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Going Beyond the Traditional Possibility Correspondence Model*
—The Empty Information Set in Recognition Correspondence Model —

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Going Beyond the Traditional Possibility Correspondence Model*

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Abstract

This note introduces a new model, termed recognition correspondence, which accounts for situations where agents fail to recognize signals, affecting their decision-making. In contrast with traditional possibility correspondence, this model addresses both technical and cognitive limitations that lead to empty information sets. These sets, interpreted as the absence of sensory signals, prevent agents from recalling or updating knowledge. Market behavior during the subprime mortgage crisis and asymptomatic COVID-19 infection illustrates the model. We also redefine the knowledge operator within this framework and explore its implications for non-trivial unawareness. The model demonstrates that, even when agents receive no signals, they default to habitual behavior rather than conscious decision-making.

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Keywords: Possibility Correspondence, Recognition Correspondence, Necessitation, Monotonicity, Unawareness.

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1. Introduction

When the subprime mortgage crisis unfolded in 2007, barely few recognized it as a precursor to the 2008 Lehman Brothers collapse. The Lehman Shock had a profound impact not only on investors and American workers but also on labor markets globally. However, the information regarding the subprime mortgage crisis, which was one of the triggers for this event, was likely not utilized by many of those deemed “irrational” in their 2008 decision-making processes. [1] How, then, can we mathematically model the recognition—or lack thereof—of the subprime mortgage crisis by these “irrational” individuals? In this note, we propose to represent such recognition not through the standard *possibility correspondence* used in economics and game theory. Instead, we introduce a new framework in this note termed *recognition correspondence*. We also formulate a knowledge operator based on the recognition correspondence model.

In a state-space model, information sets function as a form of signals that trigger an agent's knowledge or beliefs. For example, signals such as “fever” or “abdominal pain” evoke knowledge related to the common cold or food poisoning, while “fluctuations in stock prices” serve as signals that influence the formation of expectations about the future market. Now, how should we interpret an information set that is empty? Mathematically, an empty information set can be represented by relaxing the Seriality assumption in a possibility correspondence. [2] However, two interpretations of an empty

[1] See Mishkin (2011) and Wiggins, Piontek, and Metrick (2019).

[2] Seriality is a key property to consider when analyzing the possibility

information set can be considered.

The first interpretation is that the agent is unable to access information owing to technical constraints. For instance, a person trapped in an elevator during an earthquake might wish to know what is happening outside but is unable to obtain any information. In this case, even though information may exist, the technical constraints hinder access to it.

The second interpretation is that the agent is unaware of the existence of relevant information owing to cognitive factors. For people uninterested in investment or economics, stock market trends might be of little importance, and they may not seek out such information at all. Similarly, people who do not take initiative to monitor their health regularly. may not think about their health status unless a signal, such as a fever, arises.

In a standard possibility correspondence, the agent narrows down which state is true based on the information set received. The agent considers each state within the information set to be possible based on the obtained information. When the possibility correspondence does not satisfy Seriality — that is, when an information set is empty — the agent is unable to narrow down any potential state. In such a case, the agent is interpreted as considering 'no state to be possible.' In other words, an empty information

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correspondence model. Previous studies (e.g., Chellas 1980; Bonanno 2015, 2018, 2020) highlight the importance of Seriality. In broad terms, when seriality holds, all information sets that the agent can receive are non-empty. This implies that the agent is able to narrow down the set of possible true states, regardless of whether the information is actually accurate.

set is interpreted as “failing to obtain any information about which state is true.” More precisely, it indicates the inability to acquire information that narrows down the set of possible true states within the given state space. This interpretation implicitly assumes that the agent is aware of the situation or state space they are in. This interpretation corresponds to the first case described previously, where technical constraints prevent access to information about the external situation for an individual trapped in an elevator during an earthquake; this individual can still consider the possibility that they are experiencing the effects of an earthquake.

Conversely, modeling the second case based on cognitive constraints is difficult using a possibility correspondence. This is because, in this interpretation, the agent neither recognizes nor contemplates the situation or state space they face.

Thus, to address this difficulty of the traditional possibility correspondence model, this paper proposes recognition correspondence— a new correspondence to capture the second case. Recognition correspondence is interpreted as *signals to the senses*. For instance, signals such as fever or abdominal pain engage the body, whereas fluctuations in stock prices, viewed on television, engage the visual senses. This correspondence can also represent the absence of such sensory signals as an empty information set. In other words, an empty information set is interpreted as the absence of “signals that engage the senses.” In the absence of signals received through the senses, the agent fails to recognize anything. [3] For example, an

[3] As far as the author's literature review has shown, no interpretations

asymptomatic COVID-19 carrier may be unaware that they are infected with COVID-19, and consequently not pay attention to their health status owing to the absence of a fever signal. Thus, a recognition correspondence with a relaxed Seriality assumption can effectively capture the cognitive state of such an agent.

The remainder of this note is structured as follows. In Section 2, a concrete example of the subprime mortgage crisis is presented. Section 3 discusses the knowledge operator in the context of the standard possibility correspondence model. Section 4 formalizes the recognition correspondence model and redefines the knowledge operator based on it. In Section 5, we demonstrate the analysis of decision-making within the recognition correspondence model and suggest that decision-makers' behavior, particularly in the context of an empty information set, may be based on conventions. This note is closely related to research on unawareness. Therefore, Section 6 examines non-trivial unawareness within both the possibility correspondence and recognition correspondence models. Finally, the conclusions and discussions are presented in Section 7.

2. Example

Given the state space Ω and an agent's possibility correspondence $P: \Omega \rightarrow 2^{\Omega}$, for any $\omega \in \Omega$, $P(\omega)$ represents the set of possible states the agent considers. If we assume Seriality for P, meaning that for every state ω,

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similar to the recognition correspondence were identified.

$P(\omega) \neq \emptyset$, the agent is able to recognize the possibility that the true state might be within the received information set. However, when Seriality is not assumed, i.e., if we assume $P(\omega) = \emptyset$ for some ω, the agent is interpreted not only as being unable to consider that ω could be the true state but also as recognizing that no state is possible.

Let us consider an example involving two individuals, Alice and Bob. Alice is a cautious trader who closely monitors movements in the financial markets and the real economy. On detecting a signal from the market, she also considers how that signal might influence her own decision-making. By contrast, Bob is a university student with an interest in Japanese Buddhism. He is studying in Japan, and has entered as a monk-in-training in a temple, where he is not allowed access to the Internet. The temple receives Japanese newspapers; however, there is no availability of English-language newspapers. As Bob is not proficient in Japanese, he does not read these newspapers. Consequently, he receives no information about occurrences in the U.S. markets.

Now, consider a state space $\Omega = \{\omega_1, \omega_2, \omega_3\}$, where ω_1 represents a state where the subprime mortgage crisis has occurred and a global financial crisis will follow the next year; ω_2 represents a state where the subprime mortgage crisis has occurred, but its impact on the financial markets next year will be minimal; and ω_3 represents a state where the subprime mortgage crisis has not occurred, and the financial markets will remain stable next year. Let $E_1 = \{\omega_1, \omega_2\}$ denote a state where the subprime mortgage crisis has occurred, and let $E_2 = \{\omega_3\}$ denote that the subprime mortgage crisis has not occurred. Under a standard possibility correspondence, Alice's possibility correspondence can be described as $P_A(\omega_1) = P_A(\omega_2) = \{\omega_1, \omega_2\}$ and $P_A(\omega_3) = \{\omega_3\}$. Thus, her knowledge (and beliefs) is consistent.

Next, let us consider Bob's information set. Since he is at the temple and receives no information, his information set can be considered empty. However, if we use the standard knowledge operator $K_B^*: 2^\Omega \to 2^\Omega$, an interesting situation arises. If we describe Bob's possibility correspondence as $P_B(\omega_1) = P_B(\omega_2) = P_B(\omega_3) = \emptyset$, then based on the knowledge operator $K_B^*(E) = \{\mathrm{\omega} \in \Omega | \mathrm{P}(\mathrm{\omega}) \subseteq \mathrm{E}\}$, for all $\mathrm{\omega} \in \Omega$ and all $E \subseteq \Omega$, it follows that $\omega \in K_B^*(E)$. In other words, Bob knows every possible event in every state. He knows that the subprime mortgage crisis has occurred, that its impact is minimal, and that the financial markets are stable—all at once. This is clearly inconsistent. [4]

To avoid this issue, we propose a recognition correspondence $R_B: \Omega \to 2^\Omega$. Under this correspondence, we still have $R_B(\omega_1) = R_B(\omega_2) = R_B(\omega_3) = \emptyset$. However, recognition correspondence is interpreted as signals that are detected by the senses, and if no signals are received, no knowledge is recalled. In this case, if we define the knowledge operator as $K_B(E) = \{\omega \in \Omega | R_B(\omega) \subseteq E \wedge R(\omega) \neq \emptyset\}$, then for all ω and E, we have $\omega \in \Omega \setminus K(E)$. In other words, Bob has no knowledge in any state.

[4] This inconsistency arises because Bob's possibility correspondence P_B does not satisfy the assumption of Seriality.

3. Standard Possibility Correspondence Models

First, we define the standard possibility correspondence model $\langle \Omega, P \rangle$ and (standard) knowledge operator. We define Ω as the state space and $P: \Omega \to 2^{\Omega}$ as the agent's (standard) possibility correspondence. We do not assume that P satisfies Seriality, that is, $P(\omega) = \emptyset$ for some $\omega \in \Omega$. Here, the standard knowledge operator $K^*: 2^{\Omega} \to 2^{\Omega}$ is defined as follows: given any event $E \subseteq \Omega$,

$$\begin{cases} \omega \in K^*(E) \text{ if } P(\omega) \subseteq E; \text{ and} \\ \omega \notin K^*(E) \text{ otherwise.} \end{cases}$$

Considering that P might not satisfy Seriality, the agent's knowledge (or belief) might be inconsistent. Let us consider Bob's example, wherein $P_B(\omega_1) = P_B(\omega_2) = P_B(\omega_3) = \emptyset$. Then, $K_B^*(E_1) = K_B^*(E_2) = K_B^*(\Omega) = \Omega$; that is, Bob knows (and believes) that the sub-prime mortgage crisis do and do not happen![5] [6]

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5 The author thanks Bart Lipman, Spyros Galanis, and Satoshi Fukuda for pointing out the correct interpretation of empty information sets based on information functions or possibility correspondences. I am also grateful to Satoshi Fukuda for introducing me to the literature on Seriality.

6 Previous studies call the operator a (qualitative) belief, and not knowledge, if a possibility correspondence does not satisfy Reflexivity that $\omega \in P(\omega)$ for any $\omega \in \Omega$. This note does not follow such a convention. By contrast, this study refers to it as knowledge because we want to discuss unawareness as second-order ignorance based on Modica and Rustichini (1994).

As stated in Dekel, Lipman, and Rustichini (1998), this lemma follows from the definition of a knowledge operator.

Lemma 1: In a standard possibility correspondence model $\langle \Omega, P \rangle$, the knowledge operator satisfies the following properties:

- Necessitation $K^*(\Omega) = \Omega$, and
- Monotonicity $E \subseteq F \implies K^*(E) \subseteq K^*(F)$.

The proof to Lemma 1 shows that whether or not the information set is non-empty does not directly affect the establishment of Necessitation and Monotonicity.

4. Recognition Correspondence Models

Let us consider a pair $\langle \Omega, R \rangle$, where Ω is the state space and $R: \Omega \to 2^{\Omega}$ is the *recognition correspondence*, and let us call the pair a recognition correspondence model. For any $\omega \in \Omega$, $R(\omega)$ is interpreted as "an agent who recognizes or receives some information set that engages the five senses that recalls the agent's knowledge and beliefs at ω." In this case, the information set is a signal to the five senses, and the agent is expected to recall relevant knowledge according to the received information set.

Example 1. Consider a person who is asymptomatically infected with SARS-CoV-2. Over the last few years, studies have shown that people infected with

SARS-CoV-2 are not necessarily symptomatic. It has been found that asymptomatically infected people may be objectively infected with SARS-CoV-2 without being aware that they are infected with it. Let us assume that a person is actually infected with SARS-CoV-2. If they feel feverish, we might suspect a COVID-19 infection. However, what if they do not experience any symptoms at all? The homo economicus who is a fully rational individual might, at any time, consider the possibility of asymptomatic SARS-CoV-2 infection; however, most people would inadvertently "forget" that SARS-CoV-2 can cause asymptomatic infection. This can be rephrased as follows: Most people can recall their knowledge of COVID-19 if they feel feverish, but if they do not feel feverish, they may not recall their knowledge of COVID-19. Therefore, even if one has knowledge of COVID-19, if they are unaware that they are infected with SARS-CoV-2, it is as if they do not possess knowledge of COVID-19.

Let us mathematically formulate the aforementioned case of SARS-CoV-2; this concept has been borrowed from Tada (2024). Let us consider a SARS-CoV-2 infection-carrier and agent, Claire. In this state space, $\Omega = (\omega_1, \omega_2, \omega_3, \omega_4)$, ω_1 is interpreted as "Claire is infected with SARS-CoV-2 and gets a fever," ω_2 is interpreted as "Claire gets a fever, but is not infected with SARS-CoV-2," ω_3 is interpreted as "Claire is infected with SARS-CoV-2 but does not get a fever," and ω_4 is interpreted as "Claire is not infected with SARS-CoV-2." Let $R_C: \Omega \to 2^{\Omega}$ denote Claire's recognition correspondence; for any $\omega \in \Omega$, let $R_C(\omega)$ be information sets. Suppose $R_C(\omega_1) = R_C(\omega_2) = \{\omega_1, \omega_2\}$ and $R_C(\omega_3) = R_C(\omega_4) = \emptyset$ denote "Claire recognizes the fever" and "Claire does not recognize the fever," respectively. Here, we interpret $R_C(\omega_1)$ and $R_C(\omega_2)$ as signals of fever, whereas $R_C(\omega_3)$ and $R_C(\omega_4)$ do not act as signals to the five senses.

In the example with SARS-CoV-2, at ω_1 or ω_2, Claire receives an information set $R_C(\omega_1) = R_C(\omega_2) = E_1$. Therefore, Claire receives a signal to the senses ("Claire has fever"), and she can consider her physical condition. Thus, when she has a fever, Claire receives the signal "fever" from nature to the senses; accordingly, she *becomes* aware of her health status. ■

Recognition correspondence might not satisfy Seriality, that is, $R(\omega) = \emptyset$ for some ω. Thus, we interpret it as "an agent that cannot receive any signal to five senses." Considering the COVID-19 example, at ω_3 or ω_4, because $R_C(\omega_3) = R_C(\omega_4) = \emptyset$, Claire cannot receive any signal to the senses. Thus, she may not be aware of her condition. Therefore, if there is no fever, it is more plausible to interpret it as her *not having received* a "fever" signal to the senses than to interpret it as her receiving a "no fever" signal from nature. Thus, Claire would not be aware of her health status, that is, whether she is healthy or not.

Next, let us define the non-standard knowledge operator $K: 2^\Omega \to 2^\Omega$ in $\langle \Omega, R \rangle$. Given any event E, $K(E)$ is defined as follows:

$$\begin{cases} \omega \in K(E) \text{ if } R(\omega) \subseteq E \text{ and } R(\omega) \neq \emptyset; \text{ and} \\ \omega \notin K(E) \text{ otherwise.} \end{cases}$$

At ω, the agent knows E if the given information set ω is a subset of E

and is non-empty. Therefore, to know E, the agent must receive a signal to the senses regarding ω. [7]

Example 1 (Continued). Let $K_C\colon 2^\Omega \to 2^\Omega$ be Claire's redefined knowledge operator based on R. For any event $E \subseteq \Omega$, $K_C(E)$ are interpreted as "Claire knows about event E" and $\neg K_C(E)$ are interpreted as "Claire does not know about event E." Given two events $E_1 = \{\omega_1, \omega_2\}$ and $E_2 = \{\omega_3, \omega_4\}$, E_1 is interpreted as "Claire has a fever" or "Claire has no fever." Now, consider the definition of K_C, $K_C(E_1) = \{\omega_1, \omega_2\}$, $K_C(E_2) = \emptyset$, and $K_C(\Omega) = \{\omega_1, \omega_2\}$. That is, when Claire receives a fever signal, she knows that she has a fever and is aware of her situation. By contrast, at ω_1 and ω_2, she does not know every event. ■

Accordingly, the following theorem holds:

Theorem 1: In a recognition correspondence model $\langle \Omega, R \rangle$ with a non-standard knowledge operator K, K satisfies the following properties:

- (*Generalized Necessitation*) For any $\omega \in \Omega$, $R(\omega) \neq \emptyset$ if and only if $K(\Omega) = \Omega$.
- (*Monotonicity*) $E \subseteq F \implies K(E) \subseteq K(F)$

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[7] Tada (2021, 2023), and Rathke (2024) have similarly defined a knowledge operator. However, their definition is based on the possibility correspondence. Thus, if the information set is empty, the interpretation remains difficult.

Proof. Suppose for any $\omega \in \Omega$, $R(\omega) \neq \emptyset$; then, K is equivalent to the standard knowledge operator K^*. Hence, Generalized Necessitation holds. Next, suppose that there exists $\omega \in \Omega$ satisfying $R(\omega) = \emptyset$, then, $\omega \notin K(E)$ for any $E \subseteq \Omega$. Thus, $\omega \notin K(\Omega)$. Hence, $K(\Omega) \not\supseteq \Omega$.

Next, suppose $E \subseteq F$ given $\omega \in K(E)$. Thus, by the definition of K, $R(\omega) \subseteq E$ and $R(\omega) \neq \emptyset$. This is because $E \subseteq F$ and $R(\omega) \subseteq E \subseteq F$. Hence, $\omega \in K(F)$, that is, $K(E) \subseteq K(F)$. ■

Interestingly, the non-standard knowledge operator K may not satisfy Necessitation. In case an information set that is empty, then Necessitation does not hold, and vice versa. Conversely, Monotonicity holds. That is, our knowledge operator excludes only Necessitation. [8]

5. Decision Making in Recognition Correspondence Models

In this note, we propose a recognition correspondence model that assumes that agents are unable to recognize signals. However, examining the significance this model holds in decision-making contexts is crucial. This section presents decision-making using the recognition correspondence

[8] Fukuda (2024) uses *information correspondence,* another type of novel correspondence. In their model, given some state, an agent might receive multiple information sets. Thus, their belief operator also might drop Necessitation.

model.

Example 2. Let us consider two traders and define the state space as $\Omega = \{\omega_1, \omega_2\}$. Here, ω_1 represents the presence of early signs of a financial crisis, and ω_2 indicates that the market is stable. Trader 1 is a trader who carefully monitors market trends, and her possibility correspondence is given by $P_1(\omega_1) = \{\omega_1\}$, and $P_1(\omega_2) = \{\omega_2\}$. In contrast, Trader 2 is a trader who does not closely follow the news and lacks accurate information about financial market trends. His possibility correspondence is defined as $P_2(\omega_1) = P_2(\omega_2) = \{\omega_1, \omega_2\}$. In the case of ω_1, if the trader sells their stocks, they will earn a profit of 100,000 dollars, but if they buy, they will incur a loss of 200,000 dollars when the financial crisis hits. In the case of ω_2, selling the stocks will yield a profit of 100,000 dollars, while buying will result in a return of 200,000 dollars in the future. Now, let the probability of ω_1 be 0.1 and the probability of ω_2 be 0.9, with ω_1 being the true state. Trader 1, having perceived the signs of the crisis, will choose to sell her stocks. In contrast, Trader 2 underestimates the probability of a financial crisis occurring, with an expected utility for buying of $Eu_1(buy) = 0.1 \times -20 + 0.9 \times 20 = 16$, and for selling, $Eu_1 = (sell) = 10$. Hence, he will consciously choose to buy.

Now, let us consider recognition correspondence instead of possibility correspondence. Trader 1 perceives the signal of the impending crisis, while Trader 2, not paying attention to the market, neither perceives the signal nor focuses on market trends. In the absence of a financial crisis, both traders would likely not be overly concerned about the market's condition. In this scenario, Trader 1's recognition correspondence is $R_1(\omega_1) = \{\omega_1\}$, $R_1(\omega_2) = \emptyset$, while Trader 2's recognition correspondence is $R_2(\omega_1) = R_2(\omega_2) = \emptyset$. If ω_1 is the true state, Trader 1 will sell her stocks, but what about Trader 2? He receives an empty information set, meaning he is not aware of any signals regarding the market trends. Therefore, he would likely act as if the market were stable and proceed with his usual actions.

■

In this example, the key question here is whether Trader 2 is acting "consciously." Under possibility correspondence, he made a conscious decision to buy stocks. In contrast, under recognition correspondence, he would "unconsciously" buy stocks, acting in line with his habitual behavior. What does this mean?

Even when individuals fail to recognize necessary information, they still take some form of action. For instance, people who were unaware of the subprime mortgage crisis continued to make economic decisions, and asymptomatic SARS-CoV-2 carriers still made plans in their daily lives. In such cases, an empty information set might initially seem to lead to random behavior. However, individuals tend to "act as usual." In other words, those who did not recognize the subprime mortgage crisis planned their economic activities assuming the market conditions were stable, and asymptomatic SARS-CoV-2 carriers behaved as if they were not infected.

Notably, this "acting as usual" behavior happens automatically, without consciously recognizing the stability of the market or their not being infected with SARS-CoV-2. In Example 2, under ω_1, $R_2(\omega_1) = \emptyset$ suggests that when faced with an empty information set, Trader 2 may act out of habit rather than conscious decision-making.

This suggests that the absence of signals does not prompt any behavioral change—what could be termed signal absence leads to a lack of modification in behavior. In the absence of signals that trigger the recall of knowledge, agents likely rely on ingrained habits or routines. In cultural or institutional contexts, this might manifest as customary behavior. Therefore, we propose that an empty information set can be interpreted as reflecting habitual or customary actions.

Specifically, even under an empty information set, individuals follow pre-established behavior, and decision-making is still possible without the presence of signals. In everyday life, many actions—such as greetings, table manners, or choosing work attire—are automatically performed out of habit. However, the empty information set becomes especially significant when individuals cannot fully comprehend the situation they are in.

For example, consider that a person dining at a Christian household is unfamiliar with the custom of prayer before meals. The individual may be confused when others pray before eating. In such a case, the person might either sit passively or imitate the prayer gestures of those around them. Thus, behavior derived from an empty information set can be understood as driven more by the person's behavioral tendencies than by conscious decision-making.

In the context of game theory, if one anticipates that the other player has an empty information set, one may base their decision-making on the other player's behavioral tendencies. Therefore, the emptiness of the information set is an important factor in predicting the other player's actions. [9]

6. Unawareness in Recognition Correspondence Models

Dekel, Lipman, and Rustichini (1998) address the Impossibility Theorem of unawareness, noting that we cannot represent non-trivial unawareness in standard state space models. Since their research, attempts to study non-trivial unawareness in the context of standard state-space models have declined, and mainstream research on unawareness has shifted toward the use of the lattice structure model (e.g., Heifetz, Meier, and Schipper 2006, 2013). However, as pointed out by Dekel, Lipman, and Rustichini (1998), the Impossibility Theorem does not hold if its assumptions are relaxed. This is because it becomes possible to represent non-trivial unawareness even within state-space models. Ewerhart (2001), Fukuda (2021), Tada (2023, 2024), and Rathke (2023, 2024) have suggested the possibility of non-trivial unawareness in state-space models. Therefore, in this section, we explore non-trivial unawareness within the framework of the recognition

[9] This section was developed through discussions with Hirokazu Takizawa. The author gratefully acknowledges the valuable comments and insights provided by him.

correspondence model.

Let us consider the Impossibility Theorem of unawareness in a standard possibility correspondence model $\langle \Omega, P \rangle$. Given a standard knowledge operator K^* defined by P, we assume the following three properties of the unawareness operator $U: 2^\Omega \to 2^\Omega$ proposed by Dekel, Lipman, and Rustichini (1998):

Plausibility	$U(E) \subseteq \neg K^*(E) \cap \neg K^* \neg K^*(E)$,
KU Introspection	$K^* U(E) = \emptyset$, and
AU Introspection	$U(E) \subseteq UU(E)$.

Plausibility means that an agent is unaware of an event, then the agent does not know it and she or he does not know that she or he does not know it. KU Introspection means that there is no event such that an agent knows that she or he is unaware of the event. AU Introspection means that if an agent is unaware of an event, then she or he is unaware that she or he is unaware of it. Dekel, Lipman, and Rustichini (1998) have suggested the following theorems:

Theorem 2: According to Dekel, Lipman, and Rustichini (1998), in a standard information structure $\langle \Omega, P \rangle$, if the unawareness operator U satisfies Plausibility, KU Introspection, and AU Introspection, then,

- (*Triviality*) If the knowledge operator K^* satisfies Necessitation, then for any event $E \subseteq \Omega$, $U(E) = \emptyset$; and

- (*Unawareness Leads to Ignorance*) If K^* satisfies Monotonicity, then for all events $E, F \subseteq \Omega$ and $U(E) \subseteq \neg K^*(F)$.

Therefore, when Plausibility, KU Introspection, and AU Introspection hold, Necessitation implies Triviality and Monotonicity implies "Unawareness Leads to Ignorance." Triviality indicates that the agent cannot be aware of anything. "Unawareness Leads to Ignorance" suggests that the agent cannot acquire any knowledge when they are unaware of some events. Hence, both properties (Triviality and Unawareness Leads to Ignorance) are trivial, suggesting that "a non-trivial model of unawareness requires us to abandon both necessitation and monotonicity" (Dekel, Lipman, and Rustichini 1998, p. 166). However, the two properties of Necessitation and Monotonicity must be held by the knowledge operator's definition, as shown in Lemma 1. Therefore, we cannot discuss non-trivial unawareness in the context of a standard possibility correspondence model.

Now, let us consider a recognition correspondence model $\langle \Omega, R \rangle$ and our knowledge operator K defined by R. From Theorem 1, since Necessitation holds if and only if Seriality holds for R whereas Monotonicity always holds, the following corollary holds.

Corollary 1: Assume that in a recognition correspondence model $\langle \Omega, R \rangle$ with the non-standard knowledge operator K, the unawareness operator $U: 2^\Omega \to 2^\Omega$ satisfies Plausibility ($U(E) \subseteq \neg K(E) \cap \neg K \neg K(E)$), KU Introspection ($KU(E) = \emptyset$), and AU Introspection ($U(E) \subseteq UU(E)$). Thus, U

satisfies the following conditions:

- (Generalized Triviality) If $R(\omega) \neq \emptyset$ for any $\omega \in \Omega$, then $U(E) = \emptyset$.
- (Unawareness Leads to Ignorance) For any $E, F \subseteq \Omega$, $U(E) \subseteq \neg K(F)$.

As pointed out by the above corollary, when R does not satisfy Seriality, Triviality does not hold. However, Unawareness Leads to Ignorance always holds even if Seriality does not hold. Then, is Unawareness Leads to Ignorance trivial? Dekel, Lipman, and Rustichini (1998) assert that it is, while we, the authors of this note, argue otherwise.

The recognition correspondence R allows the information set to be empty. Given this information set, the agent cannot know about the relevant event. Here, given $\omega \in \Omega$, suppose $R(\omega) = \emptyset$. Thus, for any E, $\omega \notin K(E)$, that is, $\omega \in \neg K(E)$. Therefore, at ω, if the agent cannot receive relevant information (set), then they cannot know about every event. If $F = \neg K(E)$, then $\omega \in \neg K(F) = \neg K \neg K(E)$. Therefore, at ω, the agent cannot know about ignorance. Therefore, if there exists an event E such that $\omega \in U(E)$, it is consistent. Accordingly, $\omega \in U(E)$ means that, at ω, because the agent cannot receive any relevant signal to the senses, they cannot perceive or understand that they are facing a current issue. Hence, they cannot perceive every event. Accordingly, Unawareness Leads to Ignorance suggests that the agent who cannot perceive the recognition correspondence model $\langle \Omega, R \rangle$ cannot know tautology.

Let $A(E) = \neg U(E)$ be interpreted as "an agent is aware of E." Now, let us consider the COVID-19 example.

Example 1 (Continued). Let $K_C: 2^\Omega \to 2^\Omega$ be Claire's redefined knowledge operator based on R. For any event $E \subseteq \Omega$, $K_C(E)$ are interpreted as "Claire knows about event E," $\neg K_C(E)$ are interpreted as "Claire does not know about event E," $U_C(E)$ is interpreted as "Claire is unaware of event E," and $A_C(E) = \neg U_C(E)$ is interpreted as "Claire is aware of event E."

Suppose for any $E \supseteq E_1$, if the true state is ω_1 or ω_2, then Claire is aware and explicitly knows about the event E, that is, $\omega_1 \in A_C(E)$, $\omega_2 \in A_C(E)$, $\omega_1 \in K_C(E)$, and $\omega_2 \in K_C(E)$; furthermore, if the true state is ω_3 or ω_4, then Claire is unaware of *any* event and does not know about *any* event, that is, $\omega_3 \in U_C(E)$, $\omega_4 \in U_C(E)$, $\omega_3 \notin K_C(E)$ and $\omega_4 \notin K_C(E)$ for any $E \subseteq \Omega$. Thus, the unawareness operator does not satisfy Triviality and the redefined knowledge operator K satisfies Monotonicity, that is, for any $E, F \subseteq \Omega$ with $E \subseteq F$, $K_C(E) \subseteq K_C(F)$. However, this does not satisfy Necessitation because $K_C(\Omega) = \{\omega_1, \omega_2\}$.

If the unawareness operator based on the redefined knowledge operator satisfies Plausibility, KU Introspection, and AU Introspection, then Unawareness Leads to Ignorance holds according to Impossibility Theorem (Dekel, Lipman, and Rustichini, 1998); accordingly, $U(E) \subseteq \neg K(F)$ for any E and F. For example, suppose $U_C(E_2) = E_2$, then $U_C(E_2) \subseteq \neg K_C(E)$ for any $E \subseteq \Omega$ because $\omega_3 \notin K_C(E)$ and $\omega_4 \notin K_C(E)$.

At ω_3 or ω_4, because Claire's senses do not receive a fever signal, either she does not know that she has fever or she does not have a fever; also, she lacks knowledge regarding the current issue related to COVID-19. However, given ω_1 or ω_2 (i.e., her senses receive a fever signal), she becomes aware of the current issue. Therefore, when Claire has an asymptomatic SARS-CoV-2 infection, she cannot gain knowledge about COVID-19. This can be interpreted as forgetting the knowledge about COVID-19 unless there is a fever signal. ■

7. Discussion and Conclusion

In this note, we aimed to explain the recognition correspondence model and characterize the properties of knowledge and unawareness.

7.1 Explicit and Implicit Knowledge and Awareness

In this note, an information set $R(\omega)$ is interpreted as a signal to the senses that recalls the agent's knowledge and beliefs. Therefore, a non-empty information set implies that an agent is aware of some event. Conversely, $R(\omega) = \emptyset$ implies that an agent is unaware of any event, because at ω, they do not receive any signal to their senses. Hence, we can define the awareness and unawareness operators based on $R(\omega)$ as follows:

$$\begin{cases} \omega \in A(E) \text{ if } R(\omega) \subseteq E \text{ and } R(\omega) \neq \emptyset; \text{ and} \\ \omega \in U(E) \text{ otherwise.} \end{cases}$$

The following remark then clearly holds.

Remark 1: For any $\omega \in \Omega$, $P(\omega) = R(\omega)$ implies $K(E) = K^*(E) \cap A(E)$.

Remark 1 suggests that our novel knowledge operator is the explicit knowledge operator. Fagin and Halpern (1988) distinguished between *implicit and explicit knowledge operators*. An agent explicitly knows about an event if and only if they implicitly know about and are aware of it. If we interpret a standard knowledge operator $K^*(E)$ as an implicit knowledge operator, our novel knowledge operator is equivalent to an explicit knowledge operator.

Halpern and Rêgo (2013) discuss the relationships among implicit knowledge, explicit knowledge, and unawareness. They suggest that Dekel, Lipman, and Rustichini's (1998) theorem does not distinguish between explicit and implicit knowledge. Furthermore, Fagin and Halpern's (1988) awareness structures show that the implicit knowledge operator satisfies Necessitation and Monotonicity, but the unawareness operator based on the knowledge operator does not satisfy Plausibility and KU Introspection; however, the unawareness operator based on the explicit knowledge operator satisfies Plausibility and KU Introspection. The explicit knowledge operator *may not* satisfy Necessitation and Monotonicity. [10]

This feature is also inherent in our model. First, our novel knowledge operator might not satisfy Necessitation even if Monotonicity holds. Hence, even if we suppose that our knowledge operator satisfies Plausibility and KU Introspection, non-trivial unawareness can be expressed. Conversely,

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[10] Recently, Belardinelli and Schipper (2023) discussed implicit knowledge in unawareness structures.

given non-trivial unawareness, a standard knowledge operator must not satisfy Plausibility and KU Introspection. Let us consider the COVID-19 example. We assumed $K^*(E_1) = \Omega$, $K^*(E_2) = E_2$, $K^*(\Omega) = \Omega$, and $U(E_2) = E_2$. Thus, $\neg K^*(E_2) = E_1$, $K^*\neg K^*(E_2) = E_1$, and $\neg K^*\neg K^*(E_2) = E_2$. Let us suppose Plausibility; accordingly, $U(E_2) \subseteq \neg K^*(E_2) \cap \neg K^*\neg K^*(E_2) = E_1 \cap E_2 = \emptyset$. However, because we suppose $U(E_2) = E_2$, it is a contradiction. Therefore, the standard knowledge operator does not satisfy Plausibility. Additionally, because $K^*U(E_2) = K^*(E_2) = E_2$, KU Introspection does not hold. Hence, the following corollary holds:

Corollary 2: In a standard information structure $\langle \Omega, P \rangle$, given are the standard knowledge operator K^* and unawareness operator U. Suppose there exists some event E such that $U(E) \neq \emptyset$, then the two operators K^* and U do not satisfy Plausibility and KU Introspection, that is, there exists E such that $U(E) \nsubseteq \neg K^*(E) \cap \neg K^*\neg K^*(E)$ and $K^*U(E) \neq \emptyset$.

This corollary captures the characteristics noted by Halpern and Rêgo (2013). They contend that the Triviality Theorem proposed by Dekel, Lipman, and Rustichini (1998) stems from a failure to differentiate between implicit and explicit knowledge. Therefore, if we interpret the standard knowledge operator as the implicit knowledge operator, then Plausibility and KU Introspection must not hold in non-trivial unawareness.

7.2 Small World Interpretation

Certain objections may be made to the interpretation involving empty information sets and novel knowledge operators. Assume that Claire is asymptomatically infected with SARS-CoV-2, that is, the true state is ω_3 and $R_C(\omega_3) = \emptyset$ in Example 1. Our interpretation suggests that she cannot receive any knowledge about COVID-19. However, an alternative interpretation may suggest that she cannot recognize knowledge about COVID-19 or anything else in the world; for example, her breakfast she had or the fact that she is about to go shopping. The alternative interpretation is inappropriate for considering empty information sets, and if unawareness is to be discussed, it may be more appropriate to use state-space models with lattice structures (e.g., Heifetz, Meier, and Schipper 2006, 2013). However, the alternative interpretation can be seen as interpreting the state space as a "large world." In this interpretation, each state in the state space is considered to describe all events in the world. However, in a specific decision-making context, it is unnecessary to account for factors that are irrelevant to the decision at hand. For example, when predicting the outcome of a dice roll, we typically do not consider the possibility of a meteor striking the earth. In this context, our interpretation can be understood as treating the state space as a "small world," which is more appropriate. [11]

[11] The terms large world and small world in decision theory, as introduced by Savage (1954), refer to the completeness of the decision-maker's knowledge about the decision problem. In a small world, the decision-maker has well-defined outcomes and can assign probabilities to them. In a large world, uncertainty prevails, and the decision-maker may not even know all possible outcomes, let alone their probabilities. The small world assumption allows for the application of standard decision-making models, while the

7.3 Related Literature

Most economic and game theories suppose common knowledge. Common knowledge refers to a situation where everyone knows a certain event, everyone knows that everyone knows it, everyone knows that everyone knows that everyone knows it, and so on, ad infinitum. This concept was first introduced by Lewis (1969) and was mathematically formalized by Aumann (1976). In standard game theory, the assumption of common knowledge is frequently employed as a tool for analysis.

Studies on unawareness are among the studies on common knowledge criticism. There are three approaches to the studies on unawareness. The first is the approach of non-normal modal logics, as seen Fagin and Halpern (1988), Wansing (1990), Halpern (2001), Halpern and Rêgo (2009, 2013), Sillari (2008a, 2008b), Schipper (2015), and Belardinelli and Schipper (2023). The second approach comprises the standard state-space models, as demonstrated in Geanakoplos (2021), Samet (1990), Shin (1993), Modica and Rustichini (1994, 1999), Dekel, Lipman, and Rustichini (1998), and Chen, Ely, and Luo (2012). Finally, the third approach involves state-space models with lattice structures, as seen in Heifetz, Meier, and Schipper (2006, 2013), Li (2009), Schipper (2013), Galanis (2013), Fukuda (2021), and

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large world requires more sophisticated approaches that account for ambiguity and uncertainty. See Savage (1954) and Binmore (2009) for detailed discussions on this distinction.

Belardinelli and Schipper (2023). For a long time, given the Impossibility Theorem, few studies were conducted on non-trivial unawareness in standard state-space models. However, as pointed out Halpern and Rêgo (2013), their result is attributed to the failure to distinguish between explicit and implicit knowledge. Recently, several studies have succeeded in representing non-trivial unawareness using standard state-space models (e.g., Ewerhart 2001; Fukuda 2021, 2024; Tada 2023, 2024; Rathke 2023, 2024; Sasaki and Tada 2024).

This note excludes Necessitation from the knowledge operator's properties. Previous studies have analyzed the characteristics of a knowledge operator excluding Necessitation from non-normal modal logics and standard state-space approaches. The former approach (non-normal modal logics) divides knowledge operators into explicit and implicit knowledge operators, suggesting that an implicit knowledge operator satisfies Necessitation, whereas an explicit knowledge operator may not (see Halpern and Rêgo 2013). The latter approach (a non-standard state-space approach) axiomatically excludes Necessitation and characterizes non-trivial unawareness (see Tada 2024; and Ratheke 2023). Considering the former perspective, this note defines an explicit knowledge operator based on a recognition correspondence; however, from the latter perspective, it provides an axiomatic approach in standard state-space models founded on a correspondence approach.

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The authors have no competing interests to declare.

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