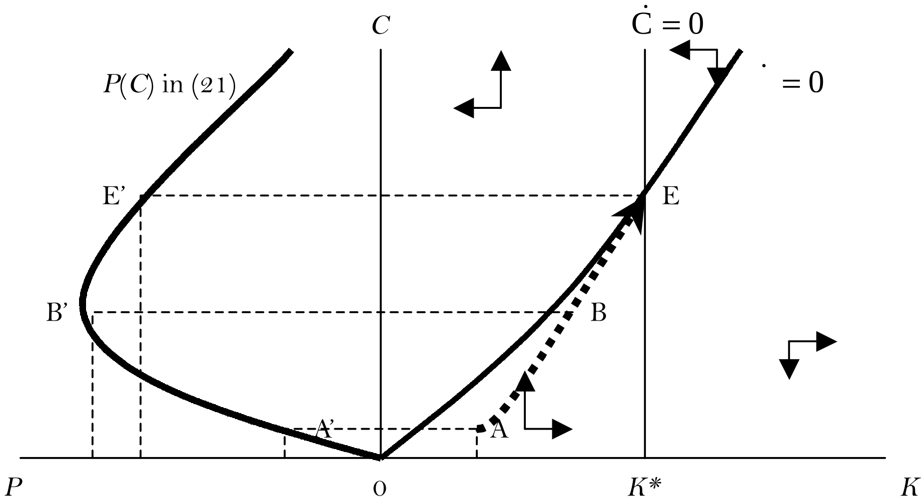

Figure 3. The phase diagram (A separable production function case)

4. Some Extended Dynamic Versions

This section will be devoted to more theoretical frameworks related to EKC. We shall follow some alternative procedures, and then compile conditions related to the possibility of an inverted

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[2] It is possible for the system to have multiple equilibriums. Although the curve $\dot{} = 0$ is upward-sloping for a low *C* as in the first quadrant in Figure 3, it can bend towards the northwest and possibly be downwards-sloping for a sufficiently large *C*. Hence, it is easy to see that there can be an upper equilibrium point where the system is unstable.

U-shaped EKC to occur.

4-1 Stock-related pollution case:

Pollution function (2) is now specified as

(23) $P = C - C^{\square} E^{\square} + a K^{\square}$, $a > 0$.

This means that the accumulated capital stock also causes pollution through the production process. On the other hand, we shall feature a simple form of production function given by

(24) $M = M(K) = K^{\square}$, $\gamma < 1$

The condition on γ implies a diminishing marginal productivity[3].

The optimization formula in this case is given by

$$(25)\qquad \to \int_0^\infty \quad {}^{-\rho} \quad = \int_0^\infty (C^\alpha E^\beta - \quad {}^{\gamma})\ {}^{-\rho} \qquad . \\ \dot{} \;\; = \quad {}^{\gamma} - E - C$$

Because the current-value Hamiltonian is simply defined as

$$H = C^\alpha E^\beta - \quad {}^{\gamma} + \lambda \quad {}^{\gamma} - E - C \ ,$$

we have the following necessary conditions for optimality: the static ones are

$$(26)\qquad \frac{\partial H}{\partial C} = \alpha C^{\alpha-1} E^\beta - \lambda = 0, \ \frac{\partial H}{\partial E} = \beta C^\alpha E^{\beta-1} - \lambda = 0 ,$$

together leading to a simple allocation between consumption and effort for pollution abatement, $E = \dfrac{\beta}{\alpha} C$. Moreover, the dynamic condition is given by

$$(27)\qquad \dot{\lambda} = \lambda \ \rho - \gamma \quad {}^{\gamma-1} \ + \ \gamma \quad {}^{\gamma-1} .$$

Differentiating the first equation in (26) with respect to time and taking (27) into consideration yields

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[3] This case is very close to Forster's model except for the specification of consumer's utility function.

$$(28)\quad \begin{cases} \dot{C} = \dfrac{C}{\alpha+\beta-1}\left((\rho - \gamma \quad {}^{\gamma-1}) + \dfrac{\gamma \quad {}^{\gamma-1}}{\alpha\left(\dfrac{\beta}{\alpha}\right)^{\beta} C^{\alpha+\beta-1}} \right). \\ \dot{} \;=\; {}^{\gamma} - \left(\dfrac{\alpha+\beta}{\alpha}\right) C \end{cases}$$

At the equilibrium point of this system, the Jacobi matrix becomes

$$(29)\qquad = \begin{bmatrix} -\dfrac{\gamma \quad {}^{\gamma-1}}{\alpha\left(\dfrac{\beta}{\alpha}\right)^{\beta} C^{\alpha+\beta-1}} & \dfrac{C}{\alpha+\beta-1}\left(-\gamma(\gamma-1) \quad {}^{\gamma-2} + \dfrac{\gamma(\gamma-1) \quad {}^{\gamma-2}}{\alpha\left(\dfrac{\beta}{\alpha}\right)^{\beta} C^{\alpha+\beta-1}} \right) \\ -\dfrac{\alpha+\beta}{\alpha} & \gamma \quad {}^{\gamma-1} \end{bmatrix}.$$

This gives the trace and determinant related to (29) as:

$$(30)\quad \begin{cases} \qquad = (1 - \dfrac{}{\Phi})\gamma \quad {}^{\gamma-1} \\ D \qquad = \dfrac{\gamma \quad {}^{2(\gamma-1)}}{(\alpha+\beta-1)\Phi} - \;\; \gamma(\alpha+\beta-1) + (1-\gamma)\Phi(1 - \dfrac{}{\Phi}) \\ \Phi \equiv \alpha\left(\dfrac{\beta}{\alpha}\right)^{\beta} C^{\alpha+\beta-1} > 0 \end{cases}$$

It is easy to prove from the first equation in (28) that the positivity of equilibrium production needs an additional condition:

$$(31)\quad 1 - \frac{}{\Phi} > 0,$$

and hence, *Trace J* in (30) must be positive. As mentioned before, the condition under which the system (28) has the optimal controls on *C* as well as *E* is for *Determinant J* in (30) to have negative sign. Table 1 summarizes possible stock-related pollution cases where this is likely to occur.

Table 1 A summary of stability conditions

Case	□+□−1	*a*	*Determinant J*
i	+	Sufficiently large (*a* is close to Φ)	−
ii	+	Small enough	+−

iii	—	0< a <Φ	—

In the case when pollution occurs in the production process so that capital accumulation becomes one of the major sources of pollutants, optimal control towards steady state exists. Unlike in the simple model version, even if □+□ is larger than unity, there can be a saddle point case whenever a is sufficiently large and close to Φ, meaning that production process causes a large-scale pollution to society (Case (i) in Table 1). What we would like to know is about an evolution process of pollution control along the optimal path as well as the possibility of the inverted U-shaped EKC in Case (i). This can be investigated by the following procedure. The slopes of $\dot{C}=0$ and $\dot{} = 0$ curves are:

(32) $$\frac{C}{}\Big|_{\dot{C}=0} = \frac{(1-\gamma)\Phi(1-\frac{}{\Phi})C}{(\alpha+\beta-1)}, \qquad \frac{C}{}\Big|_{\dot{}=0} = \frac{\alpha\gamma \quad ^{\gamma-1}}{\alpha+\beta} > 0 \quad .$$

Moreover it is notable that the stability condition, i.e. the negativity of *Determinant J* in (30), include

(33) $$\frac{C}{}\Big|_{\dot{C}=0} < \frac{C}{}\Big|_{\dot{}=0} .$$

Therefore, except for the difference in shape, the dynamic properties as well as the phase are essentially the same as the ones for the simple case, which is depicted as in Figure 2. The time derivative of (23) yields

(34) $$\dot{} = 1-(\alpha+\beta)\frac{\Phi}{\alpha}\dot{C} + \quad \gamma \quad ^{\gamma-1} \dot{} \quad .$$

Because the dynamic optimization process with a positive income growth towards steady state entails both increase in consumption and cumulative capital, a sufficient condition for pollution to decrease is clearly assured when the expression in the square brackets in (34) is negative and small enough. So long as a is large enough, evaluation of both Φ and the marginal productivity of capital in the neighborhood of (C,K) = (0,0) leads to an initial state with increases in pollution. In the early stages of economic development, pollution will increase. However, increases in C provide increases in Φ, whereas the marginal productivity of capital declines due to diminishing returns. The result can be decreases in pollution because the first term in (34), which turns negative eventually, dominates the second term., which is positive. This will lead to the possibility of an inverted U-shaped EKC[4].

4-2. Variable cost function of pollution abatement:

This subsection is devoted to a small revision of the simple case, with only the abatement cost of pollution being considered. We shall incorporate a cost function, $\bar{\ }$, in real terms related to pollution abatement and then we have a slightly different version of (8):

(35) $\dot{\ } = \quad - \bar{\ }\mathrm{E} - \mathrm{C}, \bar{\ } = \bar{\ }(\mathrm{C}) = \ \mathrm{C}^{\delta}, \quad > 0, \delta > 0; \qquad .$

In the capital accumulation process, *C* continues to grow and the social cost of efforts to reduce pollution will increase sharply and this leads to a hard restriction on capital accumulation. The marginal social cost of pollution abatement is increasing if $\delta > 1$ but decreasing with respect to consumption for $1 > \delta > 0$.

The optimization formula for this case is given by

(36)
$$\rightarrow \int_0^{\infty} \quad {}^{-\rho} \quad = \int_0^{\infty} (\mathrm{C}^{\alpha}\mathrm{E}^{\beta}) \ {}^{-\rho} \qquad .$$
$$\dot{\ } = \ (\ ) - \ \mathrm{C}^{\delta}\mathrm{E} - \mathrm{C}$$

The optimal trajectory satisfies

(37) $\alpha \mathrm{C}^{\alpha-1}\mathrm{E}^{\beta} - \lambda(\ \delta \mathrm{C}^{\delta-1}\mathrm{E} + 1) = 0$,

(38) $\beta \mathrm{C}^{\alpha}\mathrm{E}^{\beta-1} - \lambda \ \mathrm{C}^{\delta} = 0$,

and

(39) $\dot{\lambda} = -\dfrac{\partial \mathrm{H}}{\partial} + \rho\lambda = -\lambda \quad '(\ ) + \rho\lambda = \lambda \ \rho - \quad '(\ )$,

where λ is the shadow price of capital. It is notable that (37) and (38) together yield

(40) $\mathrm{E} = \left(\dfrac{\beta}{(\alpha - \beta\delta)} \right) \mathrm{C}^{1-\delta}$,

and substituting (40) into the pollution function (2) yields[5]

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[4] Selden and Song (1995) also investigate the dynamic properties of Foster's model and prove that an inverted U-shaped EKC can occur mainly due to rapid increases in pollution abatement. They enumerate three contributors by which pollution can be abated: a high marginal efficacy of abatement, $\partial P/\partial E$ and a large direct effect of growth on pollution, $(\partial P/\partial K)\times(dK/dt)$, and a large increment of the marginal disutility of pollution, $\partial^2 U/\partial P^2$. As far as our case here is concerned, the second one seems to be the most important.

[5] It is hardly acceptable that in the process of economic growth, an increase in consumption leads to a decrease in

(41) $$= C\left(1-\left(\frac{\beta}{(\alpha-\beta\delta)}\right)^{\beta} C^{\alpha+\beta(1-\delta)-1}\right).$$

Because this is a slightly modified version of the simple case in section 3-1, we have the following dynamic equations:

(42) $$\begin{cases} \dot{C} = \dfrac{C}{\alpha+\beta(1-\delta)-1}[\rho - \quad '(\quad)] \\ \dot{\ } \ = \quad (\quad) - \left(\dfrac{\alpha+\beta(1-\delta)}{\alpha-\beta\delta}\right) C \end{cases},$$

and clearly making $\delta = 0$ in (42) leads to the same system as (14). Therefore, it is easy to prove that the condition for the equilibrium to be stable is give by

(43) $$\frac{\alpha+\beta(1-\delta)}{(\alpha-\beta\delta)\ \alpha+\beta(1-\delta)-1} < 0.$$

Taking the restrictions imposed on (40) into consideration, we find that (43) is consistent with the following condition to be compared with (17):

(44) $\alpha + \beta(1-\delta) < 1$.

As far as utility arises from consumption and pollution abatement, even if we incorporate the social cost into the model as the blockage of capital growth, it seems impossible to show the existence of an inverted U-shaped EKC. All the cases show the positive correlation between income and pollution. Of course, it would be easy to say that this is a normal or usual fact in the real world. However, it is also notable that incorporating the social cost of pollution abatement into the model system leads to the lower consumption as well as lower pollution along the optimal growth path in comparison with the case of no social cost.

4-3. Conditions for an 'inverted U-shaped EKC'

It is useful to give a synopsis of the model analysis concerning conditions under which an inverted U-shaped EKC can be observed. Table 2 summarizes the cases considered in Sections 3 and 4. Table 2 shows that an inverted U-shaped EKC can occur when there is a restrictive influence of pollution upon production and/or when a changing pattern of production via capital

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effort to abate pollution. Hence, we shall assume that $1-\delta > 0$ in this connection as well as $\alpha - \beta\delta > 0$ for the positivity of *E*.

accumulation has an impact on pollution. The major reason why pollution can be reduced along the optimal control path with economic growth is clear. This is because the social planner, who controls consumption as well as pollution abatement so as to coordinate the economy on optimal growth path, tends to prefer the latter to the former, leading to more expenditure towards abatement of pollution. After all, preference of the society to be more environment-friendly may be important to curb pollution.

Table 2. Possibilities of inverted U-shaped EKC

Case	*Inverted U-shaped EKC*	*Production function* $\left(\frac{\partial}{\partial}\right)$	*Pollution function* $\left(\frac{\partial}{\partial}\right)$	*Social marginal cost of pollution abatement* $(^{-})$
3-1	Impossible	0	0	1
3-2	Possible	—	0	1
4-1	Possible	0	+	1
4-2	Impossible	0	0	Variable

5. Concluding Remarks

Theoretical analysis of EKC has been performed. Even if environmental aggravation is seen with continuous growth of consumption or income, it has been shown clearly that the expenditure on pollution abatement increases, and then an environmental improvement follows. Such a policy becomes possible because people's consciousness to pollution control is relatively high so that people ask for an environmental improvement as part of the improvement in the living standard. In the early stage of economic development, it is usually observed that governments do not pay attention to environmental degradation and people also tend not to think about environmental situation but about their income because their living standards are too low. This means that there must be a switching pattern of the environmental policy before income is sufficiently large when they can see an environmental problem ahead. Such an induced governmental control on pollution must lead to a feasible technological progress in pollution abatement. Such strengthening of the environmental policy is reflecting the change in people's environmental consciousness[6]. Possibly, the most prevailing explanations for an inverted U-shaped EKC are (i) an increasing demand for environmental quality, (ii) technological improvements to make production cleaner, (iii) de-industrialization of the economy, i.e. a structural change from manufacturing sector to service sector[7]. Among these explanations, our model analyses have emphasized the first one which leads to an increasing awareness of deterioration of environmental quality.

Even if people demand a better environmental quality, governments do not always act so as to maximize people's welfare. This occurs partly because there is no suitable socio-political organization and mainly because there is little political freedom. Boyce (2002), for example, has tested the consistency of the impacts of literacy, political rights as well as civil liberties on some pollutants. His research almost supports the hypothesis that improvements in environmental quality can be attained via better literacy and higher political and civil liberties. Bimonte (2002) has also pointed out that expanding the real freedom can make development sustainable because participation can lead to people's individual preference to be justified towards social welfare. Because political as well as social factors can influence environmental quality to a considerable extent, it may be misleading to reduce these factors to a simple relationship, like an inverted U-shaped EKC between per capita income and environmental quality. How environmental and socio-economic aspects are interrelated is still an open question.

References

Andreoni J. and A.Levinson (2001), "The Simple Analytics of the Environmental Kuznets Curve," *Journal of Public Economics*,80(2), 269-286.

Atkinson,G.R., Dubourg,K., Hamilton,M. Munasinghe, D.Pearce and C.Young, (1997) *Measuring*

6 In connection with this, a changing pattern of policy stance is analyzed by Stocky (1998). In that model, two control variables are chosen, consumption and an emission standard. The government sets the latter in order to maximize the social welfare. Because the marginal cost of pollution is assumed to increase along the growth path, the government is forced to change their environmental policy and to tighten the emission standard. This leads to a single-peaked, non-smooth, EKC.

7 For example, Hanley, *et al.* (2001), gives good text-based explanations.

Sustainable Development, Edward Elgar.

Bimonte,S. (2002) "Information access, income distribution, and the Environmental Kuznets Curve," *Ecological Economics*,41, 145-156.

Boyce,J.K. (2002) *The Political Economy of the Environment*, Edward Elgar.

Cole, M.A., Rayner, A.J. and J.M.Bates (1997) "The Environmental Kuznets curve: an empirical analysis," *Environmental and Development Economics*, 2, 401-416.

Dasgupta,S., B.Laplante, H.Wang and D.Wheeler (2002) "Confronting the Environmental Kuznets Curve," *Journal of Economic Perspectives*,16,1,147-168.

Foster,B.A.(1973) "Optimal Capital Accumulation in a Polluted Environment," *Southern Economic Journal*,39,544-547.

Freeman,A.M. (2002) "Environmental Policy Since Earth Day I: What Have We Gained?," *Journal of Economic Perspectives*,16,1,125-146.

Grossman,G.M. and A.B.Krueger, (1993) "Environmental Impacts of the North American Free Trade Agreement," in P.Garber ed., *the U.S.-Mexico Free Trade Agreement*, Cambridge MIT Press,13-56.

Grossman,G.M. and A.B.Krueger, (1995) "Economic growth and the environment," *Quarterly Journal of Economics*,112,353-377.

Hanley,N. ,Shogren,J.F. and B.White, (2001) *Introduction to Environmental Economics*, Oxford University Press.

Kelly, D.L. (2003) "On environmental Kuznets curves arising from stock externalities," *Journal of Economic Dynamics & Control*,27, 1367-1390.

Levinson,A.(2002) "The Ups and downs of the environmental Kuznets curve," in, List,J.A and Zeeuw,A.de ed. *Recent Advances in Environmental Economics*, Edward Elgar, 119-141.

Lieb,C.M.(2002) "The environmental Kuznets curve and satiation: a simple static model," *Environment and Development Economics*,7,429-448.

Neumayer, E.(1999) *Weak versus Strong Sustainability*, Edward Elgar.

Pashe,M. (2002) "Technical progress, structural change, and the environmental Kuznets curve," *Ecological Economics*,42,381-389.

Selden,T and Song D. (1994) "Environmental Quality and Development: Is there a Kuznets Curve for Air Pollution Emissions?," *Journal of Environmental Economics and Management,* 27, 142 -162.

Selden,T and Song D. (1995) "Neoclassical Growth, the J Curve for Abatement, and the Inverted U

Curve for Pollution," *Journal of Environmental Economics and Management,* 29, 162-168.

Shafik,N. (1994) "Economic Development and Environmental Quality: An Econometric Analysis," *Oxford Economic Papers,*46,757-773.

Stokey,N.L. (1998) "Are there limits to growth?," *International Economic Review*,39,No.1,1-31.

Tisdell,C.(2001) "Globalization and sustainability: environmental Kuznets curve and the WTO," *Ecological Economics*,39,185-196.