IS PRICE SQUEEZE THE OPTIMAL STRATEGY FOR INTEGRATED FIRMS?

Chuan Yang

and

Yasuo Kawashima

Faculty of Economics

Chuo University

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Address: 742-1, Higashinakano, Hachioji-shi, Tokyo 192-0393,Japan

fax:+81 426 74 3425

e-mail: yasuokaw@tamacc.chuo-u.ac.jp

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ABSTRACT

This paper seeks to analyze the effects of a price squeeze by an upstream firm in vertically related markets and to examine equilibrium in markets when one of the upstream firms integrates one of the downstream firms. We show that an integrated monopolist has no incentives to employ such a strategy, but that an integrated duopolist does have such incentives. Monopolization does not give the maximum profits to the monopolist with increased market prices. Such a strategy employed by the integrated firms is anti-competitive in its nature, but it improves market efficiency when employed by the integrated duopolist.

1 Introduction

The features of a vertically integrated firm (for example, a monopolist or a duopolist) are that the integrated firm supplies inputs to downstream rivals and at the same time competes to supply outputs in a downstream market.[1] As Armstrong (2008) pointed out, by using a pricing strategy in the vertically related markets, the integrated firm can control not only input prices, but also output prices. Then, Vickers (2008) points out that a price squeeze arises either when price for inputs is unduly high or when price for outputs is unduly low.[2] As the monopolization is expected to give the monopolist the maximum profits *in a single market*, the integrated firm will have incentives to drive rival firms out of a market by strategies such as a price squeeze.[3] In these settings, the Chicago School has long maintained that a price squeeze is not a rational strategy of the integrated firm.[4]

In fact, Joskow (1985) observes through empirical analysis that the integrated firm has incentives to employ a price squeeze to exclude rival firms from a market. King and Maddock (1999) show that the integrated firm has incentives to practice a price squeeze. Whinston (1990) shows that a price squeeze is a profitabler strategy for the integrated frim if the assumptions of the Chicago School is relaxed. Chen (2001) shows that an integrated firm raises the costs of independent firms by increasing switching costs. Thus, there is the fear that such a strategy

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[1] Note that words "inputs" and "outputs" are defined from the viewpoint of downstream firms.

[2] For an expository explanation of a price squeeze, see Tirole (1988), and see Joskow (1985) for its definition.

[3] For a survey of price squeezes, see Crocioni and Veljanovski (2003).

[4] See, for example, Bork (1978).

enables the monopolist to monopolize the downstream market.[5]

On the other hand, following Weisman (1995) and Sibley and Weisman (1998), Sappington (2006) concludes that the integrated firm does not have incentives to increase a rival's costs. Raising the rival's costs will cause profits of the rival firm to be squeezed to zero. Thus, it follows from these observations that the integrated firm may not have incentives to exclude the rival from a market by some strategies such as a price squeeze. These results are quite contrary to observations by Joskow (1985), King and Maddock (1999), Whinston (1990), and Chen (2001). Thus, the debate is not settled. From the viewpoint of regulation, it is very important to know whether the vertically integrated firm (a monopolist or a duopolist) has incentives to practice a price squeeze.[6]

The present paper will show that the integrated upstream monopolists do not have incentives to employ a price squeeze even if an downstream market is duopolistic, but that the integrated duopolist has incentives to practice a price squeeze if both of an up- and downstream market are duopoly. To be precise, if the upstream monopolist merges one of downstream firms, extra profits earned in the downstream market under a price squeeze is less than profits lost in the upstream market and market price goes up, which is equal to a monopoly price. On the other hand, when the integrated duopolist employ such a strategy, additional profits earned in a downstream market outweigh profits lost in an upstream market. Thus, the integrated duopolist has incentives to adopt a price squeeze. Moreover, it will be interesting to note that downstream prices decrease under a price squeeze by the integrated duopolist. These are quite different form the results in the integrated monopolist. Depending upon the presence of competitive pressure in the upstream market, effects of a price squeeze are quite different. These are some of the important features of vertically related markets

The model presented in this paper has different types of games. It is important to note that all palyers are constrained to play their original roles once a game starts. This means that we do not focus on problems of palyers' incentives, and that it is possible to compare the important characteristics of games. Our model is as follows. The first stage corresponds to an upstream market where an integrated upstream monopolist (or a duopolist) supplies inputs to an independent downstream firm and the second stage corresponds to a downstream market where the downstream division of the integrated firm and the independent downstream firm compete to

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[5]Recently, Grout (2001), and Bouckaert and Verboven (2004) have investigated a price squeeze for the implementation of regulation policy.

[6]This strategy is akin to predatory pricing (or limit pricing). See, for example, Kawashima (1983) for the analysis of limit pricing.

supply a final good. These markets are modeled as two-stage games with complete and perfect information. In fact, given the linear demand for the final good, the downstream firms compete to supply them. Once equilibrium outputs of these firms are determined, demand for inputs of the independent firm is derived from equilibrium outputs of the independent downstream firm. The integrated firm maximizes profits for the given demand for input. Thus, it is possible to solve for upstream equilibrium price and input, from which downstream equilibrium prices and outputs are determined.

The paper is organized as follows: Section 2 explains our basic settings and shows that the backward induction enables us to determine not only equilibrium outputs and inputs, but also equilibrium prices in the up- and down-stream market. When we turn our attention to a game in which the integrated firm has incentives to be a Stackelberg leader who can employ a price squeeze, such a strategy by the integrated monopolist is examined to see how profits earned by that strategy compare with those earned without that strategy. Then, we will show that the integrated monopolist has no incentives to employ a price squeeze. In Section 3, it is shown that the integrated duopolist, who becomes a Stackelberg leader, has incentives to employ a price squeeze after the determination of equilibrium in one-sided integration is shown. We end up with conclusion in Section 4.

2 Price Squeeze by Upstream Monopolist

Consider first the integration in vertically related markets in which downstream firms D1 and D2 supply a consumer good in the downstream market, and a monopolist supplies inputs in the upstream market. Without these inputs a consumer good can not be produced. The monopoly firm is called U.

The demand for the consumer good is given by

$$P = a - x = a - (x_1 + x_2), \tag{1}$$

where P represents the price of consumer good, and x_i the quantity of output by Di, i=1,2.

To simplify our analysis, assume that α_i units of inputs are translated into a unit of output (or a consumer good) by firm Di. It follows that

$$x_i = \frac{1}{\alpha_i} y_i, \; i = 1, \; 2, \tag{2}$$

where y_i is the quantity of inputs by Di. This is the production function of D1 and D2. To proceed with our analysis, it will be assumed that productivity of D2 is higher than that of D1, and that the difference between them is not very large. Formally, this assumption is expressed as

$$1 < \alpha = \alpha_1/\alpha_2 \leq 2. \tag{3}$$

Assume also that the monopolist (or U) produces inputs at a given constant marginal cost c. Fixed costs are assumed away in what follows. Then, c is also the average cost of producing inputs.

Together with these, it is also assumed that

$$ac_i \;\; = \;\; \alpha_i p, \; i = 1, 2$$

where ac_i stands for the unit cost of the consumer good produced by Di and p for the price of inputs. This means that Di using α_i units of inputs produces one unit of the consumer good at a cost $\alpha_i p$.

To proceed with our analysis, assume further that

$$a > 4\alpha_1 c. \tag{4}$$

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This assumption is crucial to the analysis to follow although it is enough to assume that $a > 3\alpha_1 c$. However, when games of multiple independent firms are considered, it is necessary to assume (4) to guarantee that each firm is viable in an upstream and a downstream market.

When an upstream monopolist U and D1 are integrated, D2 has to buy its essential inputs from its competitor, the integrated firm I.[7] Firm I is a bottleneck input provider and may be called a partial monopolist, which is a monopolist in the upstream market, but it faces a competitor in the downstream market. This is one of the features of vertically related markets. Although U may have incentives to merge with an efficient firm D2, merger with the efficient firm ends up with a pure monopoly and this game is not considered. This conversion of the market into a pure monopoly will be analyzed later. When the merger does not lead the downstream market to a pure monopoly, it is not known what a market will become after the integration. This is why we take up the game in which U merges with less efficient firm. This game is called the partial monopoly game (henceforth, PM game).

In the PM game, assume that transfer price of throughputs is given by the marginal (or average) cost c. Thus, unit costs $\hat{c}$ of producing final goods by the integrated firm I are given by

$$\hat{c} = \alpha_1 c.$$

On the other hand, the independent firm D2 has to purchase inputs at a market price p, which will be higher than the marginal cost c of its rival. Then, firm I can enjoy a cost advantage over the independent D2. Thus, firm I, which merges with D1, may be able to exercise power over price in the market and foreclose D2 from the market. Note also that focus here is mainly on the effects of vertical integration and that motivations for the integration are ignored.

Together with (1) and (2), and denoting by x_I, x_2 the outputs of the partial monopolist I and D2, these profits are given by

$$
\begin{aligned}
\pi_I &= \pi_d + \pi_u = (P - \hat{c})x_I + (p - c)y_2 = (P - \alpha_1 c)x_I + \alpha_2(p - c)x_2 \\
&= (a - \alpha_1 c - (x_I + x_2))x_I + \alpha_2(p - c)x_2, \\
\pi_2 &= (P - \alpha_2 p)x_2 = (a - \alpha_2 p - (x_I + x_2))x_2,
\end{aligned}
$$

where π_d and π_u stand for profits in the downstream and upstream markets by firm I. The first order conditions for the maximization of profits are

$$\frac{\partial \pi_I}{\partial x_I} = \frac{\partial \pi_d}{\partial x_I} = a - \alpha_1 c - 2x_I - x_2 = 0,$$

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[7]A game in which all firms are independent is considered by Vickers (1995). For further details, see also Yang and Kawashima (2011).

$$\frac{\partial \pi_2}{\partial x_2} = a - \alpha_2 p - x_I - 2x_2 = 0.$$

The Nash equilibrium outputs of these firms are expressed as

$$\hat{x}_I = \frac{a - 2c\alpha_1 c + \alpha_2 p}{3}, \tag{5}$$

$$\hat{x}_2 = \frac{a + \alpha_1 c - 2\alpha_2 p}{3}. \tag{6}$$

D2 has to purchase inputs from I and its demand is derived from $\hat{x}_2$. In view of (2), demand for inputs by D2 is

$$\alpha_2 \hat{x}_2 = Y = \frac{\alpha_2(a + \alpha_1 c - 2\alpha_2 p)}{3}$$

Solving this equation for p, we have the inverse demand p for inputs:

$$p = \frac{-3Y + \alpha_2(a + \alpha_1 c)}{2\alpha_2^2}.$$

The upstream division of firm I faces demand for its products derived above. Its profit is

$$\pi_I = \pi_d + \pi_u = (P - \alpha_1 c)\hat{x}_I + (p - c)Y = (P - \alpha_1 c)\hat{x}_I + (\frac{-3Y + \alpha_2(a + \alpha_1 c)}{2\alpha_2^2} - c)Y$$

$$= \frac{-5Y^2 + 4Y(\alpha_1 - \alpha_2)\alpha_2 c + \alpha_2^2(a - \alpha_1 c)^2}{4\alpha_2^2}.$$

It follows that the condition for optimality is

$$\frac{d\pi_I}{dY} = -\frac{5Y}{2\alpha_2^2} + (-1 + \frac{\alpha_1}{\alpha_2})c = 0.$$

The partial monopolist supplies products to D2. It is given by

$$Y^* = \frac{2(\alpha_1 - \alpha_2)\alpha_2 c}{5} > 0,$$

where inequality is due to (3). Price charged for this input by the bottleneck provider is given by substituting Y^* into the inverse demand function of inputs, and is given by

$$p_I^* = \frac{-3Y^* + \alpha_2(a + \alpha_1 c)}{2\alpha_2^2} = \frac{5a - \alpha_1 c + 6\alpha_2 c}{10\alpha_2} > 0, \tag{7}$$

where the sign comes from (4). It would be of some interest to check whether the equilibrium price in the upstream market is higher than the provider's marginal cost c. Subtracting c from p_I^*, we have

$$p_I^* - c = \frac{5a - \alpha_1 c + 6\alpha_2 c}{10\alpha_2} - c = \frac{5a - \alpha_1 c - 4\alpha_2 c}{10\alpha_2} > 0,$$

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which is positive under our assumptions (3) and (4). Thus, the integrated firm can make positive profits by supplying inputs to D2 and has cost advantage over its competitor because the firm can get throughputs at the marginal costs. However, it is not certain that the independent firm can reap positive profits in the market because the firm has disadvantages in costs. It follows from these that D2 may have to exit from the market on account of its cost disadvantage. This may be a prospect that D2 has to face.

Thus, we can now summarize our analysis above as:

Lemma 1. *Equilibrium prices in the upstream and the downstream markets where there is a partial monopolist in vertically related markets are given by*

$$p_I^* = \frac{5a - \alpha_1 c + 6\alpha_2 c}{10\alpha_2}.$$

$$P_I = \frac{5a + c(3\alpha_1 + 2\alpha_2)}{10},$$

where P_I is equilibrium downstream price and is higher than $\alpha_2 p_I^*$. Moreover, outputs of D2 and I are given by

$$x_2^* = \frac{2(\alpha_1 - \alpha_2)c}{5} > 0,$$

$$x_I^* = \frac{2(\alpha_1 - \alpha_2)c}{5} > 0.$$

Proof. Substituting (7) into (5) and (6) yields,

$$x_I^* = \frac{a - 2\alpha_1 c + \alpha_2 p_I^*}{3} = \frac{a - 2\alpha_1 c + \alpha_2 \frac{5a - \alpha_1 c + 6\alpha_2 c}{10\alpha_2}}{3}$$
$$= \frac{5a - 7\alpha_1 c + 2\alpha_2 c}{10} > 0,$$

and

$$x_2^* = \frac{a + \alpha_1 c - 2\alpha_2 p_I^*}{3} = \frac{a + \alpha_1 c - 2\alpha_2 \frac{5a - \alpha_1 c + 6\alpha_2 c}{10\alpha_2}}{3} = \frac{2(\alpha_1 - \alpha_2)c}{5} > 0,$$

where the inequalities above come from (3) and (4). Then, it follows from (1), x_I^* and x_2^* that we have

$$P_I = a - x_I^* - x_2^* = \frac{5a + c(3\alpha_1 + 2\alpha_2)}{10}.$$

Using P_I and p_I^* derived above, we get

$$P_I - \alpha_2 p_I^* = \frac{2c(\alpha_1 - \alpha_2)}{5} > 0,$$

where the sign comes from (3). Firm I has cost advantage over D2, but the independent firm D2 can reap profits to the extent that D2 has more efficient production technology. □

Lemma 1 shows the essential featrures of the partial monopoly game. Thus, a downstream competitor can be viable as long as it has a superior technology: *i.e.* $\alpha_2 < \alpha_1 \le 2\alpha_2$. If this assumption does not hold, the vertical integration results in excluding a rival firm from the market and the downstream market changes into a pure monopoly. Thus, technological superiority enables an independent firm to cope effectively with the partial monopolist even if the monopolist has cost advantages. As mentioned before, this is the reason why it is assumed that the upstream monopolist integrates less efficient firm D1.

Even if the independent firm D2 can make profits, the partial monopolist I has several strategies to drive a competitor out of a market. For example, the partial monopolist I may employ a price squeeze strategy through which I sets the monopoly price in the market. The partial monopolist I can set the price for the input high enough so that the downstream competitors can not compete with it. After the competitor exits from the market, firm I will set higher downstream (or monopoly) prices to maximize its profits. Thus, concern about vertical integration by the upstream monopolist or duopolist is that it exploits leverage in the downstream market to reap the maximum profits and that this eventually aggravates market efficiency.

Then, turn our attention to a game in which the integrated firm changes its role and becomes a leader. For example, the leader practices a price-squeeze strategy and its influences on market outcomes will be examined in what follows. Following Joskow (1985), this is defined as a strategy of the monopolist which charges so a high price for its input to its downstream competitors that the competitors cannot make any profits. Prices in the downstream and the upstream market under a price squeeze are denoted by P_{SQ} and p_{sq} respectively. Formally, assume that

$$P_{SQ} = \alpha_2 \ p_{sq}. \tag{8}$$

Note that individual equilibrium outputs of I and D2 are given by (5) and (6). Total output is the sum of the individual outputs. Therefore, we have

$$\hat{x}_I + \hat{x}_2 = \frac{a - 2\alpha_1 c + \alpha_2 p}{3} + \frac{a + \alpha_1 c - 2\alpha_2 p}{3} = \frac{2a - \alpha_1 c - \alpha_2 p}{3}.$$

Taking this, (1) and (8) into account, the price P_{SQ} set under the price-squeeze strategy is expressed as

$$P_{SQ} = a - \hat{x}_I - \hat{x}_2 = a - \frac{2a - \alpha_1 c - \alpha_2 p_{sq}}{3} = \frac{a + \alpha_1 c + \alpha_2 p_{sq}}{3} = \frac{a + \alpha_1 c + P_{SQ}}{3}.$$

This gives us

$$P_{SQ} = \frac{a + \alpha_1 c}{2}. \tag{9}$$

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Together with (8), the price of input, which is derived from the price-squeeze strategy by firm I, is reduced to

$$p_{sq} = \frac{a + \alpha_1 c}{2\alpha_2}. \qquad (10)$$

Note that if firm I sets input price equal to p_{sq} given above, output of D2 is equal to zero. In fact, substituting (10) into (6) reveals that $\hat{x}_2$ indeed equals zero.

By definition, under a price squeeze, outputs of D2 are equal to zero. This in turn means that the integrated firm I can be a monopolist. Thus, price under this strategy is equal to a monopoly price P_M. Then, we have

$$P_M = P_{SQ} = \frac{a + \alpha_1 c}{2}. \qquad (11)$$

Monopoly output x_m is given by

$$x_m = a - \frac{a + \alpha_1 c}{2} = \frac{a - \alpha_1 c}{2} > 0,$$

where the inequality is due to (4).

These results are summarized in

Proposition 1. *Price under a price-squeeze strategy in PM game is the same as monopoly price. Moreover, the price under a price-squeeze strategy is higher than equilibrium price in the vertical integration game. Thus*

$$P_I < P_{SQ} = P_M.$$

Proof. Note that the price set under the price-squeeze strategy is given in (9), while the monopolist charges price equal to (11). These are the same. It is easy to see that

$$P_{SQ} - P_I = \frac{a + \alpha_1 c}{2} - \frac{5a + (3\alpha_1 + 2\alpha_2)c}{10} = \frac{(\alpha_1 - \alpha_2)c}{5} > 0.$$

where the inequality comes from (3). □

It is interesting to note that monopoly price equals price set under a price squeeze. Profits earned under this strategy are shown to be equal to those under monopoly because the monopoly is a market in which the monopolist has the maximum control over the market and hence it can reap the highest profits. It follows from *Proposition* 1 that a price squeeze is anti-competitive in its nature and then market efficiency is shown to be aggravated. Moreover, our *Proposition* 1 suggests that a price squeeze by the partial monopolist I in these markets achieves the maximum

profits. In fact, profits earned in monopoly and under a price squeeze in the partial monopoly games are equal.

However, when we consider vertically related markets, results may differ because firm I has two sources of profits. Monopolization by some strategy enables the firm I to achieve the maximum profits in the downstream market, but it causes a reduction in profits in the upstream market. Then, the monopolization has positive and negative effects on profits simultaneously. These considerations suggest that a price squeeze may not be optimal for firm I. This possibility is conjectured by Sappington (2006), who suggests that profits lost in the upstream market, which are due to exclusion of the downstream rival, will not be compensated by additional profits earned in the downstream market.

It will be interesting to examine if firm I has incentives to control integrated market structure. If it has, the market structure will turn into a pure monopoly under some strategies and will then aggravate economic efficiency. Deregulation, which permits the firm I to employ foreclosing strategies, will then result in the formation of a pure monopoly and hence the partial monopoly game will change into an inefficient market structure. Then, deregulation should be banned.

Proposition 2. *The integrated monopolist would prefer to keep the market structure unchanged in the partial monopoly game because it is not optimal for the monopolist to engage in a price squeeze in the partial monopoly game.*

Proof. The integrated monopolist I can make profits not only in the downstream market, but also in the upstream market. If the monopolist I succeeds in driving competitors out of the downstream market, the resulting increase in profit earned in that market using perfect control may be greater or smaller than the profit lost in the upstream market. It is not clear which is larger. Therefore, we should make a precise comparison of profits earned under a monopolistic price squeeze with that earned by the partial monopolist.

If the monopolist I can perfectly control the downstream market, its profit is

$$\pi_m = (P_M - \alpha_1 c)x_m = (\frac{a + \alpha_1 c}{2} - \alpha_1 c)\left(\frac{a - \alpha_1 c}{2}\right) = \left(\frac{a - \alpha_1 c}{2}\right)^2.$$

On the other hand, the monopolist I in the integration game can make a profit of π_I, which consists of profits π_d in the downstream market and π_u in the upstream market. In view of *Lemma* 1 and its proof, the profit in the downstream market is

$$\pi_d = (P_I - \alpha_1 c)x_I^* = \left(\frac{5a + c(3\alpha_1 + 2\alpha_2)}{10} - \alpha_1 c\right)x_I^* = \frac{(5a - 7\alpha_1 c + 2\alpha_2 c)^2}{100}.$$

Similarly, the profit earned in the upstream market is

$$\pi_u = (p_I^* - c)\alpha_2 x_2^* = \frac{c(\alpha_1 - \alpha_2)(5a - (\alpha_1 + 4\alpha_2)c)}{25}.$$

Total profits of I are given by

$$
\begin{aligned}
\pi_I &= \pi_d + \pi_u = \frac{(5a - 7\alpha_1 c + 2\alpha_2 c)^2}{100} + \frac{c(\alpha_1 - \alpha_2)(5a - (\alpha_1 + 4\alpha_2)c)}{25} \\
&= \frac{5a^2 - 10a\alpha_1 c + (9\alpha_1^2 - 8\alpha_1\alpha_2 + 4\alpha_2^2)c^2}{20}.
\end{aligned}
$$

Therefore, the difference between profit π_m earned by a pure monopoly and profit π_I earned by the partial monopolist I is

$$\pi_m - \pi_I = \left(\frac{a - \alpha_1 c}{2}\right)^2 - \frac{5a^2 - 10a\alpha_1 c + (9\alpha_1^2 - 8\alpha_1\alpha_2 + 4\alpha_2^2)c^2}{20} = -\frac{(\alpha_1 - \alpha_2)^2 c^2}{5} < 0.$$

As monopoly profit is lower than the profit earned by I, a price-squeeze strategy results in lower profits for the partial monopolist. This means that firm I can make higher profits if it does not change this integrated market structure. □

Although our results differ from those obtained by Joskow (1985), they are quite consisitent with Sappington (2006). In fact, it has been shown here that firm I has no incentives to monopolize the market. The fear that the partial monopolist will try to monopolize a market and raise market price is a misconception under our assumptions. On the contrary, integrated market structures that include a partial monopoly lead to larger welfare relative to a pure monopoly and hence the formation of a partial monopoly should not be prevented by regulation.[8] Thus, this result shows that the presence of competitors is crucial for achieving market efficiency even if a firm has partial controlling power over markets. These observations are important features when we consider integrated market structures.

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[8]Yang and Kawashima (2011) has shown that equilibrium market prices are lower in the partial monopoly game than in a pre-integration game.

3 Price Squeeze by Upstream Duopolist

Next, consider integration in a double duopoly game (or DD game), where there are two independent firms in both markets. Upstream firms are called U1 and U2 whose marginal (and average) costs are fixed constants c_1 and c_2. Without loss of generality, assume that $c_1 \leq c_2$.

To proceed with our analysis, assume that

$$a > 4\alpha_1 c_2, \tag{12}$$

which takes the place of the previous assumption (4).

There are concerns that the integration causes the aggravation of market efficiency. In particular, when efficient firms are integrated, serious concerns will emerge. Then, a game that we take up is a game in which efficient firm D2 integrates with upstream firm U1, and the downstream division of the integrated firm F is superior in technologies to the downstream rival D1. For simplicity, call this game a one-sided integration game (or OI game).

The market structure of this model is depicted in Figure 1. It follows that F can get access to inputs at lower costs than D1. In other words, F is superior in costs and productivity. The integrated firm has stronger market power than the other pair of firms and there will be concerns that it will employ a strategy such as a price squeeze to reap more profits.

In order to analyze effects of the vertical integration, it is necessary to show the features of the OI game. F may be able to make profits by supplying outputs produced by using inputs not only from the upstream division, but also from an upstream market. However, assume that F does not purchase inputs from the upstream market because prices of inputs are higher than the marginal costs of inputs. Thus, profits of F and D1 are expressed as

$$
\begin{aligned}
\pi_F &= (P - \alpha_2 c_1)x_F + (p - c_1)y_1 = (a - \alpha_2 c_1 - x_F - x_1)x_F + (p - c_1)y_1, \\
\pi_1 &= (P - \alpha_1 p)x_1 = (a - \alpha_1 p - x_F - x_1)x_1,
\end{aligned}
$$

where x_F stands for output of F produced by through-puts and p for input price.

The first order conditions for maximum profits are

$$
\begin{aligned}
\frac{\partial \pi_F}{\partial x_F} &= a - \alpha_2 c_1 - 2x_F - x_1 = 0, \\
\frac{\partial \pi_1}{\partial x_1} &= a - \alpha_1 p - x_F - 2x_1 = 0.
\end{aligned}
$$

Solving these equations for x_F and x_1, we have

$$
\begin{aligned}
x_F^* &= \frac{a + \alpha_1 p - 2\alpha_2 c_1}{3}, \\
x_1^* &= \frac{a - 2\alpha_1 p + \alpha_2 c_1}{3}.
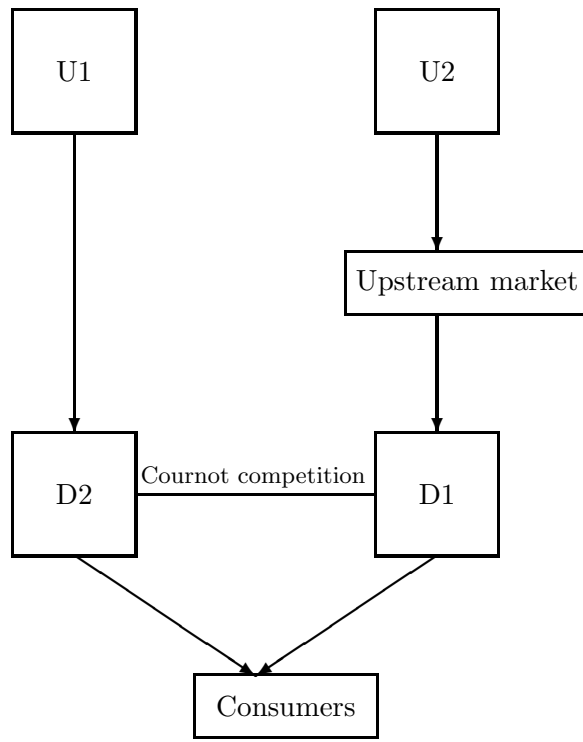\end{aligned}
$$

Figure 1: One-Sided Integration Game

As noted above, derived demand d for inputs by D1 is given by

$$d = \alpha_1 x_1^* = \alpha_1 \frac{a - 2\alpha_1 p + \alpha_2 c_1}{3}.$$

Solving this for p yields inverse demand for the inputs, and is given by

$$p = \frac{(a + \alpha_2 c_1)\alpha_1 - 3d}{2\alpha_1^2}. \tag{13}$$

Given this demand for the inputs, the upstream division of F and U2 supply the inputs to the downstream firm, and their profits are

$$
\begin{aligned}
\pi_F &= (a - \alpha_2 c_1 - x_F^* - x_1^*)x_F^* + (p - c_1)y_1 = (a - \alpha_2 c_1 - x_F^* - x_2^*)x_F^* + \left(\frac{(a + \alpha_2 c_1)\alpha_1 - 3d}{2\alpha_1^2} - c\right)y_1 \\
&= (a - \alpha_2 c_1 - x_F^* - x_1^*)x_F^* + \left(\frac{(a + \alpha_2 c_1)\alpha_1 - 3(y_1 + y_2)}{2\alpha_1^2} - c_1\right)y_1, \\
\pi_{U2} &= (p - c_2)y_2 = \left(\frac{(a + \alpha_2 c_1)\alpha_1 - 3d}{2\alpha_1^2} - c_2\right)y_2 = \left(\frac{(a + \alpha_2 c_1)\alpha_1 - 3(y_1 + y_2)}{2\alpha_1^2} - c_2\right)y_2, \\
d &= y_1 + y_2.
\end{aligned}
$$

Substituting x_F^*, x_2^*, and p derived above into π_F and π_{U2}, the first order condition for maximum profit yields

$$\frac{\partial \pi_F}{\partial y_1} = -\frac{5y_1 + 2(y_2 + \alpha_1(\alpha_1 - \alpha_2)c_1)}{2\alpha_1^2} \leq 0, \ for \ y_1, \ y_2 \geq 0,$$

where the inequality is due to (3). It follows from the Kuhn-Tucker conditions that optimal supply $\hat{y}_1$ of inputs of F is equal to 0. Thus, the integrated firm produces throughputs and does not supply inputs to the upstream market.

It follows from these arguments that U2 is the sole supplier of inputs, which are demanded by D1. U2 maximizes its profits given the derived demand for inputs. Then, we have

$$\frac{\partial \pi_{U2}}{\partial y_2} = \frac{-6y_2 + \alpha_1(a - 2\alpha_1 c_2 + \alpha_2 c_1)}{2\alpha_1^2} = 0,$$

where $(a + \alpha_2 c_1 - 2\alpha_1 c_2)$ is positive because of (12). Solving this equation for y_2, we have

$$\hat{y}_2 = \frac{\alpha_1(a - 2\alpha_1 c_2 + \alpha_2 c_1)}{6} > 0,$$

where the inequality is again due to (12). Noting that $d = y_2$ and substituting $\hat{y}_2$ into (13), equilibrium price $\hat{p}$ of inputs is determined and then substituting this $\hat{p}$ into x_F^*, x_1^* yields equilibrium outputs $\hat{x}_F, \hat{x}_1$ of the two firms.

Now we can summarize our arguments above as:

Lemma 2. *If the more productive firm D2 in the downstream market merges with the more efficient upstream firm U1, the one-sided vertically merged firm F can maximize profits by not supplying inputs to the upstream market. Equilibrium outputs, prices and inputs of firms in the post-merger game are given by*

$$
\begin{aligned}
\hat{y}_1 &= 0, \\
\hat{y}_2 &= \frac{\alpha_1(a - 2\alpha_1 c_2 + \alpha_2 c_1)}{6}, \\
\hat{p} &= \frac{a + 2\alpha_1 c_2 + \alpha_2 c_1}{4\alpha_1}, \\
\hat{x}_F &= \frac{5a + 2\alpha_1 c_2 - 7\alpha_2 c_1}{12}, \\
\hat{x}_1 &= \frac{a - 2\alpha_1 c_2 + \alpha_2 c_1}{6}, \\
\hat{P} &= \frac{5a + 2\alpha_1 c_2 + 5\alpha_2 c_1}{12}.
\end{aligned}
$$

Proof. In the post-merger game, U2 maximizes profits by supplying products for the given demand (13) for inputs. Note also that $d = y_2$ because the upstream division of F does not supply inputs to the upstream market. Then, profits of U2 are given by

$$\pi_{U2} = (p - c_2)y_2 = \left(\frac{\alpha_1(a + \alpha_2 c_1) - 3d}{2\alpha_1^2} - c_2\right)y_2,$$

Solving the first order condition for y_2 yields

$$\hat{y}_2 = \frac{\alpha_1(a - 2\alpha_1 c_2 + \alpha_2 c_1)}{2\alpha_1^2}.$$

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Substituting this into (13), we have

$$\hat{p} = \frac{a + 2\alpha_1 c_2 + \alpha_2 c_1}{4\alpha_1}.$$

It is easy to check that $\hat{p} > c_2$ under (12). In fact,

$$\hat{p} - c_2 \;=\; \frac{a + 2\alpha_1 c_2 + \alpha_2 c}{4\alpha_1} - c_2 = \frac{a - 2\alpha_1 c_2 + \alpha_2 c_1}{4\alpha_1} > 0.$$

As $\hat{y}_2$ is positive, the profits of U2 are positive and U2 can supply inputs to the upstream market. Substituting $\hat{p}$ into eqs. (5) and (6) yields

$$\hat{x}_F \;=\; \frac{a + \alpha_1 \hat{p} - 2\alpha_2 c_1}{3} = \frac{5a + 2\alpha_1 c_2 - 7\alpha_2 c_1}{12} > 0,$$

$$\hat{x}_1 \;=\; \frac{a - 2\alpha_1 \hat{p} + \alpha_2 c_1}{3} = \frac{a - 2\alpha_1 c_2 + \alpha_2 c_1}{6} > 0.$$

Finally, together with (1), equilibrium price in the downstream market is

$$\hat{P} = a - \hat{x}_F - \hat{x}_2 = \frac{5a + 2\alpha_1 c_2 + 5\alpha_2 c_1}{12}.$$

It is easy to show that $\hat{P}$ is larger than $\alpha_1 \hat{p}$. In fact, per-units profits $(\hat{P} - \alpha_1 \hat{p})$ are

$$\hat{P} - \alpha_1 \hat{p} = \frac{5a + 2\alpha_1 c_2 + 5\alpha_2 c_1}{12} - \alpha_1 \frac{a + 2\alpha_1 c_2 + \alpha_2 c_1}{4\alpha_1} = \frac{a - 2\alpha_1 c_2 + \alpha_2 c_1}{6} > 0,$$

where the sign comes from (12). This means that firm F can reap positive profits and sell outputs in the downstream market because equilibrium output $\hat{x}_1$ of firm 1 is positive. $\qquad\square$

Now, consider whether F has incentive to engage in a price squeeze. To do this, we compare profits π_F of F in the OI game with profits π_{sq} under a price squeeze. In view of Lemma 2, profits π_F^* in the OI game are given by

$$\pi_F^* = (\hat{P} - \alpha_2 c_1)\hat{x}_F = \left(\frac{5a + 2\alpha_1 c_2 + 5\alpha_2 c_1}{12} - \alpha_2 c_1\right)\frac{5a + 2\alpha_1 c_2 - 7\alpha_2 c_1}{12} = \frac{1}{144}(5a + 2\alpha_1 c_2 - 7\alpha_2 c_1)^2.$$

As F participates in the downstream market, F can manipulate downstream price to exclude the downstream rival firm from that market. It follows from Lemma 2 that F can force profits of the rival equal to zero by setting the downstream price P_{sq} equal to $\alpha_1 \hat{p}$, which can be expressed as

$$P_{sq} = \alpha_1 \hat{p} = \alpha_1 \left(\frac{a + 2\alpha_1 c_2 + \alpha_2 c_1}{4\alpha_1}\right) = \frac{a + 2\alpha_1 c_2 + \alpha_2 c_1}{4}.$$

This is a downstream price under a price squeeze by F. It then follows from this and (1) that output x_{sq} of F under this strategy is

$$x_{sq} = a - P_{sq} = a - \frac{a + 2\alpha_1 c_2 + \alpha_2 c_1}{4} = \frac{1}{4}(3a - 2\alpha_1 c_2 - \alpha_2 c_1),$$

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where it should be noted that $x_2 = 0$. Profits under such a price squeeze can be described by

$$
\begin{aligned}
\pi_{sq} &= (P_{sq} - \alpha_2 c_1) x_{sq} = (\frac{a + 2\alpha_1 c_2 + \alpha_2 c_1}{4} - \alpha_2 c_1) \frac{1}{4}(3a - 2\alpha_1 c_2 - \alpha_2 c_1) \\
&= \frac{1}{16}(a + 2\alpha_1 c_2 - 3\alpha_2 c_1)(3a - 2\alpha_1 c_2 - \alpha_2 c_1).
\end{aligned}
$$

We can summarize our results as

Lemma 3. *Profits π_{sq} under a price squeeze by the integrated duopolist F in OI game are given by*

$$
\pi_{sq} = \frac{1}{16}(a + 2\alpha_1 c_2 - 3\alpha_2 c_1)(3a - 2\alpha_1 c_2 - \alpha_2 c_1).
$$

In view of *Lemma* 2 and *Lemma* 3, it will be possible to show whether the integrated firm F has incentives to monopolize a market by a price squeeze. It has been shown that the vertically integrated firm has no incentives to exclude a rival firm out of the market when an upstream market is a monopoly. When we consider the OI game, it is not certain whether the same results will be obtained. One of the upstream duopolists may be interested in driving the rival out of the market because the duopolist can be a monopolist in the downstream market, which is expected to provide the duopolist with the maximum profits. We can be more precise in:

Proposition 3. *The integrated duopolist in a Double Duopoly game has incentives to employ a price squeeze to monopolize the market. Moreover, although a price squeeze is anti-competitive in its nature, such a strategy promotes market efficiency because it causes the downstream price to go down.*

Proof. Profits π_F^* of F in the OI game are given by $\frac{1}{144}(5a + 2\alpha_1 c_2 - 7\alpha_2 c_1)^2$. The difference between π_{sq} and π_F^* is

$$
\begin{aligned}
\pi_{sq} - \pi_F^* &= \frac{1}{16}(a + 2\alpha_1 c_2 - 3\alpha_2 c_1)(3a - 2\alpha_1 c_2 - \alpha_2 c_1) - \frac{1}{144}(5a + 2\alpha_1 c_2 - 7\alpha_2 c_1)^2 \\
&= \frac{1}{72}(a^2 + 8a\alpha_1 c_2 - 20\alpha_1^2 c_2^2 - 10a\alpha_2 c_1 + 32\alpha_2^2 c_1^2 - 11\alpha_2^2 c_1^2) \\
&= \frac{1}{72}(a^2 - 10a\alpha_2 c_1 + 25\alpha_2^2 c_1^2 + 8a\alpha_1 c_2 - 20\alpha_1^2 c_2^2 - 4\alpha_2^2 c_1^2).
\end{aligned}
$$

In what follows, consider whether the quadratic form above is positive. It is rewritten as

$$
\begin{aligned}
& (a^2 - 10a\alpha_2 c_1 + 25\alpha_2^2 c_1^2 + 8a\alpha_1 c_2 - 20\alpha_1^2 c_2^2 - 4\alpha_2^2 c_1^2) \\
&> ((a - 5\alpha_2 c_1)^2 + 8 \times (4\alpha_1 c_2)\alpha_1 c_2 - 20\alpha_1^2 c_2^2 - 4\alpha_2^2 c_1^2) \\
&= ((a - 5\alpha_2 c_1)^2 + 32\alpha_1^2 c_2^2 - 20\alpha_1^2 c_2^2 - 4\alpha_2^2 c_1^2) \\
&= ((a - 5\alpha_2 c_1)^2 + 12\alpha_1^2 c_2^2 - 4\alpha_2^2 c_1^2) > 0,
\end{aligned}
$$

where the first inequality comes from (12), and the second from (3) and $c_2 \geq c_1$. Then, profits under a price squeeze are larger than those in the OI game. It then follows that the integrated duopolist F has incentives to employ a price squeeze.

Next, it follows from *Lemma* 2 and the discussion of price squeeze that the difference between $\hat{P}$ and P_{sq} is given by

$$\hat{P} - P_{sq} = \frac{5a + 2\alpha_1 c_2 + 5\alpha_2 c_1}{12} - \frac{a + 2\alpha_1 c_2 + \alpha_2 c_1}{4} = \frac{a - 2\alpha_1 c_2 + \alpha_2 c_1}{6} > 0.$$

$\square$

These results are in a stark contrast with those in the PM game where the upstream monopolist integrates a downstream firm. The partial monopolist does not have incentives to monopolize a market. This is because additional profits earned in the downstream market are not compensated by profits lost in the upstream market due to the exit of the downstream rival. This in turn means that the monopolization of a market does not provide the firm with the maximum profits. This is the interesting feature of vertically related markets. On the other hand, it follows from *Proposition* 2 that the integrated duopolist can make more profits by excluding a rival firm from a market. Thus, strategies such as a price squeeze which enable the integrated firm to monopolize a market are adopted depending upon upstream market structure. Thus, the upstream market structure matters.

However, it is interesting to note that such a strategy by the integrated duopolist brings about a lower market price and hence promotes market efficiency. Even if the integration reduces the number of downstream firms, it enhances market welfare. Thus, this strategy is anti-competitive in the sense that a competitor is driven out of a market, but still promotes market efficiency.

The crucial assumption of the present model is that $a > 4\alpha_1 c_2$. In fact it is enough to assume that $a > 3\alpha_1 c_2$. This latter assumption will enable us to derive *Proposition* 3. Thus, the assumption $a > 4\alpha_1 c_2$ is too strong in the games which we have examined. However, when other games such as games in which all firms are independent are taken up this assumption will guarantee that all firms are viable. This is the reason why we have made the more restrictive assumption. The second question to consider is, what happens when inefficient firms are merged? It will be shown that post-merger market prices go down. However, it is not certain that the integrated firm has incentives to engage in a price squeeze. One of our results in Proposition 3 depends upon a combination of an upstream and a downstream firm.

4   Conclusions

In this paper, we have examined a price-squeeze strategy by the integrated firm in different settings and shown that the partial monopolist sets a price equal to the monopoly price under this strategy. Thus, downstream price under such a strategy is higher than the price set in a pre-merger game. The model has shown that the integrated monopolist does not have incentives to monopolize the downstream market. Thus, it is not optimal for the partial monopolist to exclude a rival firm from the market. In fact, profits lost in the upstream market due to the exclusion of a rival firm are not compensated by additional profits earned in the downstream market.

When the upstream market structure is a duopoly, the integrated duopolist has incentives to drive a rival firm from the market by engaging in a price squeeze. However, this strategy causes a decrease in market prices, which in turn advances welfare in the market. Although a price squeeze is anti-competitive in its nature, a downstream market becomes more efficient in the sense that market price becomes lower. Situations of "keiretsu" in Japanese industries emerge in the one-sided integration game. These results are in a stark contrast with those in the upstream monopoly game. Thus, upstream market structure matters when we consider the effects of exclusionary strategies of the integrated firms in vertically related markets.

There are, however, several limitations to our model. Most importantly, it has been assumed that the demand and production functions are linear. These enable us to simplify our analysis and derive interesting and explicit results. When we try to broaden our model to include non-linearity of important functions, different results will be expected with possible loss of explicit results. For example, if the production functions of downstream firms are concave, high production by a single firm results in decreasing average costs. Increased production of outputs causes the integrated firm to reap more profits. Thus, it is possible that an integrated monopolist will employ a price squeeze in such circumstances. Similarly, when we assume a non-linear demand function, different results will be obtained. As these problems are of considerable interest, their analysis will form the basis of our future research.

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